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Supersymmetry from Typicality: TeV-Scale Gauginos and PeV-Scale Squarks and Sleptons

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Supersymmetry from Typicality

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We argue that under a set of simple assumptions the multiverse leads to low energy supersymmetry with the spectrum often called spread or mini-split supersymmetry: the gauginos are in the TeV region with the other superpartners two or three orders of magnitude heavier. We present a particularly simple realization of supersymmetric grand unified theory using this idea.

INTRODUCTION

Supersymmetry is an elegant extension of spacetime symmetry, which arises naturally in string theory—the leading candidate for the fundamental theory of quantum gravity. A striking property of supersymmetry is its high capability to control quantum corrections; in particular, it can protect the mass of a scalar field, such as the Higgs field, which has sizable non-derivative interactions. This property was used to argue that the supersymmetric partners of the standard model particles are expected at the weak scale: the well-known naturalness argument [1]. On the other hand, the Large Hadron Collider (LHC) experiment has not seen any sign of superpartners so far, which is beginning to threaten this argument [2].

The plethora of string theory vacua [3] suggests that the naive naturalness argument should be modified. Because of the eternal nature of inflation [4], all these vacua are expected to be physically populated, leading to the picture of the multiverse (or quantum many-universes [5]). In this picture, our universe is one of the many universes in which low-energy physical laws take different forms, and the naturalness argument is replaced with the *typicality* argument [6]: we are typical observers among all the observers in the multiverse. Specifically, the probability that physical parameters x_i ($i = 1, 2, \dots$) are observed between x_i and $x_i + dx_i$ is given by $P(\{x_i\}) d\{x_i\}$ with

$$P(\{x_i\}) \propto \int d\{y_j\} f(\{x_i\}, \{y_j\}) n(\{x_i\}, \{y_j\}), \quad (1)$$

where y_j ($j = 1, 2, \dots$) are parameters other than x_i which vary independently of x_i 's; f is the a priori distribution function determined by the statistics of the landscape of vacua and their population dynamics, while n is the anthropic weighting factor representing the probability of finding observers for a given $\{x_i, y_j\}$. As is well known, this potentially allows us to understand the smallness of the cosmological constant (or dark energy) observed in our universe [7, 8].

In this letter, we consider what Eq. (1) implies for the masses of superpartners. (For earlier studies, see e.g. [9, 10].) We argue that under a set of simple assumptions the multiverse leads to the spectrum often called

spread or mini-split supersymmetry, which has attracted renewed interest recently [11–15]: the gauginos are in the TeV region with the other superpartners two or three orders of magnitude heavier. (For earlier work, see [16–19].) We find it encouraging that the typicality (or refined naturalness) argument, suggested by the fundamental theory, can lead—under rather simple assumptions—to a superpartner spectrum that does not have tension with the LHC results so far while allowing the possibility for future discovery. Note that this spectrum preserves the successful prediction of supersymmetric gauge coupling unification, and yet does not suffer from (or at least highly ameliorates) problems associated with low energy supersymmetry, such as the supersymmetric flavor and CP problems and the cosmological gravitino problem.

Throughout the letter, we assume that the electroweak scale is anthropically selected in the multiverse. While the physical effects that are responsible for this selection are not fully understood, we at least know that changing it by a factor of a few leads to drastic changes of the universe [20]. We therefore take

$$n(\{x_i\}, \{y_j\}) \propto \delta(v - v_{\text{obs}}), \quad (2)$$

in Eq. (1), and see what distribution of the superpartner mass scale $P(\tilde{m} \in \{x_i\})$ is obtained. Here, v is the vacuum expectation value (VEV) of the standard model Higgs field calculated in terms of parameters $\{x_i, y_j\}$, and $v_{\text{obs}} \simeq 174$ GeV is the observed value (in units of some mass scale that is fixed in the analysis). In our analysis, we vary essentially only parameters directly relevant for our question: the overall supersymmetry breaking mass scale \tilde{m} , the supersymmetric Higgs mass μ , and the VEV of the superpotential W_0 , which is needed to cancel the cosmological constant. We do not expect, however, that our basic conclusion is sensitive to this restriction.

A key element that characterizes our scenario is the assumption that the supersymmetric Higgs mass term (the μ term)—which is the only relevant supersymmetric operator in the R -parity conserving minimal supersymmetric standard model (MSSM)—is not protected by any artificial symmetry. This implies that the theoretically natural size of μ is of $O(M_*)$, where M_* is the cutoff scale of the MSSM, and μ takes a value close to \tilde{m} only because of the anthropic selection of the weak scale. This

provides a simple environmental solution to the μ problem. Together with the assumptions that supersymmetry is broken dynamically and that the supersymmetry breaking field is not a singlet, we find that the claimed spectrum is obtained after applying all the selection effects, especially that associated with the abundance of dark matter. This basic argument will be presented in the next section.

A particularly interesting realization of our scenario is obtained in the context of the minimal supersymmetric grand unified theory (GUT) [21]. In this theory, the μ term much smaller than the unified scale is obtained only as a result of a fine cancellation between different contributions, so that the setup relevant for our scenario is realized automatically. Despite the minimality of the model, there is no problem of doublet-triplet splitting or unacceptably fast dimension-5 proton decay. The precision of gauge coupling unification is also improved compared with conventional weak scale supersymmetry. This, therefore, provides one of the most attractive realizations of supersymmetric GUT.

At the end of this letter, we also comment on the case in which the supersymmetry breaking field is a singlet. In this case, the superpartner masses can be at a high energy scale with the VEVs of the two Higgs doublets being almost equal. This allows for realizing the high scale supersymmetry scenario discussed in Ref. [22].

BASIC ARGUMENT

We postulate that in the landscape of possible theories in the multiverse, probabilities relevant for observers are dominated by the branch having the following properties:

- (i) Low energy theory below some high scale M_* is the MSSM with R -parity conservation (possibly with the QCD axion supermultiplet to solve the strong CP problem). We are agnostic about the precise nature of the scale M_* here, except that we assume it is not many orders of magnitude smaller than the reduced Planck scale M_{Pl} . Later we will consider the case in which M_* is identified as the unification scale of supersymmetric GUT.
- (ii) Supersymmetry is broken dynamically as a result of dimensional transmutation associated with some hidden sector gauge group so that $\tilde{m} \propto e^{-8\pi^2/cg^2}$ [23], where g and c are the hidden sector gauge coupling and an $O(1)$ coefficient, respectively.
- (iii) The superfield X responsible for supersymmetry breaking, $\langle X|_{\theta^2} \rangle = F_X$, is not a singlet. This prohibits the terms $[XH_u H_d]_{\theta^2}$ and $[XW^\alpha W_\alpha]_{\theta^2}$ to appear in the Lagrangian at tree level, where $H_{u,d}$

and W_α represent the two Higgs doublet and gauge strength superfields of the MSSM, respectively.

In addition to these somewhat “standard” assumptions, we also postulate that the branch in the landscape dominating the probabilities has the property:

- (iv) There is no “artificial” symmetry below M_* controlling the size of operators in the MSSM (except for approximate, and perhaps accidental, flavor symmetries associated with the smallness of the Yukawa couplings). In particular, there is no approximate global symmetry that dictates the values of μ and W_0 , such as the “Peccei-Quinn” symmetry $H_{u,d} \rightarrow e^{i\alpha} H_{u,d}$, CP symmetry, or a continuous R symmetry.

As we will see, this implies that the μ problem is solved by the anthropic selection associated with electroweak symmetry breaking. In the fundamental picture in the landscape, this corresponds to postulating that having a theoretical mechanism of suppressing the μ term “costs” more than the fine-tuning needed to make $|\mu|$ small to obtain acceptable electroweak symmetry breaking.

The postulate (iii) above implies that at the leading order, supersymmetry breaking masses in the MSSM sector arise from operators of the form $[X^\dagger X \Phi^\dagger \Phi / \Lambda^2]_{\theta^4}$ and $[X^\dagger X H_u H_d / \Lambda^2]_{\theta^4}$, where Φ represents the MSSM matter and Higgs fields, as well as from tree-level supergravity effects. This yields

$$m_{\tilde{f}, H_u, H_d}^2 = c_{\tilde{f}, H_u, H_d} \tilde{m}^2, \quad b = c_b \tilde{m}^2 - \mu m_{3/2}, \quad (3)$$

where $\tilde{m} \equiv |F_X / \Lambda|$, $m_{3/2} = F_X / \sqrt{3} M_{\text{Pl}}$ is the gravitino mass, and $m_{\tilde{f}, H_u, H_d}^2$ represent the (non-holomorphic) supersymmetry breaking squared masses for the sfermion and Higgs fields while b is the holomorphic supersymmetry breaking Higgs mass squared. $c_{\tilde{f}, H_u, H_d}, b$ are coefficients, which we take to scan for values of order unity in the landscape, and Λ is the mediation scale of supersymmetry breaking, which we take to be roughly of the order of—e.g. within an order of magnitude of—the reduced Planck scale: $\Lambda \sim M_{\text{Pl}}$.

We now discuss the probability distribution of \tilde{m} , μ , and W_0 in this setup. With the assumption in (ii), it is reasonable to expect that the a priori distribution of \tilde{m} is approximately flat in logarithm: $f d\tilde{m} \propto d \ln \tilde{m}$. On the other hand, the assumption in (iv) implies that the distribution of μ is given by $f d\mu \propto d \text{Re} \mu d \text{Im} \mu \propto |\mu| d|\mu| d \arg(\mu)$ for $|\mu| \lesssim M_*$, and similarly for W_0 . We thus take $\{x_i\} = \{\tilde{m}\}$ and $\{y_j\} = \{|\mu|, |W_0|, \arg(\mu), \arg(W_0), c_{\tilde{f}, H_u, H_d}, c_b\}$ with

$$f \propto \frac{1}{\tilde{m}} |\mu| |W_0|, \quad (4)$$

and study what values of these parameters are selected by anthropic conditions. In our analysis, we ignore possible

variations of all the other parameters of the theory, but we do not expect that our basic conclusion is overturned when the full variations are performed with appropriate anthropic conditions. This approach is analogous to that adopted in the original argument for the cosmological constant in Ref. [7].

Let us first isolate the selection effects from the weak scale and the cosmological constant by writing

$$n \approx \delta(v - v_{\text{obs}}) \theta(\rho_{\Lambda, \text{max}} - |\rho_{\Lambda}|) \hat{n}(\tilde{m}, \mu), \quad (5)$$

where v and ρ_{Λ} are the values of the Higgs VEV and the vacuum energy density calculated in terms of \tilde{m} , μ , and W_0 ; $\rho_{\Lambda, \text{max}} = |\gamma \rho_{\Lambda, \text{obs}}|$ is the anthropic upper bound on the value of ρ_{Λ} , which we have taken symmetric around $\rho_{\Lambda} = 0$ for simplicity, and γ is a constant not too far from order unity. We expect that the residual anthropic weighting factor, \hat{n} , does not depend strongly on the values of W_0 or $c_{\tilde{f}, H_u, H_d, b}$ in the relevant parameter region, which we assume to be the case.

By integrating over W_0 in Eq. (1), using the expression $\rho_{\Lambda} = |F_X|^2 - 3|W_0|^2/M_{\text{Pl}}^2$ with $|F_X| = \tilde{m}\Lambda$, we obtain

$$P(\tilde{m}) \propto \int d|\mu| d\arg(\mu) d\{c_i\} \frac{1}{\tilde{m}} |\mu| \delta(v - v_{\text{obs}}) \hat{n}(\tilde{m}, \mu), \quad (6)$$

where $\{c_i\} \equiv c_{\tilde{f}, H_u, H_d, b}$, and v depends on \tilde{m} , μ , and $\{c_i\}$. An important point is that the integration of W_0 does not provide an extra probability bias for \tilde{m} , $|\mu|$, $\arg(\mu)$, or $\{c_i\}$ [10]—the effective a priori distribution function for these parameters is still given by $f_{\text{eff}} \propto |\mu|/\tilde{m}$. The value of W_0 is selected to be $|W_0| = |F_X| M_{\text{Pl}}/\sqrt{3}$ with the phase, $\arg(W_0)$, unconstrained. This leads to the gravitino mass roughly comparable to the sfermion masses:

$$m_{3/2} \sim \tilde{m}. \quad (7)$$

In fact, we may naturally expect that $\Lambda \lesssim M_{\text{Pl}}$, so that $m_{3/2}$ is comparable to or somewhat (e.g. up to an order of magnitude) smaller than \tilde{m} .

The selection from electroweak symmetry breaking acts on the mass-squared matrix of the doublet Higgs bosons

$$\mathcal{M}_H^2 = \begin{pmatrix} |\mu|^2 + m_{H_u}^2 & b \\ b^* & |\mu|^2 + m_{H_d}^2 \end{pmatrix}, \quad (8)$$

where $m_{H_{u,d}}^2 \approx O(\tilde{m}^2)$ and $|b| \approx O(\max\{\tilde{m}^2, |\mu|\tilde{m}\})$. It requires the smallest eigenvalue of \mathcal{M}_H^2 to be $\approx -v_{\text{obs}}^2$. For $\tilde{m} \ll v_{\text{obs}}$, it is not possible to satisfy this requirement. For $\tilde{m} \gtrsim v_{\text{obs}}$, on the other hand, the requirement can be met for some values of $\{c_i\}$ if $|\mu| \lesssim \tilde{m}$, while a value of $|\mu|$ much larger than \tilde{m} makes the eigenvalues of \mathcal{M}_H^2 both positive regardless of $\{c_i\}$. This therefore selects the value of $|\mu|$ to be

$$|\mu| \approx \tilde{m}, \quad (9)$$

since the probability of having $|\mu| \ll \tilde{m}$ is suppressed by the fact that $f_{\text{eff}} \propto |\mu|$. More explicitly, for a fixed \tilde{m} , the probability distribution for $|\mu|$ is given roughly by $P(|\mu|; \tilde{m}) d|\mu| \propto |\mu| \theta(\tilde{m} - |\mu|) d|\mu|$. We thus find that by integrating over $|\mu|$, $\arg(\mu)$, and $\{c_i\}$ in Eq. (6), we obtain

$$P(\tilde{m}) d\tilde{m} \propto \begin{cases} 0 & \text{for } \tilde{m} \lesssim v_{\text{obs}}, \\ n_{\text{eff}}(\tilde{m}) d \ln \tilde{m} & \text{for } \tilde{m} \gtrsim v_{\text{obs}}, \end{cases} \quad (10)$$

where $n_{\text{eff}}(\tilde{m}) \equiv \hat{n}(\tilde{m}, \mu = \tilde{m})$, and we have assumed that the anthropic weighting factor \hat{n} does not depend strongly on the precise value of μ . Interestingly, for $\tilde{m} \gtrsim v_{\text{obs}}$, the preference to smaller values of \tilde{m} by electroweak fine-tuning, $P \propto v_{\text{obs}}^2/\tilde{m}^2$, is exactly canceled by the a priori distribution of μ , which through Eq. (9) prefers larger values of \tilde{m} , i.e. $P \propto |\mu| d|\mu| \sim \tilde{m}^2 d \ln \tilde{m}$. There is, therefore, no net preference for the scale of superpartner masses before a further anthropic selection, $n_{\text{eff}}(\tilde{m})$, is applied.

What function should we consider for $n_{\text{eff}}(\tilde{m})$? Note that with Eqs. (7) and (9), the pattern of the spectrum of the superpartners and the MSSM Higgs bosons (H^0 , H^\pm , and A^0) is fixed in terms of the single mass parameter \tilde{m} : in order of decreasing masses

$$M_{\tilde{f}} \approx M_{\tilde{h}} \approx \tilde{m}, \quad M_{H^{0,\pm}, A^0} \approx \tilde{m}, \quad (11)$$

where \tilde{h} represents the Higgsinos,

$$M_{\tilde{G}} = \epsilon \tilde{m}; \quad \epsilon \approx O(0.1 - 1), \quad (12)$$

where \tilde{G} is the gravitino, and the lightest set of superpartners are the gauginos with masses

$$M_a = \frac{b_a g_a^2}{16\pi^2} m_{3/2} + \frac{d_a g_a^2}{16\pi^2} L, \quad (13)$$

($a = 1, 2, 3$), which are generated by anomaly mediation [16, 24] (the first term) and Higgsino-Higgs loops (the second term). Here, g_a are the standard model gauge couplings (in the SU(5) normalization for U(1) hypercharge), $(b_1, b_2, b_3) = (33/5, 1, -3)$, $(d_1, d_2, d_3) = (3/5, 1, 0)$, and

$$L \approx O(\tilde{m} \sin(2\beta)), \quad (14)$$

where $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$. Our question is: what environmental effects do we expect when we vary \tilde{m} keeping the relations in Eqs. (7) and (9)?

In general, there could be many subtle effects associated with a variation of \tilde{m} . For example, for fixed high energy gauge and Yukawa couplings, varying \tilde{m} leads to changes of low energy parameters such as the QCD scale and the quark and lepton masses. These changes, however, may be compensated by the corresponding variations of the high energy parameters, and their full analysis will require detailed knowledge of the statistical properties of the landscape beyond the scope of this letter.

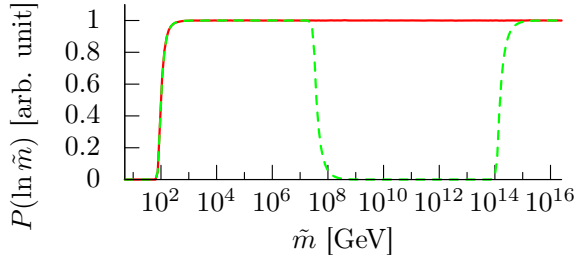


FIG. 1. A schematic depiction for the probability distribution of \tilde{m} . The red solid (green dashed) line represents the distribution before (after) the dark matter condition, $n_{\text{eff}}(\tilde{m})$, is imposed. Here, we have assumed $T_R = 10^9$ GeV and that the anthropic upper bound of the dark matter abundance is at $\Omega_{\text{DM}} h^2 = 100$.

Below, we will focus on what is arguably the dominant environmental effect of varying \tilde{m} : the change of the relic abundance of the lightest supersymmetric particle (LSP) contributing to dark matter of the universe.

As discussed in Ref. [11], this effect leads to a large “forbidden window” for the value of \tilde{m} , in which the LSP relic abundance far exceeds the observed dark matter abundance and $n_{\text{eff}}(\tilde{m}) \approx 0$. While the precise nature of the anthropic upper bound on the dark matter abundance is not well understood, we expect that there is some upper bound. This upper bound may not be sharp or close to the observed value (for discussions on possible upper bounds, see Ref. [25]), but it still excludes a large region of \tilde{m} between some values \tilde{m}_1 and \tilde{m}_2 . For the spectrum in Eqs. (11 – 13), the LSP is the wino in most of the parameter space, whose relic abundance has both thermal and non-thermal contributions; see Ref. [14] for a detailed analysis. (If ϵ in Eq. (12) is small, the second term in Eq. (13) may dominate and the bino can be the LSP; the allowed dark matter window in this case is small [14].) The probability distribution for the superpartner mass scale \tilde{m} is then given by Eq. (10) as

$$P(\tilde{m}) d\tilde{m} \approx \begin{cases} \mathcal{C} d \ln \tilde{m} & \text{for } v_{\text{obs}} \lesssim \tilde{m} \lesssim \tilde{m}_1, \tilde{m} \gtrsim \tilde{m}_2, \\ 0 & \text{for other values of } \tilde{m}, \end{cases} \quad (15)$$

where \mathcal{C} is the normalization constant. A reasonable (though highly uncertain) estimate for \tilde{m}_1 is given by

$$\tilde{m}_1 \approx O(10^6 - 10^8 \text{ GeV}), \quad (16)$$

and \tilde{m}_2 is given by the value of \tilde{m} in which the LSP becomes sufficiently heavier than the reheating temperature after inflation T_R , which we assume to take some high scale value, e.g. $T_R \gtrsim 10^9$ GeV (possibly selected by the requirement of baryogenesis). The resulting distribution is depicted schematically in Fig. 1.

With Eq. (15), it is easy to imagine that \tilde{m} is selected randomly in the logarithmic scale in the region between $v_{\text{obs}} \approx O(100 \text{ GeV})$ and \tilde{m}_1 . In particular, this can lead

to the spread or mini-split type spectrum in Eqs. (11 – 13) with

$$\tilde{m} \sim m_{3/2} \approx O(10 - 1000 \text{ TeV}), \quad (17)$$

as a typical spectrum one observes in the multiverse. The phenomenology of this spectrum is discussed in detail in Ref. [14] and references therein. In particular, the observed Higgs boson mass of $m_{h^0} \simeq 126$ GeV can be easily accommodated, and dark matter can be composed of a mixture of the wino and the QCD axion (and possibly other components as well) with some unknown ratio, which depends on the statistical distribution of the axion decay constant, among other things.

MINIMAL GUT REALIZATION

A particularly attractive and solid realization of our scenario is obtained in the context of the minimal supersymmetric SU(5) GUT [21]. In this theory, the SU(5) symmetry is broken by the VEV of $\Sigma(24)$. The superpotential relevant for the GUT breaking is

$$W_\Sigma = \frac{m_\Sigma}{2} \text{tr} \Sigma^2 + \frac{\lambda_\Sigma}{3} \text{tr} \Sigma^3, \quad (18)$$

leading to $\langle \Sigma \rangle = (m_\Sigma / \lambda_\Sigma) \text{diag}(2, 2, 2, -3, -3)$. For a non-singlet supersymmetry breaking field X , the shift of this vacuum due to supersymmetry breaking effects is small. The superpotential for the Higgs fields, $H_5 = (H_C, H_u)$ and $\bar{H}_5 = (\bar{H}_C, \bar{H}_d)$, is given by

$$W_H = \bar{H}_5 (m_H + \lambda_H \Sigma) H_5. \quad (19)$$

With the Σ VEV, this leads to the supersymmetric masses for the MSSM Higgs doublets $H_{u,d}$ and their GUT partners H_C, \bar{H}_C :

$$\mu = m_H - 3 \frac{\lambda_H m_\Sigma}{\lambda_\Sigma}, \quad \mu_C = m_H + 2 \frac{\lambda_H m_\Sigma}{\lambda_\Sigma}. \quad (20)$$

Since both terms in the right-hand side are of the order of the unification scale $M_{\text{unif}} \approx O(10^{16} \text{ GeV})$, having $|\mu| \ll M_{\text{unif}}$ requires a large fine-tuning of parameters. With this fine-tuning, μ_C is of order M_{unif} .

The situation described above is exactly the one needed to realize our scenario. In fact, we may apply the argument in the previous section without modification by identifying M_* as the unification scale M_{unif} . This realization has several virtues from the GUT point of view as well. First, the notorious doublet-triplet splitting problem is automatically “solved” because of the environmental selection for electroweak symmetry breaking. Second, the precision of gauge coupling unification is also improved with the superpartner spectrum discussed here [26], compared with the case in conventional weak scale supersymmetry. The dangerous dimension-5 proton

decay caused by exchange of H_C, \bar{H}_C is also suppressed because of the large sfermion masses, if the 1-3 element of the doublet squark mass-squared matrix is sufficiently small [27]. (This is ensured if the low energy theory has an appropriate flavor structure analogous to that in the Yukawa couplings, which also suppresses possible contributions from cutoff-scale suppressed tree-level operators [28].) This scenario, therefore, provides one of the most simple and attractive realizations of supersymmetric GUT.

COMMENT ON SINGLET X

We finally comment on the case in which the assumption (iii) is modified: the supersymmetry breaking field X is a singlet. In this case, the direct coupling of X to the Higgs fields $W \supset -\lambda X H_u H_d$ is allowed, yielding $|b| = |\lambda| \Lambda \tilde{m}$. By integrating over $\lambda \in \{y_j\}$ in the expression in Eq. (1) using the electroweak symmetry breaking condition $\delta(v - v_{\text{obs}})$, we find that the probability in the $\ln \tilde{m} - \ln |\mu|$ plane is peaked at $|\mu|^2 / \Lambda \tilde{m} \approx |\lambda|_{\text{max}}$; more explicitly,

$$P(\ln \tilde{m}, \ln |\mu|) \propto \frac{|\mu|^4}{(\Lambda \tilde{m})^2} \theta\left(|\lambda|_{\text{max}} - \frac{|\mu|^2}{\Lambda \tilde{m}}\right) \hat{n}(\tilde{m}, |\mu|), \quad (21)$$

where $|\lambda|_{\text{max}}$ is the largest value of $|\lambda|$ in the landscape, which we expect is not too far from $O(1)$, and $\hat{n}(\tilde{m}, |\mu|)$ is the anthropic weighting factor without including that for electroweak symmetry breaking.

An important element in $\hat{n}(\tilde{m}, |\mu|)$ arises from the fact that for $|b| \simeq |\mu|^2 \gg \tilde{m}^2$, a Higgs loop makes the mass squared for the top squark negative $m_{\tilde{t}}^2 \sim -(y_t^2 / 16\pi^2) |\mu|^2$. Assuming that such a region, $\tilde{m}^2 \lesssim 10^{-(2-3)} |\mu|^2$, is environmentally disallowed, we find

$$\tilde{m} \gtrsim 10^{-(2-3)} |\lambda|_{\text{max}} \Lambda, \quad (22)$$

so we expect that the superpartner mass scale is high. Moreover, we find that

$$\tan \beta - 1 \approx O\left(\frac{\tilde{m}^2}{|\mu|^2}\right) \approx O\left(\frac{\tilde{m}}{|\lambda|_{\text{max}} \Lambda}\right) \ll 1, \quad (23)$$

in most of the selected parameter space. This, therefore, provides a simple realization of the high scale supersymmetry scenario discussed in Ref. [22] with $m_{h^0} \simeq 126$ GeV. A more detailed analysis of the singlet X case will be presented elsewhere.

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