Dependence of the Radiation Pressure on the Background Refractive Index

Kevin J. Webb

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Kevin J. Webb

School of Electrical and Computer Engineering,

Purdue University,

465 Northwestern Avenue,

West Lafayette, IN 47907, USA

webb@purdue.edu

Abstract

The 1978 experiments by Jones and Leslie showing that the radiation pressure on a mirror depends on the background medium refractive index have yet to be adequately explained using a force model and have provided a leading challenge to the Abraham form of the electromagnetic momentum. Those experimental results are predicted for the first time using a force representation that incorporates the Abraham momentum by utilizing the power calibration method employed in the Jones and Leslie experiments. Extending the same procedure, the polarization and angle independence of the experimental data is also explained by this model. Prospects are good for this general form of the electromagnetic force density to be effective in predicting other experiments with macroscopic materials. Furthermore, the rigorous representation of material dispersion makes the representation important for metamaterials that operate in the vicinity of homogenized material resonances.

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Radiation pressure is related to a change in momentum (between incident and emitted photons), and it was first measured for a silver mirror more than one century ago [1]. There has been substantial attention given to models (see, for example, [2–7]), but some important issues remain. A key dilemma has been the determination of a satisfactory model, or perhaps more explicitly, the correct interpretation of an existing model, to explain the measured dependence of the force on the background refractive index in experiments by Jones and Leslie [8], which follow earlier work by Jones and Richards [9].

Explaining the Jones and Leslie experiments [8] has become tantamount to resolving whether to use the Abraham [10, 11] or Minkowski [12, 13] momentum forms. Jones [14] presents a nice historical summary of various contributions, making the point that prediction of force experiments is the requirement of any theory. Thus far, there has not been an adequate explanation for the Jones and Leslie experiments, beyond the apparent consistency with the canonical momentum [15].

Associating the Abraham form of the electromagnetic momentum ($\mathbf{E} \times \mathbf{H}/c^2$, with c the speed of light in vacuum) with the electromagnetic energy in nondispersive media yields the single photon momentum magnitude of $\hbar k_0/n$, where n is refractive index, $\hbar = h/2\pi$, with h being Planck's constant, and k_0 is the free space wave number. Doing likewise with the Minkowski momentum ($\mathbf{D} \times \mathbf{B}$) gives a momentum of $n\hbar k_0$. Atoms have been measured to have a recoil momentum of $n\hbar k_0$ [16], important in atom interferometry with optical gratings and consistent with the deBroglie momentum.

A key point in the assignment of an electromagnetic momentum is the coupling of various physical systems and the fact that conservation principles apply to the superposition of these. This has been treated nicely with a virtual power concept [2], which provides basic insight into the separation of mechanical and electromagnetic effects [4]. The delineation into a quantum mechanical canonical momentum that produces spatial translations [17] and kinetic momenta associated with Abraham or Minkowski have been proposed [15, 18]. However, it has been noted that only the (dispersionless) canonical momentum of the photon appears to explain the Jones and Leslie experiments [15]. Of general significance, the influence of dispersion must be incorporated to determine the momentum imparted [19]. At this point, the Jones and Leslie radiation pressure experiments [8] remain unexplained, except for the qualitative similarity to the photon momentum. More generally, there remain basic questions about how to describe electromagnetic forces in dispersive materials, or more specifically, whether current descriptions can explain experiments.

General expressions for the electromagnetic force are developed in this Letter that can be applied to inhomogeneous, anisotropic and dispersive media. It is shown that an interpretation of the resulting force expression that incorporates the Abraham momentum can explain the dependence of the radiation pressure on refractive index in the experiments of Jones and Leslie [8]. This picture is expanded to describe why Jones and Leslie concluded from their experiments that there was no dependence of the force on polarization or angle of incidence [8]. While it has been noted that the Abraham momentum fails to predict the Jones and Leslie results [15], the analysis here shows that the Abraham momentum incorporated into a general force expression explains these experiments. Importantly, this model could be used to predict other force results, whereas the observation that the near-normal incidence results from Jones and Leslie are consistent with the incident photons carrying a canonical momentum of $n\hbar k_0$ [15], while appealing, cannot directly be applied to determine electromagnetic forces. Likewise, the separation of the photon momentum into canonical (identified as Abraham) and kinetic (identified as Minkowski) momenta [18] does not provide a description for force. Other work has investigated issues surrounding the Jones and Leslie experiments, including the force on a perfect conductor in a background medium [20], and that on other materials as being due to the impedance mismatch [21, 22]. This leads to the primary contribution of this Letter, the explanation of all experimental results obtained by Jones and Leslie for the first time and using a rigorous and general force model stemming from Maxwell's equations with homogenized material parameters.

The Jones and Leslie [8] experiments had various improvements over earlier work by Jones and Richards [9]. The more recent experiments were designed to use a laser (632.8 nm) and explore the dependence of the radiation pressure on a mirror on the background dielectric material in a precise way. The ratio of the force on the mirror in the dielectric to that with the mirror in air was measured for carefully selected liquid dielectrics that had distinct phase and group velocities. The measured data were calibrated to compensate for various lossless. All results suggested that pressure is proportional to the background refractive index (the phase velocity ratio) and that it is independent of polarization.

A significant body of literature supports the use of the Abraham form of the momentum (see [2, 23], for example). While Abraham did not consider the frequency dependence of the material properties and in fact asked the question as to the impact [10], Einstein and

Laub [24], Penfield and Haus [2] and others [22, 25–29] have done so. The development of the non-relativistic form of the force density based on the Abraham momentum is presented here because this forms a basis for the description of the Jones and Leslie results.

Maxwell's equations with all source terms on the right-hand side can be written

$$\nabla \times \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} = -\mu_0 \frac{\partial \mathbf{M}}{\partial t} \tag{1a}$$

$$\nabla \times \mathbf{H} - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \frac{\partial \mathbf{P}}{\partial t} + \mathbf{J} \tag{1b}$$

$$\epsilon_0 \nabla \cdot \mathbf{E} = -\nabla \cdot \mathbf{P} + \rho \tag{1c}$$

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}, \tag{1d}$$

with $\mathbf{E}$ the electric field, $\mathbf{H}$ the magnetic field, $\mathbf{P}$ the polarization, $\mathbf{M}$ the magnetization, $\mathbf{J}$ the source electric current density, ρ the free electric charge density, μ_0 the permeability of free space, and ϵ_0 the permittivity of free space. Note that material dispersion and loss is incorporated into the polarization and magnetization through the frequency domain representation for these quantities. Taking the cross product of $\epsilon_0\mathbf{E}$ with (1a) and $\mu_0\mathbf{H}$ with (1b), and adding the resulting equations, gives

$$\begin{aligned}\epsilon_0 \mathbf{E} \times (\nabla \times \mathbf{E}) + \mu_0 \mathbf{H} \times (\nabla \times \mathbf{H}) \; + \; \mu_0\epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{H}}{\partial t} - \mu_0\epsilon_0 \mathbf{H} \times \frac{\partial \mathbf{E}}{\partial t} \\ = -\mu_0\epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{M}}{\partial t} + \mu_0 \mathbf{H} \times \frac{\partial \mathbf{P}}{\partial t} + \mu_0 \mathbf{H} \times \mathbf{J}.\end{aligned} \tag{2}$$

The momentum-flow tensor of the electromagnetic field, the Maxwell stress tensor, is [2]

$$\mathbf{T_e} = \frac{1}{2}\left(\epsilon_0 E^2 + \mu_0 H^2\right) \mathbf{I} - \epsilon_0 \mathbf{E}\mathbf{E} - \mu_0 \mathbf{H}\mathbf{H}. \tag{3}$$

The triple cross product terms in (2) can therefore be expressed as

$$\epsilon_0 \mathbf{E} \times (\nabla \times \mathbf{E}) + \mu_0 \mathbf{H} \times (\nabla \times \mathbf{H}) = \nabla \cdot \mathbf{T_e} + \epsilon_0 \left(\nabla \cdot \mathbf{E}\right) \mathbf{E} + \mu_0 \left(\nabla \cdot \mathbf{H}\right) \mathbf{H}. \tag{4}$$

Taking the Abraham form of the momentum density associated with the electromagnetic field [2],

$$\mathbf{G_e} = \mu_0\epsilon_0 \mathbf{E} \times \mathbf{H}, \tag{5}$$

leads to

$$\frac{\partial \mathbf{G_e}}{\partial t} = \mu_0\epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{H}}{\partial t} - \mu_0\epsilon_0 \mathbf{H} \times \frac{\partial \mathbf{E}}{\partial t}. \tag{6}$$

Using (4) and (6), (2) can re-written as

$$\nabla \cdot \mathbf{T_e} + \epsilon_0 \left(\nabla \cdot \mathbf{E}\right) \mathbf{E} + \mu_0 \left(\nabla \cdot \mathbf{H}\right) \mathbf{H} + \frac{\partial \mathbf{G_e}}{\partial t} = -\mu_0 \epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{M}}{\partial t} + \mu_0 \mathbf{H} \times \frac{\partial \mathbf{P}}{\partial t} + \mu_0 \mathbf{H} \times \mathbf{J}. \quad (7)$$

The kinetic force density due to the electromagnetic fields, or the conservation of momentum (density), can now be written in terms of $\mathbf{T_e}$ and $\mathbf{G_e}$ as

$$\begin{aligned} \mathbf{f} &= -\mathbf{f}_e = -\nabla \cdot \mathbf{T_e} - \frac{\partial \mathbf{G_e}}{\partial t} \\ &= \mu_0 \epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{M}}{\partial t} - \mu_0 \mathbf{H} \times \frac{\partial \mathbf{P}}{\partial t} + \epsilon_0 \left(\nabla \cdot \mathbf{E}\right) \mathbf{E} + \mu_0 \left(\nabla \cdot \mathbf{H}\right) \mathbf{H} - \mu_0 \mathbf{H} \times \mathbf{J}. \end{aligned} \quad (8)$$

In an open system, partner polarization and magnetization charges outside the (differential) volume would produce additional forces [2]. Substituting $\epsilon_0 \left(\nabla \cdot \mathbf{E}\right)$ and $\nabla \cdot \mathbf{H}$ from (1c) and (1d) into (8), using

$$-\left(\nabla \cdot \mathbf{P}\right) \mathbf{E} = -\nabla \cdot \left(\mathbf{PE}\right) + \left(\mathbf{P} \cdot \nabla\right) \mathbf{E} \quad (9)$$

$$-\left(\nabla \cdot \mathbf{M}\right) \mathbf{H} = -\nabla \cdot \left(\mathbf{MH}\right) + \left(\mathbf{M} \cdot \nabla\right) \mathbf{H}, \quad (10)$$

and identifying $\mathbf{T}_p = -\mathbf{PE}$ and $\mathbf{G}_p = 0$ for polarization and $\mathbf{T}_m = -\mu_0 \mathbf{MH}$ and $\mathbf{G}_m = 0$ for magnetization leads to a kinetic force density

$$\begin{aligned} \mathbf{f} &= -\left(\mathbf{f}_e + \mathbf{f}_p + \mathbf{f}_m\right) \\ &= \frac{\partial \mathbf{P}}{\partial t} \times \mu_0 \mathbf{H} - \frac{\partial \mu_0 \mathbf{M}}{\partial t} \times \epsilon_0 \mathbf{E} \\ &\quad + \rho \mathbf{E} - \mu_0 \mathbf{H} \times \mathbf{J} + \left(\mathbf{P} \cdot \nabla\right) \mathbf{E} + \mu_0 \left(\mathbf{M} \cdot \nabla\right) \mathbf{H}. \end{aligned} \quad (11)$$

The force density in (11) is due to Einstein and Laub [24], and has been used by others [2, 22, 25–29], and the relativistic form has been derived [2]. The three coupled systems in (11), each with a stress tensor, result in an net stress tensor given by $\mathbf{T}_e + \mathbf{T}_p + \mathbf{T}_m$. Note that arbitrary material dispersion can be incorporated through the time derivatives of $\mathbf{P}$ and $\mathbf{M}$ in (8) and (11). For plane waves in locally homogeneous isotropic media and referring to (8), $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{H} = 0$. However, for general material arrangements and beam profiles, $(\mathbf{P} \cdot \nabla)\mathbf{E}$ and $(\mathbf{M} \cdot \nabla)\mathbf{H}$ in (11) can be nonzero. In the discussion here for the dielectric case, $(\mathbf{P} \cdot \nabla)\mathbf{E}$ at the surface or within the liquid is assumed not to contribute to the force on the mirror in the Jones and Leslie experiments [8], in which case from (8) or (11),

$$\mathbf{f} = \frac{\partial \mathbf{P}}{\partial t} \times \mu_0 \mathbf{H} - \frac{\partial \mu_0 \mathbf{M}}{\partial t} \times \epsilon_0 \mathbf{E} - \mu_0 \mathbf{H} \times \mathbf{J}, \quad (12)$$

with SI units of Nm^{-3}. Equation (12) is used to explain the Jones and Leslie experiments.

A single plane wave representation is considered with illumination of a perfect mirror at angle θ_i in a background (liquid) medium of refractive index $n = \sqrt{\epsilon_l}$, with ϵ_l the dielectric constant, as in the Jones and Leslie experiments [8]. The laser coherence is assumed to be sufficiently high so as to allow a monochromatic picture. Consider then time harmonic fields and the incident wave vector as $\mathbf{k} = k_x\hat{x} + k_z\hat{z} = k_l\ \hat{k}$, so $\cos\theta_i = k_z/k_l$. The multilayer dielectric stack mirror used by Jones and Leslie is modeled as a perfect electric conductor (PEC). Doing so assumes that the light is totally reflected and the specific structure of the fields near to the surface of this one-dimensional photonic crystal can be neglected. In another view, the problem is treated by a Huygen's equivalent electric current source existing on the $z = 0$ surface. From (12), this results in a time-averaged force density directly applied to the mirror of

$$\langle \mathbf{f}_j \rangle = -\frac{\mu_0}{2}\Re\left(\mathbf{H}\times\mathbf{J}^*\right), \tag{13}$$

where $\Re$ is the real part, and $\mathbf{H}$ and $\mathbf{J}$ are phasor frequency domain quantities that are distinguished from the temporal form by context. The mirror surface is assumed to have global coordinates such that the normal into the incident field space is $-\hat{z}$ and the mirror surface is at $z = 0$. From the boundary condition on the tangential magnetic field applied in (13), the pressure on the mirror (Nm^{-2}) is

$$\begin{aligned}\langle \mathbf{F}_j \rangle &= -\frac{\mu_0}{2}\Re\left(\mathbf{H}_{z=0}\times\mathbf{J}_s^*\right)\\ &= \frac{\mu_0}{2}\Re\left[\mathbf{H}_{z=0}\times\left(\hat{z}\times\mathbf{H}_{z=0}^*\right)\right],\end{aligned} \tag{14}$$

where $\mathbf{J}_s$ A/m is the surface electric current density.

For a TE plane wave (E_y, H_x, H_z) in the temporal frequency domain,

$$\mathbf{E} = \hat{y}E_0 e^{ik_x x}\left(e^{ik_z z} + \Gamma_e e^{-ik_z z}\right) \tag{15}$$

$$\mathbf{H} = -\hat{x}\frac{E_0}{Z_z}e^{ik_x x}\left(e^{ik_z z} - \Gamma_e e^{-ik_z z}\right) + \hat{z}\frac{E_0}{Z_x}e^{ik_x x}\left(e^{ik_z z} + \Gamma_e e^{-ik_z z}\right), \tag{16}$$

with $\Gamma_e = -1$ for a PEC mirror and TE impedances $Z_z = \omega\mu_0/k_z$ and $Z_x = \omega\mu_0/k_x$. For TM polarization (H_y, E_x, E_z),

$$\mathbf{H} = \hat{y}H_0 e^{ik_x x}\left(e^{ik_z z} + \Gamma_h e^{-ik_z z}\right) \tag{17}$$

$$\mathbf{E} = \hat{x}H_0 Z_z e^{ik_x x}\left(e^{ik_z z} - \Gamma_h e^{-ik_z z}\right) - \hat{z}H_0 Z_x e^{ik_x x}\left(e^{ik_z z} + \Gamma_h e^{-ik_z z}\right), \tag{18}$$

with $\Gamma_h = 1$ for a PEC mirror and TM impedances $Z_z = k_z/(\omega\epsilon_0\epsilon)$ and $Z_x = k_x/(\omega\epsilon_0\epsilon)$. Considering monochromatic waves, the time domain fields corresponding to (15) - (18) are defined by $\mathbf{E}(x,z,t) = \Re\left[\mathbf{E}(x,z,\omega)\exp(-i\omega t)\right]$, with ω the Fourier conjugate variable associated with t and for the electric field.y

For the TE case, substituting (16) with $\Gamma_e = -1$ and $z = 0$ into (14) gives

$$\begin{aligned}\langle \mathbf{F}_j \rangle_{TE} &= \hat{z} 2\mu_0 \frac{|E_0|^2}{|Z_z|^2} \\ &= \hat{z} 2\mu_0 |E_0|^2 \frac{\cos^2\theta_i}{|\eta_l|^2},\end{aligned} \tag{19}$$

where η_l is the wave impedance of the liquid. From (17) and (14), the TM pressure is

$$\begin{aligned}\langle \mathbf{F}_j \rangle_{TM} &= \hat{z} 2\mu_0 |H_0|^2 \\ &= \hat{z} 2\mu_0 \frac{|E_0|^2}{|\eta_l|^2}.\end{aligned} \tag{20}$$

Note that the forces in (19) and (20) depend on the impedance of the background, so changing the sign of the refractive index does not change the sign of the force.

The exact time-averaged force density in the liquid in which the mirror is inserted, from (12) and for monochromatic light, is [29]

$$\begin{aligned}\langle \mathbf{f}_d \rangle &= \langle \frac{\partial \mathbf{P}}{\partial t} \times \mu_0 \mathbf{H} \rangle \\ &= \frac{\omega\mu_0\epsilon_0}{2} \Im\left(\chi_E \mathbf{E} \times \mathbf{H}^*\right),\end{aligned} \tag{21}$$

where $\Im$ is the imaginary part, and we have set $\mathbf{P} = \epsilon_0\chi_E\mathbf{E}$, with $\chi_E = \epsilon_l - 1$ the assumed isotropic electric susceptibility. There are two contributors to $\langle \mathbf{f}_d \rangle$, that due to χ_E'', where $\chi_E = \chi_E' + i\chi_E''$, and that associated with the standing wave within the beam region where the incident and reflected fields overlap in the neighborhood of the mirror. Any force imparted to the liquid could produce a mechanical force on the mirror. However, the liquids used in the experiments had small loss [8], so the force on the mirror due to absorption in the background liquid is neglected and the dielectric constant of the background is assumed to be real. The $\hat{z}$-component of the force density near the mirror for the TE case, from (21) for a lossless liquid, is

$$\begin{aligned}\langle \mathbf{f}_d \rangle &= -\hat{z} \frac{\omega\mu_0\epsilon_0}{2} \Im\left(\chi_E' E_y H_x^*\right) \\ &= \hat{z}\epsilon_0(\epsilon_l' - 1)|E_0|^2 k_z \sin(2k_z z),\end{aligned} \tag{22}$$

where $\epsilon_l' = \Re(\epsilon_l)$ and $k_z = \sqrt{k_l^2 - k_x^2}$ has been assumed real. The total z-dependent pressure on the liquid becomes

$$\langle \mathbf{F}_d \rangle = \int_{z \to -\infty}^{0} \langle \mathbf{f}_d \rangle \, \mathrm{d}z. \tag{23}$$

In the Jones and Leslie experiment [8], the compact beam (with a 0.6 mm spot) is incident on the mirror at 6.4°, meaning that there is a standing wave pattern with triangular support that reduces as one moves away from the mirror (in the $-z$-direction, referring to (22)). Consequently, there is a small net negative pressure for each period of the standing wave at the extremities. Small loss may also enhance the standing wave near the mirror. The resulting net negative force on the liquid ($\langle \mathbf{F}_d \rangle < 0$) is related to the picture of the momentum delivered to the mirror and described by (11). The key aspect here is that this suggests there is no net pressure from the liquid applying a force to the mirror in the Jones and Leslie experiment, leading to the position that $\langle \mathbf{f}_d \rangle$ can be neglected in the determination of the force on the mirror.

It would thus appear that the pressure $\langle \mathbf{F}_j \rangle$ N/m^2 on a PEC mirror should explain the Jones and Leslie experiments [8]. However, the ability to do so is by no means obvious by looking at the relevant equations, (19) and (20). Jones and Leslie [8] find that the force is proportional to the background refractive index in a set of TE experiments with background liquids having various refractive indices and $\theta_i = 6.4°$, unclear from either equation, and that the force is independent of polarization, certainly not evident from these equations.

The laser power was fixed by a correction method [8]. This was the major component of the calibration and accounted for reflections at the glass window, liquid interface - the reflection from which varied due the various liquids in which the mirror was immersed. The free space to glass window interface had an antireflection coating, presumably for either normal incidence or TE polarization at $\theta_i = 6.4°$, and the initial set of experiments were done with TE polarization. In the single plane wave picture, this calibration corresponds to fixing the Poynting vector magnitude

$$S = \Re\left[\frac{|E_0|^2}{2\eta_l}\right] = \frac{|E_0|^2}{2\eta_0}\sqrt{\epsilon_l}, \tag{24}$$

leading to

$$|E_0|^2 = \frac{2\eta_0 S}{\sqrt{\epsilon_l}}. \tag{25}$$

Substituting (25) into (19) and (20) gives

$$\langle \mathbf{F}_j \rangle_{TE} \; = \; \hat{z}\frac{4S}{c}\cos^2\theta_i\sqrt{\epsilon_l} \tag{26}$$

and

$$\langle \mathbf{F}_j \rangle_{TM} \; = \; \hat{z}\frac{4S}{c}\sqrt{\epsilon_l}. \tag{27}$$

Equations (26) and (27) fulfill the first requirement of predicting that the radiation pressure on the mirror is proportional to the refractive index of the background medium, $n = \sqrt{\epsilon_l}$, provided that the Poynting vector is constant. Hence, the most important conclusion here is that the Jones and Leslie experiment showing the radiation pressure is proportional to the background refractive index can be predicted with a force formulation stemming from Maxwell's equations with use of the Abraham momentum. With a $\hat{z}$-directed detector aperture, (26) and (27) hold with $S \to S_z$, where the quantity preserved is S_z, the z-component of the Poynting vector.

The Jones and Leslie experiments [8] show the rather surprising result that the radiation pressure is independent of polarization. The data show the ratio of the observed pressure for the two linear polarizations (TE and TM) with $\theta_i = 6.4°$ for various background liquids, and for $\theta_i = 20°$ for one case. The results indicate to two significant figures that the pressure is independent of polarization for $\theta_i \neq 0$. In the simplified example geometry, consider that $\hat{z}$ is the unit vector normal to the detector. In comparing the pressure due to TE and TM, the correction factors differ because of the polarization-dependent loss, notably from the antireflection coating. Therefore, the argument here is that S_z is maintained constant in the power calibration.

For the TE case and assuming lossless propagating fields,

$$S_z = \frac{|E_0|^2}{2Z_z}. \tag{28}$$

Using (28) in (19),

$$\begin{aligned} \langle \mathbf{F}_j \rangle_{TE} \; &= \; \hat{z}\frac{4\mu_0 S_z}{Z_z} \\ &= \; \hat{z}\frac{4S_z k_z}{\omega}, \end{aligned} \tag{29}$$

assuming Z_z and k_z real. For TM polarization with real Z_z, from (20),

$$\begin{aligned}\langle \mathbf{F}_j \rangle_{TM} &= \hat{z}\frac{4\mu_0 S_z Z_z}{\eta_l^2} \\ &= \hat{z}4\mu_0 S_z \left(\frac{k_z}{\omega\epsilon_l\epsilon_0}\right)\frac{1}{\eta_l^2} \\ &= \hat{z}\frac{4S_z k_z}{\omega}. \end{aligned} \tag{30}$$

Notice that the TE pressure (29) and the TM pressure (30) with fixed S_z are identical. Consequently, it can be conjectured that the normalization used in the Jones and Leslie experiment [8] also resulted in a polarization-independent and angle-independent force.

The development of the electromagnetic force from Maxwell's equations and using the Abraham momentum leads to the important force expressions given as (8) and (11), and the simpler form for plane waves in homogeneous isotropic media in (12) that was used here to explain the Jones and Leslie experiments. The equivalent explanation of the experimental results can be built using the Lorentz force (the $q\mathbf{v}\times\mu_0\mathbf{H}$ component, where $\mathbf{v}$ is velocity and q is charge). A primary observation from the treatment presented here is that the measured force on a mirror can be explained using the Abraham momentum as being proportional to the refractive index of the background medium, provided the Poynting vector is constant. Therefore, the Jones and Leslie experiments do not necessarily support the validity of the Minkowski momentum. This classical picture presented here is also consistent with the measured atomic recoil being proportional to the background refractive index. One might therefore expect that the force density expressions in (8) or (11), or their relativistic forms, may explain observable macroscopic forces. Of importance in a number of applications, arbitrary dispersion and loss can thus be rigorously treated. This is particularly important in the treatment of metamaterials, where the time derivative terms in (8) and (11) allow the incorporation of material dispersion for an arbitrary electromagnetic temporal field, and the anisotropy common in lattice-based metamaterials can be included in $\mathbf{P}$ and $\mathbf{M}$. Provided that local homogenization applies [30], forces on dispersive structured material, even when homogenization does not hold, can thus be determined by integrating the force density.

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