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The forces exerted on small objects by beams of light have attracted considerable interest because of their application to optical micromanipulation. Gradients in the light's intensity give rise to the conservative forces that are responsible for optical trapping [1]. Gradients in the phase control radiation pressure [2]. No such role was ascribed to light's polarization until Albaladejo, Marqués, Laroche and Sáenz reported that the curl of the spin angular momentum density contributes an additional term to the radiation pressure [3]. Contrary to this claim, however, no spin-curl force acts on small illuminated objects, as we now demonstrate.

The vector potential of a monochromatic beam of light of frequency ω may be written as

$$\mathbf{A}(\mathbf{r}, t) = u(\mathbf{r}) \, e^{-i\omega t} \, \hat{\epsilon}(\mathbf{r}), \qquad (1)$$

where $u(\mathbf{r})$ is the amplitude of the beam, and

$$\hat{\epsilon}(\mathbf{r}) = \sum_{j=1}^{3} a_j(\mathbf{r}) \, e^{i\varphi_j(\mathbf{r})} \, \hat{e}_j \qquad (2)$$

is the polarization. The polarization's component along coordinate $\hat{e}_j$ has relative amplitude $a_j(\mathbf{r})$ and phase $\varphi_j(\mathbf{r})$. Normalization requires $\sum_{j=1}^{3} a_j^2(\mathbf{r}) = 1$.

The force that such a beam of light exerts on a small optically isotropic object is given in the Rayleigh dipole approximation by [4]

$$\mathbf{F}(\mathbf{r}) = \frac{\omega^2}{2} \, \Re \left\{ \alpha \sum_{j=1}^{3} A_j(\mathbf{r}, t) \, \nabla A_j^*(\mathbf{r}, t) \right\}, \qquad (3)$$

where $\alpha = \alpha' + i\alpha''$ is the object's polarizability. Equation (3) is valid for particles small enough that spatial variations in the light's instantaneous electromagnetic field may be ignored. It appears as Eq. (7) in Ref. [3] and is the basis for that Letter's results.

Substituting Eqs. (1) and (2) into Eq. (3) yields an equivalent expression for the time-averaged force,

$$\mathbf{F}(\mathbf{r}) = \frac{\omega^2}{4}\alpha' \, \nabla u^2(\mathbf{r}) + \frac{\omega^2}{2}\alpha'' \, u^2(\mathbf{r}) \sum_{j=1}^{3} a_j^2(\mathbf{r}) \, \nabla\varphi_j(\mathbf{r}). \quad (4)$$

The first term on the right-hand side of Eq. (4) is the intensity-gradient force. The second is a generalization of the phase-gradient force reported in Ref. [2] that is valid for arbitrary polarizations. It may be identified with the scattering force experienced by the particle [1–3].

Equation (4) reveals that the spin angular momentum plays no role in $\mathbf{F}(\mathbf{r})$. This may be appreciated because the time-averaged spin angular momentum density [5],

$$\mathbf{s}(\mathbf{r}) = i \frac{\omega}{2\mu c^2} \, u^2(\mathbf{r}) \, \hat{\epsilon}(\mathbf{r}) \times \hat{\epsilon}^*(\mathbf{r}), \qquad (5)$$

involves cross terms in the components of the polarization, whereas Eq. (4) does not. Here, c is the speed of light in a medium of permeability μ.

Spin-curl forces do arise for particles that are larger than the wavelength of light [6]. Their absence in the dipole approximation is remarkable because it means that the radiation pressure experienced by a Rayleigh particle is not simply proportional to the Poynting vector, as is usually assumed. This surprising insight was brought to light in Ref. [3]. Reference [3] goes on to suggest, however, that the curl of the spin angular momentum density contributes to the force experienced by a small illuminated object, The correct expression for the dipole force in Eq. (4) demonstrates that it does not.

[1] A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, and S. Chu, Opt. Lett. **11**, 288 (1986).

[2] Y. Roichman, B. Sun, Y. Roichman, J. Amato-Grill, and D. G. Grier, Phys. Rev. Lett. **100**, 013602 (2008).

[3] S. Albaladejo, M. I. Marqués, M. Laroche, and J. J. Sáenz, Phys. Rev. Lett. **102**, 113602 (2009).

[4] P. C. Chaumet and M. Nieto-Vesperinas, Opt. Lett. **25**, 1065 (2000).

[5] M. V. Berry and M. R. Dennis, Proc. Royal Soc. A **457**, 141 (2001).

[6] D. B. Ruffner and D. G. Grier, Phys. Rev. Lett. **108**, 173602 (2012).