Comment on “Formation of Phase Space Holes and Clumps”

I. Y. Dodin

Phys. Rev. Lett. **113**, 179501 — Published 21 October 2014
DOI: 10.1103/PhysRevLett.113.179501

Comment on "Formation of Phase Space Holes and Clumps"

I. Y. Dodin
Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543, USA

PACS numbers: 52.35.-g, 52.25.Dg

Ref. [1] revisits formation of phase space holes and clumps (HC) in a one-dimensional plasma model. The plasma contains a dilute collisionless electron beam with a plateau centered at the plasma resonance. The linear dispersion relation (DR) is studied for waves near that resonance. The plateau instability is reported, and HC formation is attributed to dissipative destabilization of negative-energy modes at the plateau edges. But this analysis is not quite complete. A more detailed calculation shows, for weak dissipation, that: (i) the plateau instability is the standard bump-on-tail (BTI) instability, whose rate continues analytically to that of the plateau-less BTI; and (ii) destabilization of edge modes is not universal to HC formation.

Suppose some bulk-plasma dielectric $\epsilon_p(\omega,k)$. Adding a beam with velocity distribution $F_0(v)$ $[\int_{-\infty}^{+\infty} F_0(v)\,dv = 1]$ produces the dielectric $\epsilon(\omega,k) = \epsilon_p(\omega,k) - (\omega_b^2/k^2)\,\mathsf{L}\int[F_0'(v)/(v-\omega/k)]\,dv$; $\omega_b^2 \doteq 4\pi n_b e^2/m$, $\doteq$ denotes definitions, and L stands for a Landau contour. Suppose $F_0 = \bar{F} + \delta F_0$, where $\bar{F}$ is a distribution with scale $u_0 \gg \Delta v$, and $\delta F_0(v) \doteq [\bar{F}(v_c) - \bar{F}(v)]\,H(\Delta v - |v - v_c|)$ describes the plateau centered at $v = v_c$ [$H(v)$ is the unit step function]; i.e., the beam is flat $[F_0(v) = \bar{F}(v_c)]$ at $|v - v_c| < \Delta v$. Then $\epsilon(\omega,k) \approx \bar{\epsilon}(\omega,k) - i\pi\beta + \delta\epsilon(\omega,k)$, assuming $|\omega/k - v_c| \ll u_0$ (and $k > 0$). Here $\beta \doteq \omega_b^2 \bar{F}'(v_c)/k^2$, $\bar{\epsilon}(\omega,k) \doteq \epsilon_p(\omega,k) - (\omega_b^2/k^2)\,\mathsf{P}\int[\bar{F}'(v)/(v-\omega/k)]\,dv$, P denotes the principal-value integral through $v = \omega/k$. Also, $\delta\epsilon \doteq 2\beta w/(w^2-1) + \beta J$ is due to δF_0; $w \doteq (\omega - kv_c)/(k\,\Delta v)$, and $J \doteq \mathsf{L}\int_{-1}^{+1}(z-w)^{-1}\,dz = \ln(1-w) - \ln(-1-w) + i\pi H(1-|w_r|)\,(1 - \operatorname{sgn} w_i)$, $w_r \doteq \operatorname{Re} w$, and $w_i \doteq \operatorname{Im} w$. Assume the branch cut of $\ln z$ to be at real $z \in (-\infty, 0)$. Then, $\ln(-1-w) = -i\pi \operatorname{sgn} w_i + \ln(1+w)$, so $\epsilon(\omega,k) \approx \bar{\epsilon}(\omega,k) + \beta\,[g(w) + \sigma(w)]$, $g(w) \doteq \ln(1-w) - \ln(1+w) + 2\beta w/(w^2-1)$, and $\sigma(w) \doteq -2\pi i\,H(-w_i)\,H(|w_r| - 1)$. Expanding $\bar{\epsilon}(\omega,k)$ in w gives $\epsilon(\omega,k) \approx \beta\,[\kappa + \mu w + g(w) + \sigma(w)]$, and the DR is $\epsilon = 0$, where $\kappa \doteq \beta^{-1}\bar{\epsilon}(kv_c,k)$, $\mu \doteq (k\,\Delta v/\beta)\,[\partial_\omega \bar{\epsilon}(\omega,k)]_{\omega = kv_c}$.

In contrast with Ref. [1], this DR contains an additional term, $\sigma(w)$. (Ref. [1] is also limited to $\kappa = 0$ and specific $\bar{\epsilon}$.) That is a far-reaching issue. On one hand, nonzero σ ensures that $\epsilon(\omega,k)$ is analytic for small w_i at $w_r \neq \pm 1$, as it should be. On the other hand, nonzero σ renders $\epsilon(\omega,k)$ discontinuous for all $w_i \leq 0$ at $w_r = \pm 1$. This is also understood. Indeed, for $\operatorname{Im}\omega < 0$, $\epsilon(\omega,k)$ was constructed by analytic continuation. That requires continuation of $F_0(v)$ to complex v; but $F_0(v)$ is nonanalytic already at real v, so its continuation is ambiguous. Predictions based on the abrupt-plateau model for $\operatorname{Im}\omega < 0$ thereby must be taken with caution, except at small $|\operatorname{Im}\omega|$ *and* far from $\operatorname{Re}\omega = k(v_c \pm \Delta v)$. This fact is also missed in Ref. [1], but, if it is taken into account, one discovers the following. For a narrow plateau ($\mu \ll 1$), the nonzero ws found in Ref. [1] continue analytically into $w \approx (-\kappa \mp i\pi)/\mu \gg 1$. These are recast as $\bar{\epsilon}(\omega,k) \pm i\pi\beta \approx 0$. But the one with $w_i < 0$ is unphysical, as was explained. Hence we have just $\bar{\epsilon}(\omega,k) - i\pi\beta \approx 0$, and that is the plateau-less BTI dispersion. This is not a coincidence: in v space, the real axis is then far from $v = \omega/k$; hence the fine structure of the real-velocity distribution, such as a narrow plateau, cannot matter. In contrast, at $\mu \gtrsim 1$, the distance $\operatorname{Im}\omega/k$ from the pole to the axis is $\lesssim \Delta v$; then the BTI can be suppressed.

In other words, a detailed calculation shows that there are no plateau-specific linear collisionless instabilities (unless the BTI is restricted, unjustly, to smooth F_0). Also, dissipative destabilization of edge modes is not universal to HC formation. Suppose zero dissipation. As phase space volume is conserved, field oscillations cannot affect the beam distribution F *inside* the plateau, even if $\operatorname{Re}\omega = kv_c$. Perturbations of F are thus confined to the edges. That is where wave breaking occurs then, so F is impregnated with holes at $v \gtrsim v_c + \Delta v$ and clumps at $v \lesssim v_c - \Delta v$, and they hence act as independent waves. Edge modes are not required for this (away from the BTI threshold). Only a friction force is needed, later, to drag HC away from the plateau.

The work was supported by the U.S. DOE under the Contract No. DE-AC02-09CH11466.

[1] M. K. Lilley and R. M. Nyqvist, Phys. Rev. Lett. **112**, 155002 (2014).