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Next-to-leading order forward hadron production in the small- x regime: rapidity factorization

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Single inclusive hadron production at forward rapidity in high energy p+A collisions is an important probe of the high gluon density regime of QCD and the associated small- x formalism. We revisit an earlier one-loop calculation to illustrate the significance of the “rapidity factorization” approach in this regime. Such factorization separates the very small- x unintegrated gluon density evolution and leads to a new correction term to the physical cross section at one-loop level. Importantly, this rapidity factorization formalism remedies the previous unphysical negative next-to-leading order contribution to the cross section. It is much more stable with respect to “rapidity” variation when compared to the leading-order calculation and provides improved agreement between theory and experiment in the forward rapidity region.

Introduction. As the theory of strong interactions, Quantum Chromodynamics (QCD) [1] has been extensively tested and verified. In particular, QCD in the weak coupling regime has been very successful in predicting and interpreting high energy scattering processes in fixed target and collider experiments. Such a success is based on the well-established QCD collinear factorization formalism [2], which describes the hadron as a dilute system of partons. It was subsequently found that the parton densities (especially the gluon density) grow dramatically when the longitudinal momentum fraction x carried by a parton in a proton becomes very small due to bremsstrahlung processes. Such a fast growth would violate the fundamental principle of unitarity and cannot be sustained. It is, thus, expected that the gluon density will eventually become so large that a non-linear regime, called a saturation regime [3], will be reached. Another characteristic of the small- x regime is that external hard probes will interact with the partons in a nucleon or a nucleus coherently rather than independently [4, 5]. In recent years, the high parton density limit has become one of the most active research topics for QCD theory. The quest to identify the quantum coherent scattering regime is a critical goal for the ongoing experiments at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). It is a corner stone of the physics program for the planned Electron Ion Collider (EIC) [6].

Single forward hadron production in high energy proton-nucleus (p+A) collisions constitutes one of the key observables in searching for gluon saturation. The observed suppression of inclusive hadrons at forward rapidity in d+Au collisions at RHIC [7] has provided evidence for the significance of cold nuclear matter effects, among them coherent multiple scattering. However, in the small- x formalism, experimental data are still mostly interpreted via leading order (LO) calculations [8, 9]. A significant step forward is the first calculation of forward hadron production at next-to-leading order (NLO) [10]. However, the resulting one-loop correction in this ap-

proach is negative. At moderate and large transverse momenta it dominates the cross sections, which become negative (and unphysical) [11].

In this paper we demonstrate that besides the well-known standard collinear factorization, which separates the short-distance dynamics from the long-distance physics, one has to pay close attention to the so-called “rapidity factorization” regime. It necessitates a rapidity cut-off to separate the very small- x unintegrated gluon density evolution from the finite one-loop contributions. We revisit the NLO calculation for forward hadron production in high energy p+A collisions to show that such a procedure leads to a new NLO correction term. This term remedies the unphysical negative one-loop cross section obtained in [11]. The new formalism also leads to much less sensitivity to the choice of “rapidity” factorization scale at NLO in comparison to LO results and improved agreement between data and theory.

Rapidity factorization. The mechanism of inclusive hadron production at forward rapidities in p+A collisions, $p+A \rightarrow h+X$, in the small- x regime at LO can be described as follows: an energetic parton (either quark or gluon) from the proton scatters coherently on the gluon field of the nucleus, as it penetrates the target, and then fragments into the final-state hadron. Let us focus on the situation where a quark from the proton undergoes such scattering ($qA \rightarrow q$) to demonstrate the formalism. The differential cross section at forward rapidity y and transverse momentum $p_\perp$ is given by [8, 9]

$$\frac{d\sigma}{dyd^2p_\perp} = \int_\tau^1 \frac{dz}{z^2} D_{h/q}(z) x_p f_{q/p}(x_p) \mathcal{F}(x_g, k_\perp), \quad (1)$$

where the sum over quark flavors is suppressed for simplicity, $k_\perp = p_\perp/z$, and $\tau = \frac{p_\perp}{\sqrt{s}} e^y$. $f_{q/p}(x_p)$ is the collinear parton distribution function (PDF) in the proton with $x_p = \tau/z$ and $D_{h/q}(z)$ is the fragmentation function (FF). x_g is the longitudinal momentum fraction of the probed gluons in the nucleus and is given by $x_g = x_A$

with $x_A = \frac{p_\perp}{z\sqrt{s}}e^{-y}$. All the information for the transverse momentum transfer from coherent multiple scattering is contained in $\mathcal{F}(x_g, k_\perp)$, the so-called unintegrated gluon distribution defined as

$$\mathcal{F}(x_g, k_\perp) = \int \frac{d^2 b_\perp d^2 b'_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (b_\perp - b'_\perp)} S^{(2)}(b_\perp, b'_\perp), \quad (2)$$

where $S^{(2)}(b_\perp, b'_\perp)$ is the dipole scattering amplitude given by $S^{(2)}(b_\perp, b'_\perp) = \frac{1}{N_c} \langle \text{Tr} [U(b_\perp) U^\dagger(b'_\perp)] \rangle$. Here

$U(b_\perp) = \mathcal{P} \exp \left\{ ig_s \int_{-\infty}^{+\infty} dx^+ t^c A_c^-(x^+, b_\perp) \right\}$ is the Wilson line in the small- x formalism.

Let us now concentrate on the NLO calculation, in which we have to consider both real and virtual corrections. The calculation is standard in the so-called light-front perturbation theory [12], and the result can be written as the sum of three terms [10], $d\sigma/dy d^2 p_\perp = I^R + I^V + I^Y$. The expressions are given by

$$I^R = \alpha_s C_F \int_\tau^1 \frac{dz}{z^2} D_{h/q}(z) \int_{\tau/z}^1 d\xi \frac{1+\xi^2}{(1-\xi)_+} x f_{q/p}(x) \int \frac{d^2 b_\perp d^2 b'_\perp d^2 x_\perp}{(2\pi)^4} e^{-ik_\perp \cdot (b_\perp - b'_\perp)} \frac{2(x_\perp - b_\perp) \cdot (x_\perp - b'_\perp)}{(x_\perp - b_\perp)^2 (x_\perp - b'_\perp)^2} \\ \times \left[S^{(2)}(b_\perp, b'_\perp) + S^{(2)}(v_\perp, v'_\perp) - S^{(3)}(b_\perp, x_\perp, v'_\perp) - S^{(3)}(v_\perp, x_\perp, b'_\perp) \right], \quad (3)$$

$$I^V = -2\alpha_s C_F \int_\tau^1 \frac{dz}{z^2} D_{h/q}(z) x_p f_{q/p}(x_p) \int_0^1 d\xi \frac{1+\xi^2}{(1-\xi)_+} \int \frac{d^2 v_\perp d^2 v'_\perp d^2 u_\perp}{(2\pi)^4} e^{-ik_\perp \cdot (v_\perp - v'_\perp)} \frac{2}{u_\perp^2} \\ \times \left[S^{(2)}(v_\perp, v'_\perp) - S^{(3)}(b_\perp, x_\perp, v'_\perp) \right], \quad (4)$$

$$I^Y = \int_\tau^1 \frac{dz}{z^2} D_{h/q}(z) x_p f_{q/p}(x_p) \int \frac{d^2 b_\perp d^2 b'_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (b_\perp - b'_\perp)} \\ \times \left\{ \frac{\alpha_s N_c}{2\pi^2} \int_0^1 \frac{d\xi}{1-\xi} \int d^2 x_\perp \frac{(b_\perp - b'_\perp)^2}{(b_\perp - x_\perp)^2 (x_\perp - b'_\perp)^2} \left[S^{(4)}(b_\perp, x_\perp, b'_\perp) - S^{(2)}(b_\perp, b'_\perp) \right] \right\}, \quad (5)$$

where $S^{(3)}$ and $S^{(4)}$ are multi-point gluon correlators defined in [10, 13]. Both I^R and I^V are finite when $\xi \rightarrow 1$ but they contain collinear divergences. To see the collinear divergences explicitly and, thus, regularize them, it is useful to work in momentum space. We use dimensional regularization in $n = 4 - 2\epsilon$ dimensions with the change

$$\int \frac{d^2 q_\perp}{(2\pi)^2} \rightarrow \mu^{2\epsilon} \int \frac{d^{2-2\epsilon} q_\perp}{(2\pi)^{2-2\epsilon}}. \quad (6)$$

In momentum space the collinear divergences are manifest, as demonstrated in [10, 14]. They lead to the standard DGLAP splitting functions accompanied by $(-1/\epsilon)$ with $1/\epsilon = 1/\epsilon - \gamma_E + \ln 4\pi$ and can be absorbed into the redefinition of either the PDF $f_{q/p}(x)$ or the FF $D_{h/q}(z)$, which leads to the well-known DGLAP evolution equations for PDFs and FFs [10, 15]. Such a collinear factorization procedure introduces a factorization scale (μ) dependence [16]. Since μ is an artificial scale, the physical cross section should not depend on it in an all-order result. In practice, since one can only calculate to finite order, some residual μ -dependence remains. However, it should be reduced in the NLO calculation compared with the LO result and the cross section at NLO is expected to have smaller uncertainty [11].

On the other hand, I^Y is divergent in the limit $\xi \rightarrow 1$. This is the so-called rapidity divergence. As we will

demonstrate below, I^Y contains no collinear divergence. It is instructive to realize that for forward hadron production we have

$$\int_0^1 \frac{d\xi}{1-\xi} = \int_0^1 \frac{d\xi_g}{\xi_g} = \int_0^\infty dy_g = \int_{-\infty}^Y dy_A, \quad (7)$$

where $\xi_g = 1 - \xi$ is the momentum fraction of the projectile quark carried by the radiated gluon, with $y_g = \ln 1/\xi_g$ the rapidity of the radiated gluon w.r.t. the projectile proton. On the other hand, $y_A = Y - y_g$ is the rapidity of the radiated gluon w.r.t. the target nucleus, where $Y = \ln(s/m_p^2)$ is the rapidity interval between the projectile proton and the target nucleus¹, with s (m_p) the center-of-mass energy squared (nucleon mass). The divergence occurs when $y_A \rightarrow -\infty$, thus the name “rapidity divergence”. Rapidity divergence is a general feature [17] when one uses the transverse momentum dependent distributions, e.g. $\mathcal{F}(x_g, k_\perp)$ in our case. It is

¹ Strictly speaking, Y should be the rapidity interval between the projectile quark and the target nucleus. However, we are using the so-called hybrid formalism [8–10], in which the projectile quark is purely collinear to the parent proton without transverse momentum. In this case we have quark momentum $p_q \approx x_p p$ with p the proton momentum, and thus the quark rapidity is the same as the proton rapidity.

very easy to see from Eq. (5) that such a divergence disappears when one integrates over $k_\perp$ [10]. Realizing that

$$\int_{-\infty}^Y dy_A = \int_{-\infty}^{Y_0} dy_A + \int_{Y_0}^Y dy_A, \quad (8)$$

following the ideas of collinear factorization, we compare I^Y to the LO result in Eq. (1) and see that one should absorb this divergence into the redefinition of the dipole scattering amplitude

$$\begin{aligned} S_{Y_0}^{(2)}(b_\perp, b'_\perp) &= S^{(2)}(b_\perp, b'_\perp) + \frac{\alpha_s N_c}{2\pi^2} \int_{-\infty}^{Y_0} dy_A \\ &\times \int d^2 x_\perp \frac{(b_\perp - b'_\perp)^2}{(b_\perp - x_\perp)^2 (x_\perp - b'_\perp)^2} \\ &\times \left[S^{(4)}(b_\perp, x_\perp, b'_\perp) - S^{(2)}(b_\perp, b'_\perp) \right]. \end{aligned} \quad (9)$$

Here, the rapidity cut-off Y_0 is introduced to separate the “fast” and “slow” gluon fields [18–20]. The Y_0 -dependence for the renormalized dipole gluon distribution $S_{Y_0}^{(2)}(b_\perp, b'_\perp)$ leads to the well-known Balitsky-Kovchegov (BK) evolution equation [20, 21]

$$\begin{aligned} \frac{\partial}{\partial Y_0} S_{Y_0}^{(2)}(b_\perp, b'_\perp) &= \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_\perp \frac{(b_\perp - b'_\perp)^2}{(b_\perp - x_\perp)^2 (x_\perp - b'_\perp)^2} \\ &\times \left[S_{Y_0}^{(4)}(b_\perp, x_\perp, b'_\perp) - S_{Y_0}^{(2)}(b_\perp, b'_\perp) \right]. \end{aligned} \quad (10)$$

In other words, we subtract the standard BK term in Eq. (9) following [18–20] (not the BK with kinematic constraint (kcBK) advocated in [22, 23]). After the subtraction, a finite correction appears from the rapidity factorization procedure

$$\begin{aligned} \Delta H_Y &= \frac{\alpha_s N_c}{2\pi^2} \int_\tau^1 \frac{dz}{z^2} D_{h/q}(z) x_p f_{q/p}(x_p) \int_{Y_0}^Y dy_A \\ &\times \left\{ \int \frac{d^2 b_\perp d^2 b'_\perp d^2 x_\perp}{(2\pi)^2} \frac{(b_\perp - b'_\perp)^2}{(b_\perp - x_\perp)^2 (x_\perp - b'_\perp)^2} \right. \\ &\times e^{-ik_\perp \cdot (b_\perp - b'_\perp)} \left[S^{(4)}(b_\perp, x_\perp, b'_\perp) - S^{(2)}(b_\perp, b'_\perp) \right] \left. \right\}. \end{aligned} \quad (11)$$

To elaborate further on the rapidity factorization procedure, let us emphasize that since the target nucleus has rapidity close to $-\infty$, the radiated soft gluon with rapidity $-\infty < y_A < Y_0$ should naturally be part of the gluon distribution function of the target nucleus, while the gluon with rapidity $y_A > Y_0$ becomes the finite NLO hard-part correction. As we will show later, it is this new correction term that was missed in [11] and which ensures that the NLO cross section is positive definite². Similarly to the collinear factorization case, a rapidity

cut-off scale Y_0 is introduced in rapidity factorization. The physical cross section should also be independent of such a rapidity cut-off in the all-order result. In our finite order calculation some residual Y_0 -dependence is expected to remain. However, it should be reduced at NLO when compared to the LO result. One can choose the gluon rapidity cut-off to be the one related to the gluon momentum fraction from the LO kinematics, e.g. $x_g = x_A$. Unlike the usual collinear factorization, which can be seen as separating perturbative from nonperturbative physics, both rapidity separated parts have perturbative and nonperturbative contributions at the same time [19].

Let us now better understand the rapidity correction term ΔH_Y in Eq. (11). In particular, we would like to know whether it contains any collinear divergence. For this purpose, we transform the result to momentum space. The term $\propto S^{(2)}(b_\perp, b'_\perp)$ in the bracket $\{\dots\}$ can be written as follows

$$\begin{aligned} I_2 &= 2 \int \frac{d^2 q_\perp}{q_\perp^2} \mathcal{F}(x_g, k_\perp) - 2 \int \frac{d^2 q_\perp}{(k_\perp - q_\perp)^2} \mathcal{F}(x_g, q_\perp) \\ &\rightarrow 2\pi \int \frac{d^2 b_\perp d^2 b'_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (b_\perp - b'_\perp)} S^{(2)}(b_\perp, b'_\perp) \\ &\times \left[\frac{1}{\epsilon} + \ln \mu^2 - \ln \frac{c_0^2}{(b_\perp - b'_\perp)^2} \right], \end{aligned} \quad (12)$$

where in the second step we use dimensional regularization following Eq. (6) with $1/\epsilon = 1/\epsilon - \gamma_E + \ln 4\pi$ and $c_0 = 2e^{-\gamma_E}$. On the other hand, the second term $\propto S^{(4)}(b_\perp, x_\perp, b'_\perp)$ in Eq. (11) is given by

$$\begin{aligned} I_4 &= \int d^2 \ell_\perp d^2 q_\perp \frac{(\ell_\perp - q_\perp) \cdot (\ell_\perp - k_\perp)}{(\ell_\perp - q_\perp)^2 (\ell_\perp - k_\perp)^2} (\mathcal{G}(x_g, q_\perp, k_\perp) \\ &+ \mathcal{G}(x_g, k_\perp, q_\perp)) - 2 \int d^2 \ell_\perp d^2 q_\perp \\ &\times \frac{(q_\perp - k_\perp) \cdot (\ell_\perp - k_\perp)}{(q_\perp - k_\perp)^2 (\ell_\perp - k_\perp)^2} \mathcal{G}(x_g, q_\perp, \ell_\perp) \\ &\rightarrow \int \frac{d^2 b_\perp d^2 b'_\perp}{2\pi} e^{-ik_\perp \cdot (b_\perp - b'_\perp)} S^{(2)}(b_\perp, b'_\perp) \left[\frac{1}{\epsilon} + \ln \mu^2 \right] \\ &- \pi \int d^2 q_\perp \ln(k_\perp - q_\perp)^2 (\mathcal{G}(x_g, q_\perp, k_\perp) \\ &+ \mathcal{G}(x_g, k_\perp, q_\perp)), \end{aligned} \quad (13)$$

where $\mathcal{G}(x_g, k_\perp, q_\perp)$ is defined as

$$\begin{aligned} \mathcal{G}(x_g, k_\perp, q_\perp) &= \int \frac{d^2 b_\perp d^2 b'_\perp d^2 x_\perp}{(2\pi)^4} e^{-ik_\perp \cdot (b_\perp - x_\perp)} \\ &\times e^{-iq_\perp \cdot (x_\perp - b'_\perp)} S^{(4)}(b_\perp, x_\perp, b'_\perp). \end{aligned} \quad (14)$$

such that the correction term ΔH_Y vanishes. However, in this case, one has to allow the dipole gluon distribution $S^{(2)}(b_\perp, b'_\perp)$ evolves up to rapidity $Y = \ln(s/m_p^2)$ instead to the typical gluon rapidity in the target nucleus $\sim \ln 1/x_g$ as we will show below. Nevertheless, we could regard such a choice as a different scheme [24].

² In principle, one could choose the rapidity cut-off $Y_0 = Y$

Finally, we can write the rapidity factorization correction term in Eq. (11) as

$$\begin{aligned} \Delta H_Y = & \frac{\alpha_s N_c}{\pi} \int_{\tau}^1 \frac{dz}{z^2} D_{h/q}(z) x_p f_{q/p}(x_p) \int_{Y_0}^Y dy_A \\ & \times \left[\int \frac{d^2 b_{\perp} d^2 b'_{\perp}}{(2\pi)^2} e^{-iq_{\perp} \cdot (b_{\perp} - b'_{\perp})} S^{(2)}(b_{\perp}, b'_{\perp}) \ln \frac{c_0^2}{(b_{\perp} - b'_{\perp})^2} \right. \\ & - \frac{1}{2} \int d^2 q_{\perp} \ln(k_{\perp} - q_{\perp})^2 (\mathcal{G}(x_g, q_{\perp}, k_{\perp}) + \mathcal{G}(x_g, k_{\perp}, q_{\perp})) \\ & \left. - \frac{1}{\pi} \int d^2 \ell_{\perp} d^2 q_{\perp} \frac{(q_{\perp} - k_{\perp}) \cdot (\ell_{\perp} - k_{\perp})}{(q_{\perp} - k_{\perp})^2 (\ell_{\perp} - k_{\perp})^2} \mathcal{G}(x_g, q_{\perp}, \ell_{\perp}) \right]. \end{aligned} \quad (15)$$

In other words, the $1/\epsilon + \ln \mu^2$ term cancels between Eqs. (12) and (13). This indicates that the rapidity divergence and collinear divergence are well separated, and thus can be factorized independently.

Numerical results. To illustrate our NLO calculation, we use the GBW model [25] to parametrize the dipole scattering amplitude: $S^{(2)}(b_{\perp}, b'_{\perp}) = \exp[-(b_{\perp} - b'_{\perp})^2 Q_s^2(x)/4]$. The saturation scale in a nucleus with atomic number A is given by $Q_s^2(x) = cA^{1/3}Q_{s0}^2(x_0/x)^{\lambda}$, with $Q_{s0} = 1$ GeV, $x_0 = 3.04 \times 10^{-4}$ and $\lambda = 0.288$. We use $c = 0.56$ [26] for minimum bias p+A collisions.

We first show that, within our rapidity factorization scheme, the full NLO results with ΔH_Y in Eq. (15) remedies the negative cross section from the calculation in Ref. [11]. In Fig. 1 we present comparison to the BRAHMS h^- data at $y = 3.2$ in d+Au collisions at RHIC [7]. For consistency with [11], we choose the collinear factorization scale $\mu^2 = 10$ GeV². The red dashed curve is the LO result, the blue solid curve is our NLO calculation (including the new rapidity correction term ΔH_Y), while the black dotted curve is the previous NLO result that becomes negative for $p_{\perp} \gtrsim 2.5$ GeV [11]. We have checked that the formalism presented here yields positive-definite cross sections for variety of rapidities and center of mass energies in the physical kinematic $p_{\perp}$ region.

Of course, one should choose the collinear factorization scale μ to be related to the typical momentum scale in the hard process (e.g. $p_{\perp}$ of the hadron). In Fig. 2 we plot a new comparison to the BRAHMS data with $\mu = p_{\perp}$. The red dashed curve shows the LO result, the blue solid curve shows our NLO calculation (with ΔH_Y included). At one loop we find a good description of the experimental data. At higher $p_{\perp}$ our NLO corrections enhances the cross section as expected, since it includes the gluon radiation processes.

As we emphasized earlier, the factorization scale μ -dependence should be largely reduced in the NLO cross section when compared to the LO results. We have verified that this is indeed the case, consistent with previous findings [11]. What is much more important is to

FIG. 1. Comparison of h^- spectra obtained in the small- x formalism with fixed $\mu^2 = 10$ GeV² to BRAHMS data [7].

FIG. 2. Comparison of the LO and NLO results to BRAHMS data [7]. We choose the collinear factorization scale $\mu = p_{\perp}$.

demonstrate the reduction in sensitivity to the rapidity factorization scale $Y_0 = \ln 1/x_g$. We plot in Fig. 3 the ratio $R = \frac{dN}{dy d^2 p_{\perp}} \Big|_{x_g = \kappa x_A} / \frac{dN}{dy d^2 p_{\perp}} \Big|_{x_g = x_A}$ as a function of $\kappa = x_g/x_A$, with x_A being the typical gluon momentum fraction at LO. It can be seen that for $\kappa \in (0.25, 2)$ the LO result has a variation of $\pm 50\%$, while our NLO result with the new rapidity correction ΔH_Y shows only $\pm 10\%$ variation. On the other hand, the previous result from [11] shows more than a factor of 2 variation. In other words, the full NLO calculations provide predictions that are much more stable with respect to variation of both collinear factorization and rapidity factorization scales.

Summary. In this paper we studied forward hadron production in high energy p+A collisions within the small- x formalism. We revisited the previous one-loop calculation and demonstrated that besides the well-known collinear factorization, which separates the short-distance from the long-distance physics, one has to pay close attention to the “rapidity factorization” regime.

It separates the small- x dynamics of “fast” and “slow” gluon fields. The rapidity factorization procedure results in a new next-to-leading order correction which remedies the unphysical negative cross section from the one-loop calculation of [11]. We also demonstrated that such factorization formalism leads to much more stable and reliable cross section predictions at next-to-leading order. We expect that our results will have important applications for small- x gluon saturation phenomenology.

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FIG. 3. The rapidity factorization scale $Y_0 = \ln 1/x_g$ dependence.

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