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Dynamics of Majorana States in a Topological Josephson Junction

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Topological Josephson junctions carry 4π-periodic bound states. A finite bias applied to the junction limits the lifetime of the bound state by dynamically coupling it to the continuum. Another characteristic time scale, the phase adjustment time, is determined by the resistance of the circuit "seen" by the junction. We show that the 4π-periodicity manifests itself by an even-odd effect in Shapiro steps only if the phase adjustment time is shorter than the lifetime of the bound state. The presence of a peak in the current noise spectrum at half the Josephson frequency is a more robust manifestation of the 4π-periodicity, as it persists for an arbitrarily long phase adjustment time. We specify, in terms of the circuit parameters, the conditions necessary for observing the manifestations of 4π-periodicity in the noise spectrum and Shapiro steps measurements.

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Topological Josephson junctions have attracted much interest lately as a means of probing the zero-energy Majorana fermion states that exist at the surface of topological superconductors. Such topological superconductors may be realized via the proximity effect by combining conventional superconductors with two-dimensional (2D) topological insulators [1] or with nanowires in the presence of both strong spin-orbit coupling and a magnetic field [2, 3]. Recently several experiments have reported evidence of zero-energy states in nanowire-based systems [4–6]. To confirm their Majorana nature, additional experimental signatures are desirable.

In a topological Josephson junction, the Majorana bound states localized on either side of the junction hybridize and form an Andreev bound state whose energy $\epsilon_A(\varphi)$ is 4π-periodic in the phase difference φ between the two superconductors. Depending whether the state is occupied or empty, the energy of the junction is $\pm\epsilon_A(\varphi)/2$. In the presence of parity-changing processes, the occupation of the state may change. Thus, the equilibrium Josephson current displays the usual 2π-periodicity as the system follows the ground state. By contrast, if upon phase variation the system follows one branch of the spectrum, then 4π-periodicity should appear indeed. As a result, under dc bias voltage $V_{\rm dc}$, such a system has been predicted to manifest a fractional ac Josephson effect [1, 7, 8] at frequency $\omega_J/2 = eV_{\rm dc}/\hbar$, that is at half of the "usual" Josephson frequency. By the same token, in the presence of an additional ac bias with frequency Ω, one would expect an even-odd effect: namely only the even Shapiro steps at $eV_{\rm dc} = k\Omega$ ($k \in \mathbb{Z}$) should be visible in the current-voltage characteristics [8–10]. However, the application of a bias voltage inevitably couples the bound state to the continuum, thus causing its occupation to switch. The corresponding switching rate determines the lifetime of the bound state, τ_s. In addition to these intrinsic processes, the evolution of the phase difference across the junction depends on the properties of the circuit connecting the Josephson junction to the voltage source. Any non-zero resistance $\mathcal{R}$ of the connection allows for an adjustment of the phase difference over some characteristic time [11, 12] $\tau_R \propto \mathcal{R}^{-1}$.

In this work, we evaluate the lifetime of Majorana bound states, τ_s, limited by their dynamic coupling to the continuum. This mechanism gains importance in nearly-ballistic junctions and leads to a strong dependence of τ_s on the applied voltage. We show that the transport properties of the junction crucially depend on two characteristic time scales, τ_s and τ_R. If $\tau_s \gg \tau_R$, Majorana states lead to an even-odd effect in the height of Shapiro steps, in agreement with Refs. [8–10]. By contrast, if $\tau_s \ll \tau_R$, all Shapiro steps are suppressed. However, signatures of the 4π-periodicity are still visible in the noise spectrum which, under dc voltage bias, displays peaks at $\omega = \pm\omega_J/2$, as was seen in numerical simulations [13]. Here we develop an analytical theory for the noise spectrum and find the dependence of the peak widths on τ_s. While noise measurements in the GHz-range are not easy to realize, the low-frequency noise is more accessible. The down-conversion of the noise peak to $\omega = \pm(\omega_J/2 - \Omega)$ may be achieved by adding a small ac bias of frequency Ω.

To examine the non-adiabatic transitions between the Majorana state and quasiparticles continuum, we consider the helical edge state of a 2D topological insulator in which superconductivity has been induced by two superconducting contacts in order to create a topological Josephson junction of length L. The system is described by the Hamiltonian

$$\mathcal{H} = vp\sigma_z\tau_z - eU(x,t)\tau_z + M(x)\sigma_x + \Delta(x)e^{i\phi(x,t)\tau_z}\tau_x. \tag{1}$$

Here, v is the Fermi velocity, p is the momentum operator, $U(x,t) = V(t)[\theta(-x) - \theta(x-L)]/2$ is the electric potential, $M(x) = M\theta(x)\theta(L-x)$ is a transverse magnetic field within the junction, $\Delta(x) = \Delta[\theta(-x)+\theta(x-L)]$ and $\phi(x,t) = \varphi(t)[\theta(-x) - \theta(x-L)]/2$, with $\dot{\varphi}(t) = 2eV(t)$, are the amplitude and phase of the superconducting order parameter in the left and right leads, and σ_i, τ_j ($i, j = x, y, z$) are Pauli matrices acting in the spin and particle/hole spaces, respectively. All energies are measured from the chemical potential.

We concentrate on the case of a short junction, $L \ll \xi$, where $\xi = v/\Delta$ is the superconducting coherence length. In equilibrium ($V = 0$), such a junction hosts a single Andreev bound state with energy

$$\epsilon_A(\varphi) = \sqrt{D}\Delta\cos(\varphi/2), \tag{2}$$

where D is the transmission probability of the junction which depends on its length and on the magnitude of the transverse field. Thus, the minimal gap δ between the bound state and the continuum at $\varphi = 2n\pi$ ($n \in \mathbb{Z}$) is given as $\delta = \Delta(1-\sqrt{D})$. In the following we consider a highly transmitting junction where $\delta \approx \Delta R/2$ and $R = 1 - D \sim (ML/v)^2$ is the reflection probability.

Out of equilibrium, non-adiabatic transitions between the Andreev bound state and the continuum are induced. These transitions change the occupation of the bound state and thus lead to switching between the two current branches, $I(\varphi) = \pm I_J \sin(\varphi/2)$, where $I_J = e\sqrt{D}\Delta/2$. At dc bias $eV_{\rm dc} \ll \Delta$, violation of the adiabaticity occurs in narrow intervals $|\varphi - 2\pi n| \ll \pi$ of the time-varying phase $\varphi = 2eV_{\rm dc}t$. To find the corresponding probability of a non-adiabatic transition between the localized Majorana state and continuum, we concentrate on the case $n = 0$, corresponding to the time interval $|t| \ll \pi/(eV_{\rm dc})$. Using a gauge transformation $\mathcal{H} \to \tilde{\mathcal{H}} = U^\dagger \mathcal{H} U - iU^\dagger \dot{U}$ with $U = \exp[i\phi\tau_z/2]$ and taking the limit $L \to 0$ (keeping R fixed), we obtain for the said interval of φ the simplified Hamiltonian $\tilde{\mathcal{H}} = vp\sigma_z\tau_z + \Delta\tau_x + v[(\varphi/2)\sigma_z + \sqrt{R}\sigma_x]\delta(x)$. Furthermore, at $eV_{\rm dc} \ll \Delta$, only states close to the continuum edge, $v|p| \ll \Delta$, are relevant. Thus, after diagonalizing the bulk Hamiltonian, we can restrict ourselves to a 2×2 subspace of the initial spin and particle/hole space,

$$H = \Delta + \frac{v^2p^2}{2\Delta} + v\left(\frac{1}{2}\varphi\sigma_z + \sqrt{R}\sigma_x\right)\delta(x). \tag{3}$$

Eq. (3) describes a spin-degenerate continuum with quadratic dispersion, in the presence of a spin-dependent local potential. The first term in this potential accounts for the phase shift across the barrier in a gauge with zero electric potential in the leads and a vector potential localized at the barrier. The second term describes the magnetic barrier.

For a fixed phase, the Hamiltonian (3) accommodates a single bound state with energy $\epsilon_A(\varphi) = \Delta\left(1 - \varphi^2/8 - R/2\right)$, in agreement with Eq. (2) at $R, \varphi^2 \ll 1$. A particle occupying this bound state at time $t \to -\infty$ (within the simplified model) has a probability s to escape to the continuum as the phase increases. The problem is, thus, a generalization to a two-band model of the transition from a discrete state to a continuum, considered by Demkov and Osherov [14]. Dimensional analysis shows that the transition (or switching) probability is determined by the adiabaticity parameter $\lambda = R^{3/2}\Delta/(eV_{\rm dc})$. Below we find this probability in two limiting cases of the parameter λ.

Let us start with the anti-adiabatic regime, $\lambda \ll 1$. At $\lambda = 0$ the spin bands in Eq. (3) are decoupled. At times $t < 0$, the bound state belongs to the spin-up band whereas, at times $t > 0$, the bound state belongs to the spin-down band. The spin-up bound state is described by a wavefunction $|\psi_\uparrow(t)\rangle$ whose projection on the position of the local potential is

$$\langle x = 0|\psi_\uparrow(t)\rangle = \frac{\tau}{\sqrt{2\pi\ell}}\int_{\mathcal{C}} d\omega\, e^{i\omega t}\, e^{i(-2\omega\tau)^{3/2}/3}|\uparrow\rangle. \tag{4}$$

The wave function $|\psi_\downarrow(t)\rangle$ of the spin-down bound state is related to $|\psi_\uparrow(t)\rangle$ by time reversal. Here, the characteristic length and time scales are given by $\ell = v/[\Delta^2 eV_{\rm dc}]^{1/3}$ and $\tau = 1/[\Delta(eV_{\rm dc})^2]^{1/3}$, respectively. Furthermore, $\mathcal{C}$ is a contour in the complex ω-plane [14] that starts and ends at infinity, with arguments $\pi < \theta < 5\pi/3$ and $0 < \theta < \pi/3$, respectively, and avoids the branch cut along the positive real axis. As the spin bands are decoupled, a particle occupying the (spin-up) bound state at $t = -\infty$, has probability $1 - s = 0$ to occupy the (spin-down) bound state as $t \to \infty$.

A finite λ couples the two bands and, thus, enables spin flips. The switching probability s can be obtained from the overlap $c_\downarrow(t) = \langle\psi_\downarrow(t)|\psi(t)\rangle$ of the exact wavefunction, $|\psi(t)\rangle$, where $|\psi(-\infty)\rangle = |\psi_\uparrow(-\infty)\rangle$, with the wavefunction of the spin-down bound state, $|\psi_\downarrow(t)\rangle$, through $s = 1 - |c_\downarrow(\infty)|^2$. At $\lambda \ll 1$, $c_\downarrow$ can be computed perturbatively using

$$\dot{c}_\downarrow(t) \approx i\langle\psi_\downarrow(t)|v\sqrt{R}\sigma_x\delta(x)|\psi_\uparrow(t)\rangle. \tag{5}$$

Solving the differential equation (5) to obtain $c_\downarrow(\infty)$ and computing s, we find $s \approx 1 - 1.05\,\lambda^{2/3}$. The time scale over which the transition happens is $\tau_{\rm t} \sim \tau$.

In the quasi-adiabatic regime, $\lambda \gg 1$, it is convenient to expand the exact wavefunction in the adiabatic basis of Eq. (3),

$$|\psi(t)\rangle = c_A(t)|\psi_A(t)\rangle + \sum_{p\sigma} c_{p\sigma}(t)|\psi_{p\sigma}(t)\rangle. \tag{6}$$

Here $|\psi_A(t)\rangle$ and $|\psi_{p\sigma}(t)\rangle$ are the adiabatic wavefunctions for the bound state and the doubly degenerate states of the continuum, respectively. [Note that $|\psi_A(\mp\infty)\rangle = |\psi_{\uparrow,\downarrow}(\mp\infty)\rangle$.] The switching probability s is related to the amplitudes $c_{p\sigma}(\infty)$ of the continuum states in Eq. (6)

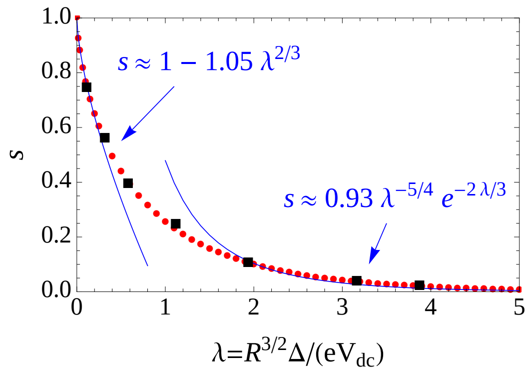

FIG. 1: Switching probability s as a function of the adiabaticity parameter λ. Dots: s found from a numerical solution of the Schrödinger equation with Hamiltonian (3). Lines: asymptotic expressions for s, see text. Squares: s extracted from the "brute-force" evaluation of the noise spectrum by solving numerically the problem of multiple Andreev reflections [13] and fitting the result by Eq. (14), see [16] for details.

through $s = \sum_{p\sigma} |c_{p\sigma}(\infty)|^2$, using the initial conditions $c_A(-\infty) = 1$ and $c_{p\sigma}(-\infty) = 0$. At $\lambda \gg 1$, using

$$\dot{c}_{p\sigma}(t) \approx i\dot{\varphi}(t)\,\frac{\langle\psi_{p\sigma}(t)|\frac{\partial H}{\partial\varphi(t)}|\psi_A(t)\rangle}{\epsilon_{p\sigma} - \epsilon_A(t)}\,e^{-i\int^t ds\,[\epsilon_{p\sigma}-\epsilon_A(s)]}\,, \tag{7}$$

the amplitudes $c_{p\sigma}(\infty)$ are expressed through integrals that may be evaluated by a saddle point method. We obtain $s \simeq 0.93\,\lambda^{-5/4}e^{-2\lambda/3}$. Furthermore, we can identify the time scale over which the transition happens, $\tau_{\rm t} \sim \sqrt{R}/(eV_{\rm dc})$.

At arbitrary λ, the switching probability can be obtained numerically by discretizing Eq. (3) on a tight-binding lattice and solving the corresponding Schrödinger equation numerically. The result, together with the asymptotes obtained above, is shown in Fig. 1.

Using the fact that the transition time $\tau_{\rm t}$ is much shorter than the Josephson oscillation period, $\tau_{\rm t} \ll \pi/(eV_{\rm dc})$, we may now write an effective discrete Markov model for the bound state dynamics, cf. Fig. 2. Using a discrete time evolution we assume that if the state is filled, at phase $\varphi_{2n} = 4n\pi$, there is a probability s of the particle to escape from the bound state to the continuum, whereas if the state is empty, at time $\varphi_{2n+1} = (4n+2)\pi$, there is a probability s of a particle from the continuum filling the bound state. Thus,

$$\begin{pmatrix} P_{2n+1} \\ Q_{2n+1} \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1-s \end{pmatrix} \begin{pmatrix} P_{2n} \\ Q_{2n} \end{pmatrix}, \tag{8a}$$

$$\begin{pmatrix} P_{2n} \\ Q_{2n} \end{pmatrix} = \begin{pmatrix} 1-s & 0 \\ s & 1 \end{pmatrix} \begin{pmatrix} P_{2n-1} \\ Q_{2n-1} \end{pmatrix}. \tag{8b}$$

Here P_n is the probability for the state to be occupied, and $Q_n = 1 - P_n$ is the probability for the state to be empty at phases $\varphi_n < \varphi(t) < \varphi_{n+1}$, corresponding to $n = \mathrm{Int}\,[\varphi(t)/(2\pi)]$. Solving these equations iteratively,

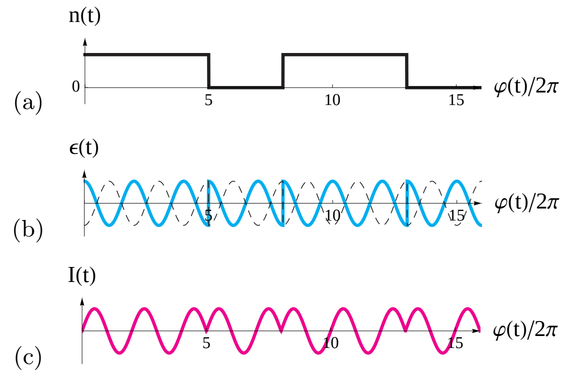

FIG. 2: Schematic view of the switching processes due to the coupling with the continuum: (a) occupation n of the bound state, (b) energy ϵ of the system, and (c) Josephson current I as a function of time t under dc bias voltage $V_{\rm dc}$.

we obtain

$$P_{2(n+k)} = P^\infty_{2n} + (1-s)^{2k}\,(P_{2n} - P^\infty_{2n})\,, \tag{9a}$$

$$P_{2(n+k)+1} = P^\infty_{2n+1} + (1-s)^{2k+1}\,(P_{2n} - P^\infty_{2n}). \tag{9b}$$

The long-time probabilities at $k \gg -1/\ln(1-s)$, corresponding to $t \gg \tau_s = -2\pi/[eV_{\rm dc}\ln(1-s)]$, are 4π-periodic and independent of the initial state: $P^\infty_n = [1-(-1)^n s/(2-s)]/2$.

In order to determine the transport properties of the junction, the switching time τ_s has to be compared to other characteristic time scales of the system. In particular, if the junction is embedded into a circuit with a resistance $\mathcal{R}$ in series, the phase difference across the junction may adjust over a typical time scale $\tau_R \propto \mathcal{R}^{-1}$.

If $\tau_s \gg \tau_R$, switching may be neglected (i.e., the current may be obtained using the initial occupations P_0/Q_0). Computing the dc current in the presence of an applied voltage $V(t) = V_{\rm dc} + V_{\rm ac}\cos(\Omega t)$ with $V_{\rm ac} \ll V_{\rm dc}$ then yields the even-odd effect discussed in Refs. [8–10]. Namely, taking into account a finite resistance $\mathcal{R}$, the average current reads

$$I_{\rm dc} = \sum_k \frac{\delta V_k}{\mathcal{R}} \left\{ 1 - \theta\left[1 - \left(\frac{\mathcal{R}I_k}{\delta V_k}\right)^2\right] \sqrt{1 - \left(\frac{\mathcal{R}I_k}{\delta V_k}\right)^2} \right\}, \tag{10}$$

where $I_k = I_J|J_k(\alpha)|$ is the height of the Shapiro step at $eV_{\rm dc} = k\Omega$ and $\delta V_k = V_{\rm dc} - k\Omega/e$. Here J_k are the Bessel functions and $\alpha = eV_{\rm ac}/\Omega$. The characteristic time scale may be identified as [15, 16] $\tau_R^{(k)} = 1/(e\mathcal{R}I_k)$ at $eV_{\rm dc} \sim k\Omega$.

A small resistance satisfying the relation $\mathcal{R}I_J \ll \Omega$ is advantageous for the resolution of Shapiro steps. Furthermore, long switching times require $s \ll 1$. In the small-s regime, the time scale τ_s decreases as $\exp[2R^{3/2}\Delta/(3k\Omega)]$ with increasing k. On the other hand, at small ac perturbation, $\alpha \ll 1$, the time scale $\tau_R^{(k)}$ increases exponentially with k. The crossover from

$\tau_s \gg \tau_R$ to the opposite limit, $\tau_s \ll \tau_R$, upon increasing $V_{\rm dc}$ may occur without violation of the condition $s \ll 1$. This restricts the number of observable Shapiro steps in the current-voltage characteristics, as we show now. At $\tau_s \ll \tau_R$, we may use the long-time probabilities P^∞/Q^∞ to compute the current and take the limit $\tau_R \to \infty$. At long times, the average current is 2π-periodic,

$$\begin{aligned} \langle I(t)\rangle &= I_J \sin\frac{\varphi(t)}{2}\left\{Q^\infty_{{\rm Int}[\varphi(t)/(2\pi)]} - P^\infty_{{\rm Int}[\varphi(t)/(2\pi)]}\right\} \\ &= \frac{sI_J}{2-s}\left|\sin\frac{\varphi(t)}{2}\right|. \end{aligned} \tag{11}$$

More importantly, Eq. (11) shows that the current is proportional to the switching probability s, when $s \ll 1$.

The result (11) remains valid in the presence of microwave irradiation as long as $V_{\rm ac} \ll V_{\rm dc}$ and $\Omega \ll \delta$. The first condition ensures that the ac bias only weakly perturbs the phase velocity $\dot\varphi$. The second condition ensures that ionization of the Majorana level by the ac perturbation would require the absorption of a large number of photons $\sim \delta/\Omega$ and, thus, has a small probability. Note that, for $s \ll 1$, the second condition is always satisfied at $\Omega \sim eV_{\rm dc}$. Then, s may be approximated by its value at dc bias only. As a consequence, Eq. (11) implies that Shapiro steps are strongly suppressed, $\langle I_k\rangle \propto s$.

In order to reveal signatures of the 4π-periodicity in the regime $\tau_s \ll \tau_R$, we now turn to the current noise spectrum,

$$S(\omega) = 2\int_0^\infty d\tau \cos(\omega\tau)\overline{\langle \delta I(t)\delta I(t+\tau)\rangle}, \tag{12}$$

where $\delta I = I - \langle I\rangle$ and the bar denotes time-averaging. It may be obtained from the correlator $\langle I(\varphi_1)I(\varphi_2)\rangle = I_J^2 \sin(\varphi_1/2)\sin(\varphi_2/2)[Q^\infty_{n_1}x_{n_2}(P_{n_1}=0) - P^\infty_{n_1}x_{n_2}(P_{n_1}=1)]$ at $\varphi_1 < \varphi_2$, where $n_i = {\rm Int}[\varphi_i/(2\pi)]$. Using the conditional probabilities obtained from Eqs. (9), we find

$$\langle \delta I(\varphi_1)\delta I(\varphi_2)\rangle = \frac{4I_J^2(1-s)}{(2-s)^2}\sin\frac{\varphi_1}{2}\sin\frac{\varphi_2}{2}(1-s)^{n_2-n_1}. \tag{13}$$

At dc bias only, the noise spectral density evaluates to

$$S(\omega) = \frac{4sI_J^2}{\pi(2-s)}\frac{(eV_{\rm dc})^3}{[\omega^2-(eV_{\rm dc})^2]^2}\frac{4\cos^2\frac{\pi\omega}{2eV_{\rm dc}}}{4\cos^2\frac{\pi\omega}{2eV_{\rm dc}}+\frac{s^2}{1-s}}. \tag{14}$$

If $s \ll 1$, it has sharp peaks at $\omega = \pm eV_{\rm dc}$, i.e., at half of the "usual" Josephson frequency:

$$S(\omega) \simeq \frac{I_J^2}{2}\frac{seV_{\rm dc}/\pi}{(\omega \mp eV_{\rm dc})^2 + (seV_{\rm dc}/\pi)^2} \tag{15}$$

at $|\omega \mp eV_{\rm dc}| \ll eV_{\rm dc}$. In particular, the peak width is $2seV_{\rm dc}/\pi$. The position of the peak reveals the 4π-periodicity of the Andreev bound state whereas the inverse width characterizes its lifetime $\tau_s \propto s^{-1}$. The peak in the noise is due to the transient 4π-periodic behavior [17] of the current at times smaller than the lifetime of the bound state.

Under microwave irradiation, the peak may be shifted to smaller frequencies. In particular, in the limit $V_{\rm ac} \ll V_{\rm dc}, \Omega/e$, we find

$$S(\omega) \simeq \frac{I_J^2}{2}J_k^2(\alpha)\frac{seV_{\rm dc}/\pi}{[\omega \mp (eV_{\rm dc} - k\Omega)]^2 + (seV_{\rm dc}/\pi)^2} \tag{16}$$

at $|\omega \mp (eV_{\rm dc} - k\Omega)| \ll eV_{\rm dc}$. As above, the peak width is set by the lifetime of the bound state which, thus, may be probed by noise measurements. Eq. (16) holds for frequencies ω not too close to zero. In the limit $\omega \to 0$, additional features related to the Shapiro steps may appear [16].

While we considered the helical edge states of 2D topological insulators, the model is also applicable to nanowires [2, 3] with strong spin-orbit coupling and a Zeeman energy much larger than Δ. Note that, in addition to the non-adiabatic processes that we considered, non-adiabatic processes in the vicinity of $\varphi = (2n+1)\pi$ become important if the zero-energy crossing is split due to the presence of additional Majorana modes at the ends of the wire [10, 17–19]. In particular, in order to see signatures associated with the 4π-periodicity, the probability of Landau-Zener tunneling across the gap at $\varphi = (2n+1)\pi$ would have to be large while the switching probability due to the coupling with the continuum, discussed in this work, remains small.

To summarize, we analyzed the electron transport through a topological Josephson junction imbedded in a realistic circuit. The Majorana states associated with the junction may lead to two effects, namely (1) the even-odd effect in the Shapiro steps, and (2) a peak in the current noise spectrum at half of the usual Josephson frequency. We found the conditions for these effects to occur. For that we identified the characteristic relaxation time scales for the junction: the lifetime of the bound state originating in its dynamic coupling to the continuum, and the phase adjustment time caused by the resistive environment provided by the circuit. The even-odd effect in the Shapiro steps requires the phase adjustment time to be shorter than the lifetime. For longer phase adjustment times, the even-odd effect is lost. The charactersitic peak in the noise spectrum is less sensitive to the ratio of the two relaxation times. In the limit of long phase adjustment time, the width of the peak provides a measure for the rate of parity-changing processes. The peak at $\omega = eV_{\rm dc}/\hbar$ should be seen easily if the dc voltage satisfies the condition $eV_{\rm dc} < R^{3/2}\Delta$, where R is the reflection probability. The peak position can be down-shifted in frequency by applying an additional ac bias to the circuit.

In the final stages of preparing the manuscript, we became aware of Ref. [20] considering related effects in nanowire-based topological Josephson junctions.

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