Floquet Majorana Fermions for Topological Qubits in Superconducting Devices and Cold-Atom Systems

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Floquet Majorana Fermions for Topological Qubits

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We develop an approach to realizing a topological phase transition and non-Abelian braiding statistics with dynamically induced Floquet Majorana fermions (FMFs). When the periodic driving potential does not break fermion parity conservation, FMFs can encode quantum information. Quasi-energy analysis shows that a stable FMF zero mode and two other satellite modes exist in a wide parameter space with large quasi-energy gaps, which prevents transitions to other Floquet states under adiabatic driving. We also show that in the asymptotic limit FMFs preserve non-Abelian braiding statistics and, thus, behave like their equilibrium counterparts.

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Introduction — Proposals of solid state [1–7] and cold atomic [8–10] systems hosting Majorana fermions (MFs) have been a recent focus of attention. These systems present novel prospects for quantum computation since a widely separated pair of MF bound states, that formally correspond to zero-energy states of an effective Bogoliubov-de Gennes (BdG) Hamiltonian, forms a nonlocal fermionic state that is immune to local decoherence. Moreover, Majorana zero-energy modes obey non-Abelian braiding statistics and thus have potential for topological quantum information processing. Among the key signatures of MFs are a zero-bias resonance in tunneling [11, 12], half-integer conductance quantization [13, 14], a 4π Josephson effect [15], and interferometric schemes [16–19]. Some of these predictions have already received possible experimental support [20–24].

Topological states of matter can be induced dynamically by time-periodic driving, the so-called Floquet approach [25–27]. This brought to the agenda the new concept of Floquet Majorana fermion (FMFs) [10, 28]. It turns out that even if the system is initially in the topologically trivial state, its Floquet version may exhibit topological properties. Floquet methods may help, for instance, in realizing MFs without a magnetic field [28]. A realization of FMF states where they can be manipulated and tuned in a wide parameter space is therefore highly desirable. The natural questions for FMF systems are: whether they are robust and tunable, whether they can encode quantum information, and whether they follow non-Abelian braiding statistics as for their equilibrium counterparts. Our study aims to answer these questions.

We consider a generic platform to investigate non-Abelian braiding statistics and potentially to realize topological quantum computation based on FMFs. The model is broadly applicable to both semiconductor-superconductor heterostructures with strong spin-orbit interaction and in-plane magnetic field [6, 7], and to cold atomic systems where superconducting order is controlled by Feshbach resonances while spin-orbit coupling and Zeeman field effects are induced by an optical Raman transition [10]. The latter realization is practically more promising since it allows a greater degree of control. Furthermore, cold atom systems can be isolated thus suppressing dissipation on long time scales.

We show, first, that if FMFs exist, they will exist at any instantaneous time. Therefore, FMFs can encode quantum information if the driving potential does not break fermion parity conservation. We study the quasi-energy spectrum of the problem analytically in the limit that the frequency is large compared to the band width. We also perform exact numerical calculations which capture certain features of the spectrum beyond this limit. A broad range of parameters supporting FMFs is identified as a function of driving frequency ω and amplitude K for two specific driving scenarios: periodic modulation of either the chemical potential or the Zeeman field. Finally, by using a two-time formalism [29, 30], we show that FMFs follow the same non-Abelian braiding statistics as their stationary counterparts. This conclusion stems from the observation that a generalized Floquet Berry matrix does not affect the non-Abelian braiding statistics of FMFs since a large quasi-energy gap ensures no transitions to other Floquet quasi-energy states in the adiabatic movement.

Floquet Theorem for Majorana Fermion — Let us consider Floquet theory [31]. Suppose that the Hamiltonian has an explicit time dependence $\hat{H}(t) = \hat{H}(t+T)$ with period $T = 2\pi/\omega$, where ω is the driving frequency. The solution of the Schrödinger equation can be described by a complete set of time-dependent states $|\Phi_\alpha(t)\rangle = e^{-i\epsilon_\alpha t}|\phi_\alpha(t)\rangle$, where the quasi-energies ϵ_α satisfy the equation $[\hat{H}(t) - i\partial_t]|\phi_\alpha(t)\rangle = \epsilon_\alpha|\phi_\alpha(t)\rangle$ and $|\phi_\alpha(t)\rangle = |\phi_\alpha(t+T)\rangle$ are Floquet states (hereafter $\hbar = 1$). The evolution operator $\hat{U}(t) = \mathbb{T}\exp(-i\int_0^T \hat{H}(t)dt)$ has the following property

$$\hat{U}(t+T,t)|\phi_\alpha(t)\rangle = e^{-i\epsilon_\alpha T}|\phi_\alpha(t)\rangle. \tag{1}$$

One can define an effective stationary Hamiltonian $\hat{H}_{\rm eff}$ through the relation (generalizing the notion in Refs. [26, 27])

$$\hat{U}(t+T,t) \equiv e^{-i\hat{H}_{\rm eff}T}, \quad 0 \le t < T, \tag{2}$$

with $\hat{H}_{\text{eff}}(t)|\phi_\alpha(t)\rangle = \epsilon_\alpha|\phi_\alpha(t)\rangle$. Here, t is a parameter of this eigenvalue problem. The effective Floquet Hamiltonian is defined at each value of the time parameter, and the topological properties of each of these Hamiltonians is the same [26, 27].

If the system is described by a BdG Hamiltonian, the quasi-particle excitation spectrum will possess a particle-hole symmetry even if a time-dependent potential is added [26]. If we define the operator $\hat{\gamma}^\dagger_\epsilon(t)$ as the creation operator for the quasi-energy state $|\phi_\epsilon(t)\rangle$, the relation $\hat{\gamma}_\epsilon(t) = \hat{\gamma}^\dagger_{-\epsilon}(t)$ is guaranteed. So, the zero quasi-energy state reveals the existence of a FMF [10]. The full wavefunction for $\epsilon_0 = 0$ can be written as $|\Phi_0(t)\rangle = e^{-i\epsilon_0}|\phi_0(t)\rangle = |\phi_0(t)\rangle = \hat{\gamma}_0(t)|0\rangle$, with $\hat{\gamma}_0(t) = \hat{\gamma}^\dagger_0(t)$. This argument shows that if the zero quasi-energy state exists, a zero energy FMF mode $\gamma(t)$ exists for every value of the time parameter t. While the MF operator evolves periodically in time $\hat{\gamma}(t) = \hat{\gamma}(t+T)$, it is in general different at different instantaneous times, $\hat{\gamma}(t) \neq \hat{\gamma}(t')$. For any fixed t, FMFs have the same properties as their equilibrium counterparts, and thus are localized in space [28, 38].

Quasi-Energy Spectrum and Floquet Majorana Fermion — To demonstrate the existence of FMFs consider a one dimensional wire with Rashba spin-orbit interaction λ_{SO}, Zeeman splitting V_z, and proximity-induced superconducting term Δ. The system can be described by a tight-binding Hamiltonian [6, 7, 10]:

$$\begin{aligned}
\hat{H}_0 \;=\; & \sum_{i,\sigma}\left[-\eta\left(\hat{c}^\dagger_{i+1\sigma}\hat{c}_{i\sigma} + h.c.\right) + \mu_L \hat{c}^\dagger_{i\sigma}\hat{c}_{i\sigma}\right] \\
+\; & \sum_i V_z\left(\hat{c}^\dagger_{i\uparrow}\hat{c}_{i\uparrow} - \hat{c}^\dagger_{i\downarrow}\hat{c}_{i\downarrow}\right) + \Delta\sum_i\left(\hat{c}^\dagger_{i\uparrow}\hat{c}^\dagger_{i\downarrow} + h.c.\right) \\
+\; & \lambda_{\text{SO}}\sum_i\left(\hat{c}^\dagger_{i+1\uparrow}\hat{c}_{i\downarrow} - \hat{c}^\dagger_{i+1\downarrow}\hat{c}_{i\uparrow} + h.c.\right), \qquad (3)
\end{aligned}$$

Here, i and $\sigma = \uparrow\downarrow$ denote fermion site and spin indices while $\hat{c}_{i\sigma}(\hat{c}^\dagger_{i\sigma})$ are corresponding operators, η is the hopping term along the chain which yields a band width $D = 4\eta$, and μ_L is the chemical potential of the lattice model which is set to the particle-hole symmetric point [32]. Note that Hamiltonian Eq. (3) is equally generic for a system of cold atoms [10].

To add time dependence, it is natural to consider modulating one of the parameters in $\hat{H}_0$: the chemical potential and the Zeeman field. We first consider periodic modulation of the chemical potential; the Hamiltonian is $\hat{H}(t) = \hat{H}_0 + \hat{H}_\mu(t)$ with

$$\hat{H}_\mu(t) = K\cos(\omega t)\sum_i(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow}), \qquad (4)$$

where $\hat{n}_{i\sigma} = \hat{c}^\dagger_{i\sigma}\hat{c}_{i\sigma}$. To calculate the quasi-energy, one can choose a Floquet basis [33]

$$|\{n_{i\sigma}\}; m\rangle = e^{-\frac{iK\sin(\omega t)}{\omega}\sum_i(\hat{n}_{i\uparrow}+\hat{n}_{i\downarrow}) + im\omega t}\,|\{n_{i\sigma}\}\rangle\,, \quad (5)$$

where $|\{n_{i\sigma}\}\rangle$ is the basis of the unperturbed system, and m labels the photon sector of the Floquet basis. The quasi-energy can be obtained by diagonalizing the Floquet operator $\hat{H}(t) - i\partial_t$ in this basis. The orthonormality condition of the Floquet states is only defined in an extended Hilbert space [34], so the inner product must include an extra time integral over a full period: $\langle\langle\cdot|\cdot\rangle\rangle = (1/T)\int_0^T dt\langle\cdot|\cdot\rangle$. The matrix elements read

$$\begin{aligned}
& \langle\langle\{n_{i\sigma}\}; m|\hat{H}(t) - i\partial_t|\{n'_{i\sigma}\}; m'\rangle\rangle \\
=\; & \frac{1}{T}\int_0^T dt\langle\{n_{i\sigma}\}|e^{\frac{iK\sin(\omega t)}{\omega}\sum_i(\hat{n}_{i\uparrow}+\hat{n}_{i\downarrow})}\left(\hat{H}_0 + m\omega\right) \\
& \times e^{-\frac{iK\sin(\omega t)}{\omega}\sum_i(\hat{n}_{i\uparrow}+\hat{n}_{i\downarrow})}|\{n'_{i\sigma}\}\rangle e^{-i(m-m')\omega t}. \qquad (6)
\end{aligned}$$

Since different photon sectors are separated by an energy gap of order ω, in the limit $\omega \gg D$, the admixture of photon sectors can be neglected; this is in essence the rotating wave approximation. Then, we can consider only the zero photon sector and obtain an effective Floquet Hamiltonian by computing the $m = m' = 0$ matrix element. The key point to notice is that only the superconducting term in (3) fails to commute with the chemical potential operator $\sum_i(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow})$. Evaluation of Eq. (6) within this rotating wave approximation (RWA) yields an effective Floquet Hamiltonian with exactly the same form as $\hat{H}_0$ with the pairing coupling Δ effectively renormalized to

$$\Delta_{\text{eff}} = \Delta J_0(2K/\omega). \qquad (7)$$

($J_0(x)$ is the zero order Bessel function of the first kind.)

We conclude from Eq. (7) that in Floquet systems one may induce a topological phase transition dynamically. Indeed, recall that the regime for a topological superconducting phase of $\hat{H}_0$, which supports MFs, requires the condition $V_z^2 > \Delta^2 + \mu^2$ ($\mu = \mu_L + 2\eta$) [6, 7, 32]. Even if initially this condition is not satisfied so that the system is in the topologically trivial state, the renormalization $\Delta \to \Delta_{\text{eff}}$ may make a topological phase possible since $\Delta_{\text{eff}} < \Delta$. Thus, periodic modulation of the chemical potential provides a way to tune the topological phase and so realize MFs by varying the parameter K/ω. The rescaling Eq. (7) holds only, of course, to the extent that off-diagonal couplings can be neglected; we address the generic case numerically below and show that more dramatic changes in behavior are entirely possible.

For periodic modulation of the Zeeman field, a similar analysis can be carried out by adding $\hat{H}_z(t) = K\cos(\omega t)\sum_i(\hat{n}_{i\uparrow} - \hat{n}_{i\downarrow})$ to $\hat{H}_0$. Since only the spin-orbit term in Eq. (3) does not commute with the Zeeman term, the spin-orbit parameter is modified in the effective Floquet Hamiltonian: $\lambda_{\text{SO}} \to \lambda_{\text{SO}}J_0(2K/\omega)$. Thus, periodic Zeeman modulation cannot induce a topological phase transition if one keeps only the zero photon sector. However, numerical investigation beyond the RWA [keeping all off-diagonal blocks of the effective Floquet Hamiltonian $\propto J_{m-m'}(2K/\omega)$] reveals that FMFs do, in fact, appear, and so we now turn to our numerical results.

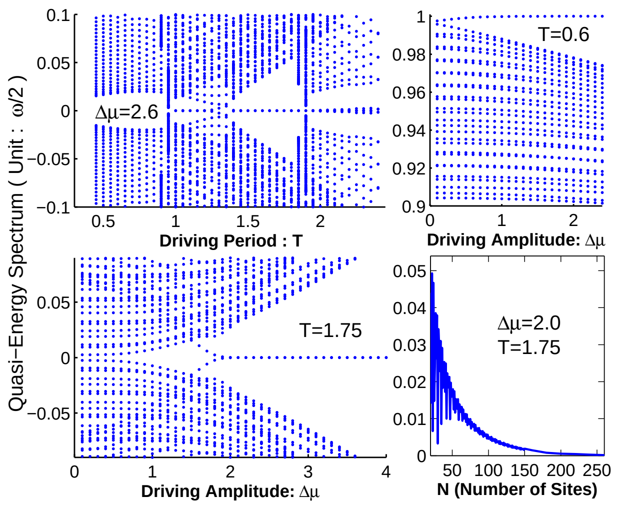

FIG. 1: (color online) Quasi-energy spectrum for square-wave driven chemical potential. Note the appearance of an $\epsilon_0 = 0$ solution for a wide range of parameters; this corresponds to the FMF localized at the end of the wire. Parameters: $\eta = 1.5$ (full band-width $D = 4\eta = 6.0$), $V_z = 1.0$, $\Delta = 1.0$, $\lambda_{\rm SO} = 1.2$, and $(\mu_1 + \mu_2)/2 = 0.5$. [Left panels]: quasi-energy near $\epsilon = 0$, as a function of driving period T for $\Delta\mu = |\mu_1 - \mu_2| = 2.6$ (upper), and as a function of driving amplitude $\Delta\mu$ for $T = 1.75$ (lower). [Right upper panel]: quasi-energy near $\epsilon = \omega/2$ as a function of driving amplitude $\Delta\mu$ for $T = 0.6$. [Right lower panel]: Finite size splitting (indicating the coupling between two FMFs at the two ends) for $\epsilon = 0$ mode as a function of the number of sites in the chain ($T = 1.75$, $\Delta\mu = 2.0$). The finite size splitting shows exponential suppression accompanied by oscillations. There are $N = 260$ sites in the chain. The experimental availability of those parameters is discussed in [38]. Note: the unit used for the quasi-energies is $\omega/2 = \pi/T$.

For numerical convenience we consider square-wave driving of the chemical potential or Zeeman field: $\mu = \mu_1$ for $nT < t < (n + 1/2)T$, and $\mu = \mu_2$ for $(n + 1/2)T < t < (n + 1)T$ (with $n = 0, 1, 2, ...$), and similarly for V_z. The evolution operator for the full period then reads $\hat{U}(T, 0) = e^{-i\frac{\hat{H}_2 T}{2\hbar}} e^{-i\frac{\hat{H}_1 T}{2\hbar}}$, and the quasi-energy spectrum ϵ_α is obtained numerically using Eq. (1) directly from Eq.(3) without RWA. In all cases here, the parameters at any instantaneous time correspond in the static system to the topologically trivial phase.

The numerical results ($\omega \sim D$) for periodically modulated chemical potential are shown in Fig. 1. Clearly, one obtains stable $\epsilon = 0$ Floquet Majorana zero modes for a large range of parameters (left panels). Since quasi-energy is only defined in an interval of ω (e.g. from $-\omega/2$ to $\omega/2$), another type of FMF exists at $\epsilon = \pm\omega/2$ with $e^{-i\omega t/2}\gamma_{\omega/2} = [e^{-i\omega t/2}\hat{\gamma}_{\omega/2}]^\dagger$ [10], as observed in the right panel of Fig. 1 (and SI [38]). Note that the parameters used in Fig. 1 are very far from those for which the RWA result Eq. (7) yields a FMF: here $V_z^2 - \mu^2 < 0$ at all times, so no renormalized Δ can yield a non-trivial phase. Nevertheless, FMF appear once $\Delta\mu$ surpasses a threshold $\Delta\mu_c$. The figure shows that the threshold for an $\epsilon = \omega/2$ FMF can be very small compared to that for an $\epsilon = 0$ FMF, and also that the quasi-energy gap can be tuned by varying $\Delta\mu$. The splitting of a $\epsilon = 0$ mode due to finite size effects is plotted in the right lower panel; it shows the expected decay of the level splitting as the number of sites, and hence the separation between the two FMF, increases.

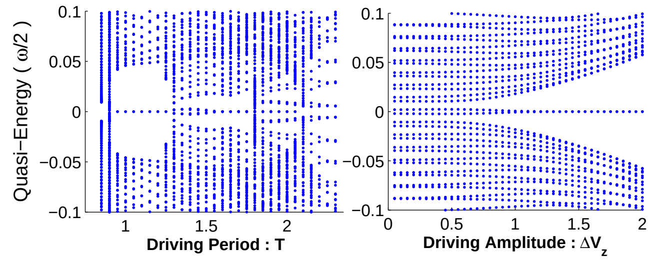

FIG. 2: (color online) Quasi-energy spectrum for square-wave driving of the Zeeman splitting, near $\epsilon = 0$. Parameters: $\eta = 1.4$ (full band-width $D = 4\eta = 5.6$), $(V_{z1} + V_{z2})/2 = 1.0$, $\Delta = 2.0$, $\lambda_{\rm SO} = 1.5$, and $\mu = 0.0$. Left panel: quasi-energy as a function of driving period T, for $\Delta V_z = |V_{z1} - V_{z2}| = 1.8$. Right panel: quasi-energy as a function of driving amplitude ΔV_z, for $T = 1.1$. There are 260 sites in the chain.

The quasi-energy spectrum with periodic Zeeman splitting is shown in Fig. 2. It also reveals FMFs. We stress once again that to obtain FMF in this case, the RWA is not enough and off-diagonal blocks of the Floquet Hamiltonian are crucial.

Floquet Topological Qubit and Non-Abelian braiding Statistics — A natural question is whether FMFs can form topological qubits, as their static counterparts do. FMF can certainly encode quantum information: an FMF exists at all instantaneous times, and neither chemical potential driving nor Zeeman driving changes the total fermion parity. Then, a more difficult question is whether FMFs obey non-Abelian braiding statistics. We will provide an argument for a 2D system, which can then be generalized to a 1D network following the argument for static MF [35].

Suppose that FMFs are moved (which can be achieved by tuning the driving potential on and off, or changing the driving amplitude,) along a path $R(t)$ with the Schrödinger equation $[\hat{H}(R(t), t) - i\partial_t]|\Phi(t)\rangle = 0$. The position of the FMF $R(t)$ is assumed to vary on a very slow time scale compared to the fast periodic driving. Then, it is convenient to separate the fast and slow time scales, and apply the two-time formalism of Floquet theory [29, 30]: $i\partial_t \to i\partial_t + i\partial_\tau$, where t indicates the fast time and τ denotes the slow time. Then the Schrödinger equation becomes

$$i\partial_\tau|\Phi(R(\tau), t)\rangle = \left[\hat{H}(R(\tau), t) - i\partial_t\right]|\Phi(R(\tau), t)\rangle. \quad (8)$$

It was pointed out by Breuer and Holthaus [29] (see also [36]) that a Floquet system follows a generalized adiabatic theorem. Define the instantaneous (for τ) quasi-energy eigenstates using the Floquet operator

$$\left[\hat{H}(R(\tau),t) - i\partial_t\right] |\phi_\alpha(R(\tau),t)\rangle = \epsilon_\alpha(R(\tau))|\phi_\alpha(R(\tau),t)\rangle. \tag{9}$$

Suppose the system is initially in a Floquet state $|\Phi(R(\tau=0),t)\rangle = |\phi_\alpha(R(\tau=0),t)\rangle$. Standard procedures in quantum mechanics can be applied to Floquet states as long as the extended inner product mentioned above, $\langle\langle\cdot|\cdot\rangle\rangle$, is used. Second order perturbation theory then yields [29, 36]

$$|\Phi(R(\tau),t)\rangle = e^{-i\theta_\alpha(\tau)}e^{-i\chi_\alpha(\tau)}\Big(|\phi_\alpha(R(\tau),t)\rangle - \sum_{\beta\neq\alpha}|\phi_\beta(R(\tau),t)\rangle\frac{\langle\langle\phi_\alpha(R(\tau))|i\partial_\tau|\phi_\beta(R(\tau))\rangle\rangle}{\epsilon_\beta(R(\tau))-\epsilon_\alpha(R(\tau))}\Big), \tag{10}$$

where $\theta_\alpha(\tau) = \int_0^\tau d\tau'\epsilon_\alpha(R(\tau'))$ is the dynamical phase, and $\chi_\alpha(\tau) = \int_0^\tau d\tau'\langle\langle\phi_\alpha(R(\tau'))|i\partial_{\tau'}|\phi_\alpha(R(\tau'))\rangle\rangle$ is the generalized Berry phase. Therefore, to avoid transitions to other quasi-energy states, the change in time scale τ must be slow and the quasi-energy gap should be large: $|\epsilon_\beta(R(\tau)) - \epsilon_\alpha(R(\tau))| \gg |\langle\langle\phi_\alpha(R(\tau))|i\partial_\tau|\phi_\beta(R(\tau))\rangle\rangle|$. We assume this condition is satisfied so that the system will stay in its initial Floquet state.

The Floquet Majorana excitations can be described by a Bogoliubov quasi-particle operator,

$$\hat{\gamma}^\dagger(t) = \int d\mathbf{r}\left[u(\mathbf{r},R(\tau),t)\hat{\psi}^\dagger(\mathbf{r}) + v(\mathbf{r},R(\tau),t)\hat{\psi}(\mathbf{r})\right], \tag{11}$$

where $\hat{\psi}^\dagger(\mathbf{r})$ ($\hat{\psi}(\mathbf{r})$) creates (annihilates) a fermion at $\mathbf{r}$, and $v = u^*$ for a MF. A $U(1)$ gauge transformation which changes the superconducting order parameter phase by 2π [37] is allowed by using the extended space of the Floquet system [38]. This causes a minus sign on both $\hat{\psi}^\dagger(\mathbf{r})$ and $\hat{\psi}(\mathbf{r})$, changing the sign of the FMF operator as well. Due to such multivaluedness, a branch cut is necessary to define the phase of the wave function. So, the exchange of two FMFs $\hat{\gamma}_i(t)$ and $\hat{\gamma}_j(t)$ can induce a transformation: $\hat{\gamma}_i(t) \to \hat{\gamma}_j(t)$ and $\hat{\gamma}_j(t) \to -\hat{\gamma}_i(t)$ (since one of the FMF, say $\hat{\gamma}_j(t)$, must pass through the branch cut). For a 1D network, the exchange of two FMFs (through a T-junction, for instance) flips the sign of the superconducting pairing term, which results in exactly the same transformation as in the 2D case [35].

Given two FMFs $\hat{\gamma}_1(t)$ and $\hat{\gamma}_2(t)$, one can form a non-local regular fermion $\hat{d}^\dagger(t) = (\hat{\gamma}_1(t) + i\hat{\gamma}_2(t))/\sqrt{2}$. Let $|G(t)\rangle$ be the Floquet BCS state which is annihilated by any Floquet quasi-energy operators. $|G(t)\rangle$ and $\hat{d}^\dagger(t)|G(t)\rangle$ form a two-fold degenerate space. The exchange of two MFs results in $|G(t)\rangle \to e^{i\varphi}|G(t)\rangle$ and $\hat{d}^\dagger(t)|G(t)\rangle \to e^{i\varphi}e^{i\pi/2}\hat{d}^\dagger(t)|G(t)\rangle$. The $\pi/2$ phase difference after the transformation signifies non-Abelian braiding statistics [39, 40].

The exchange of two MF can also induce an extra unitary evolution involving a non-Abelian Berry matrix [41]. The form of the matrix can be generalized to a Floquet system [38] by replacing $\langle\cdot|\cdot\rangle$ with $\langle\langle\cdot|\cdot\rangle\rangle$; the unitary evolution then reads

$$\hat{U}(\tau) = \mathbb{P}\exp\left[i\int_0^\tau \mathbf{M}(\tau')d\tau'\right] \tag{12}$$

where $\mathbb{P}$ denotes path-ordering and $\mathbf{M}_{\alpha\beta}(\tau) = \langle\langle\phi_\alpha(R(\tau))|i\partial_\tau|\phi_\beta(R(\tau))\rangle\rangle$ is the generalized non-Abelian Berry matrix [38]. We want to test whether $\mathbf{M}_{\alpha\beta}$ causes any extra phase difference that breaks the non-Abelian braiding statistics of FMFs. First, the non-diagonal matrix elements of $\mathbf{M}_{\alpha\beta}$ are zero since fermion parity is conserved (as emphasized above this is true for all driving scenarios). Second, we follow a procedure similar to that for a stationary MF [39, 40] where the odd parity element $i\langle\langle G|\hat{d}\ \partial_\tau\left(\hat{d}^\dagger|G\rangle\right)\rangle$ is written as the sum of the even parity element $i\langle\langle G|\partial_\tau|G\rangle\rangle$ and an extra term $i\langle\langle G|(\hat{d}\partial_\tau\hat{d}^\dagger)|G\rangle\rangle$. It is just this term that might affect the the phase difference $\pi/2$ and so the non-Abelian braiding statistics. By using Eq. (11) and the MF condition $v_i = u_i^*$ one finds

$$\langle\langle G|(\hat{d}\partial_\tau\hat{d}^\dagger)|G\rangle\rangle = \frac{2i}{T}\int_0^T dt\int d\mathbf{r}\mathrm{Re}(\mathrm{u}_1^*\partial_\tau\mathrm{u}_2 - \mathrm{u}_2^*\partial_\tau\mathrm{u}_1). \tag{13}$$

For two completely spatially separated FMFs (infinite wire), this term is zero, and the non-Abelian Berry phase does not affect the desired statistics of FMFs. For finite wires, a small correction due to the overlap of the two wave functions can induce error in FMF qubit manipulation, but it is exponentially small as the spatial separation of two FMFs increases [38].

Summary — Periodic modulation of the chemical potential or the Zeeman field appears to be a promising way to produce FMFs, both of which can be realized in 1D cold atom condensates. We find that Floquet MFs are robust and can be generated in a wide parameter range. This system may have potential for topological quantum computation since FMFs obey the same non-Abelian braiding statistics as their equilibrium counterparts.

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