Universality, Maximum Radiation, and Absorption in High-Energy Collisions of Black Holes with Spin

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We explore the impact of black hole spins on the dynamics of high-energy black hole collisions. We report results from numerical simulations with γ–factors up to 2.49 and dimensionless spin parameter $\chi = +0.85$, $+0.6$, 0, -0.6, -0.85. We find that the scattering threshold becomes independent of spin at large center-of-mass energies, confirming previous conjectures that structure does not matter in ultrarelativistic collisions. It has further been argued that in this limit all of the kinetic energy of the system may be radiated by fine tuning the impact parameter to threshold. On the contrary, we find that only about 60% of the kinetic energy is radiated for $\gamma = 2.49$. By monitoring apparent horizons before and after scattering events we show that the "missing energy" is absorbed by the individual black holes in the encounter, and moreover the individual black-hole spins change significantly. We support this conclusion with perturbative calculations. An extrapolation of our results to the limit $\gamma \to \infty$ suggests that about half of the center-of-mass energy of the system can be emitted in gravitational radiation, while the rest must be converted into rest-mass and spin energy.

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I. Introduction. Numerical relativity simulations have begun to shed light on problems of fundamental interest in high-energy physics, such as trans-Planckian scattering and gauge-gravity dualities [1]. A scenario of particular interest in this context is the collision of two black holes (BHs) near the speed of light, which we will use here to test the validity of two key assumptions made in Monte Carlo event generators [2–5] for the modeling of microscopic BH production in trans-Planckian scattering [6, 7]: (i) that the spins of the colliding objects have a negligible effect on the dynamics, and (ii) that the mass of the formed BH is (up to a factor $\lesssim 1$) given by the center of mass energy of the colliding particles, i.e. that a significant fraction of the kinetic energy of the system cannot be lost in the form of gravitational waves. For this purpose we perform a systematic analysis of ~ 160 collisions of spinning and nonspinning BH binaries in $D = 4$ (see e.g. [8, 9] for early results in $D > 4$) to answer the following two questions: i) is the internal structure of the colliding objects, here consisting of their spin angular momentum, relevant? ii) is it possible (as suggested in [10]) to radiate all of the kinetic energy in fine-tuned encounters?

Our simulations answer both questions in the negative. Spin effects become negligible for large γ: both the scattering threshold and the maximum energy radiated become universal functions of γ (independent of spins). For our largest boost ($\gamma = 2.49$), grazing encounters radiate $\lesssim 60\%$ of the available kinetic energy. In fact this percentage *decreases* with increasing boost velocity, and barely varies with spin for $v \gtrsim 0.8$. We show that the "missing" kinetic energy is accounted for by an increase in the BH mass during the encounter. These observations justify the use of semianalytical calculations in classical general relativity that neglect spins to understand properties of the collisions, and constrain the amount of GWs radiated, which will determine the initial mass spectrum of formed BHs.

Our results reinforce earlier evidence that "matter does not matter": e.g. in [11] it was shown that collisions of two bosonic solitons at sufficiently high energies lead to BH formation, and similar conclusions were reached when colliding self-gravitating fluid objects [12, 13]. Compact fluids formed in head-on collisions of neutron stars were also found to exhibit type I critical collapse in [14].

High-energy collisions of BHs have been investigated extensively in $D = 4$ spacetime dimensions for equal-mass, nonspinning BHs, where the problem is characterized by the boost factor $\gamma = (1 - v^2)^{-1/2}$ and impact parameter $b = L/P$, with v the center-of-mass velocity, L the initial orbital angular momentum and P the initial linear momentum of a single BH (here and below we use geometrical units $G = c = 1$). In the head-on case ($b = 0$), high-energy BH collisions can radiate up to $14 \pm 3\,\%$ of the center-of-mass (CM) energy, and they always produce a nonspinning remnant [15]. Grazing collisions with $b \neq 0$, on the other hand, result in one of the following three outcomes [16]: (i) a prompt merger for small $b < b^*$, (ii) a "delayed" merger for $b^* \leq b < b_{\rm scat}$, or (iii) scattering of the holes to infinity for $b \geq b_{\rm scat}$. Here $b_{\rm scat}$ denotes the scattering threshold and $b^* < b_{\rm scat}$ the "threshold of immediate merger": by fine-tuning around

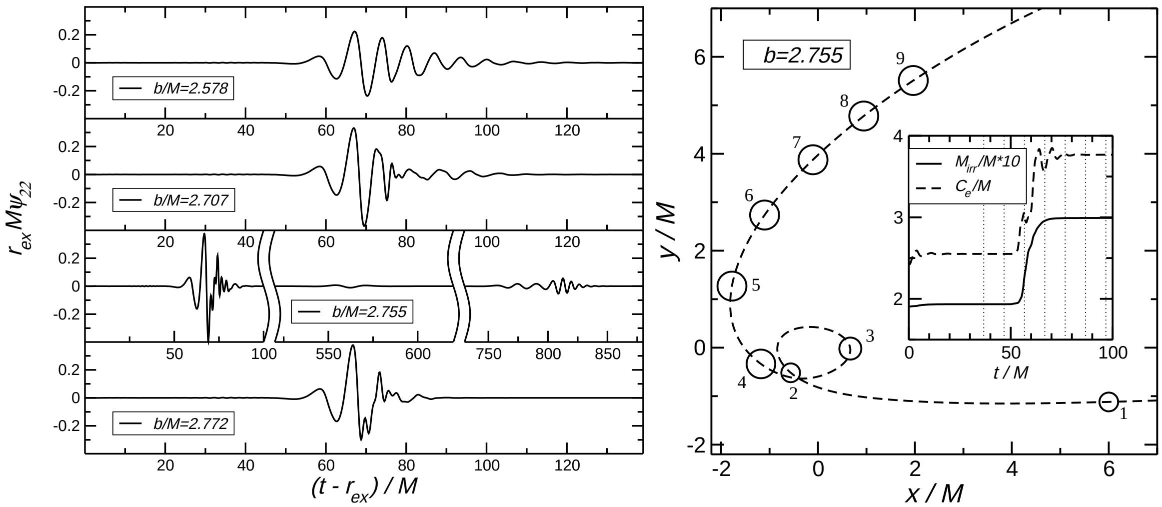

FIG. 1. Left: Waveforms for $\gamma = 2.49$, antialigned spins $\chi = 0.6$ and selected values of b. The $b/M = 2.755$ case is a triple encounter (two periastron passages followed by a merger). Right: Trajectory of one BH from the simulation with $b/M = 2.755$. Inset: time evolution of the irreducible mass $M_{\rm irr}$ and of the circumferential radius $C_{\rm e}$ of each hole. The circles represent the BH location at intervals $\Delta t = 10\ M$ (corresponding to vertical lines in the inset) and have radius equal to $M_{\rm irr}$.

b^* a binary approaches a near-circular orbit for a time $T \propto \log|b - b^*|$, before separating or merging to form a single Kerr BH [10]. In $D = 4$, grazing collisions with $\gamma \leq 2.9$ were found to radiate as much as $35 \pm 5\,\%$ of the CM energy in gravitational waves (GWs) and form BHs with near-, yet sub-extremal spins [16]. A parallel study investigated the scattering threshold $b_{\rm scat}$ for nonspinning binaries as a function of the CM energy $M = \gamma M_0$ up to $\gamma = 2.3$, finding $b_{\rm scat} \sim 2.5\,(M/v)$ [17]. Comparisons with BH perturbation theory and point-particle collisions in the zero-frequency limit provide a satisfactory understanding of the qualitative features of these simulations [10, 18, 19], but several outstanding questions remain.

II. Setup. Our simulations have been performed with the Lean code described in [20]; see also [21–25]. We obtain stable evolutions by applying two modifications to the numerical infrastructure employed in our previous studies [15, 16, 26]: (i) we evolve the conformal factor as described in Sec. II of [27], and (ii) we reduce the Courant factor to 0.45. The holes start on the x axis with radial momentum P_x and tangential momentum P_y, separated by a distance d. The impact parameter is $b \equiv L/P = P_y d/P$. We extract gravitational radiation by computing the Newman-Penrose scalar Ψ_4 at different radii $r_{\rm ex}$ from the center of the collision. Ψ_4 is decomposed into multipoles ψ_{lm} as described in Ref. [16], but measuring the polar angle θ relative to the x axis.

Spurious "junk" radiation in the initial data is quite insensitive to b, and comparable to our recent findings [15, 16]. We remove it from reported results in the same manner. Errors due to discretization and finite extraction radius are comparable to those reported in [15, 16]. We estimate uncertainties in radiated quantities of 3 % and 15 % for low and high boosts, respectively. These are dominated by discretization errors in the wave zone, which may be addressed in future work using multi-patch techniques [28].

Contrary to our recent investigation of ultrarelativistic encounters of spinning BHs in "superkick" configurations [26], here we expect the dynamics to be most strongly affected by the "hang-up" effect [29] typical of spins (anti)aligned with the orbital angular momentum $\boldsymbol{L}$. We study its boost dependence by evolving three sequences of equal-mass BH binaries: (i) a sequence with zero spins, (ii) a sequence with dimensionless spin parameters $\chi = \chi_1 = \chi_2 = 0.6$ aligned with $\boldsymbol{L}$, and (iii) a sequence with spins $\chi = 0.6$ antialigned with $\boldsymbol{L}$. For each sequence we consider four boost parameters ($\gamma = 1.22,\ 1.42,\ 1.88,\ 2.49$) and for each γ we simulate encounters with about 10 different values of b to bracket the scattering threshold. In addition, we study the boost values $\gamma = 1.22,\ 2.49$ in the same manner using larger spins $\chi = 0.85$ aligned or anti-aligned with L.

III. Scattering threshold. We expect a given initial binary configuration to result in either a prompt merger, a delayed merger or scattering to infinity. Our new simulations confirm this scenario. This is illustrated in the left panel of Fig. 1, where we plot a subset of representative waveforms from the $\gamma = 2.49$ sequence with antialigned spins $\chi = 0.6$. For small impact parameter (top) the BHs merge promptly, and the signal is a clean merger/ringdown waveform, leading to formation of a BH with dimensionless spin $\chi_f \simeq 0.87$. For the second waveform from the top the merger is not quite prompt: it shows a pattern similar to the scattering waveforms with $b/M = 2.772$ (bottom panel), followed by a ringdown. The third waveform for $b/M = 2.755$ is a rare *triple* en-

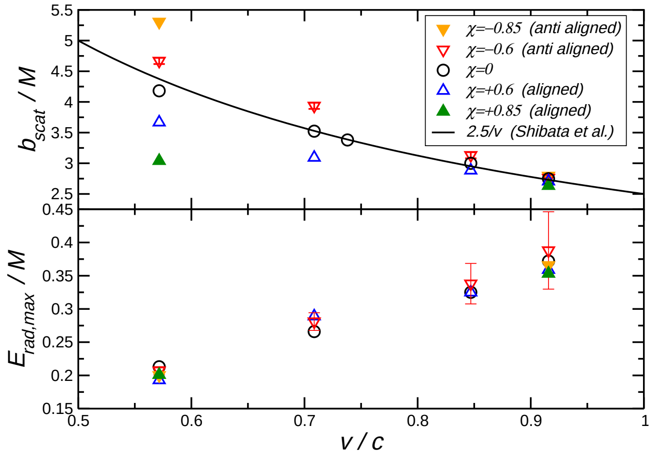

FIG. 2. Critical scattering threshold (upper panel) and maximum radiated energy (lower panel) as a function of v. Colored "triangle" symbols pointing up and down refer to the aligned and antialigned cases, respectively. Black "circle" symbols represent the thresholds for the four nonspinning configurations studied in this paper, complemented (in the upper panel) by results from [16] for $v = 0.753$. For clarity, we only plot error bars for the antialigned-spin sequence; for $b_{\rm scat}$ they are comparable in size to the symbols.

Spin	γ	b/M	K/M	$E_{\rm rad}/K$	$E_{\rm abs}/K$	$\lvert\chi_{\rm i}\rvert$	$\lvert\chi_{\rm s}\rvert$
$\downarrow 0.85$	1.22	5.322	0.179	0.870	0.088	0.84	0.66
$\downarrow 0.85$	2.49	2.784	0.598	0.602	0.329	0.82	0.12
$\downarrow 0.6$	1.22	4.671	0.179	0.899	0.088	0.60	0.45
$\downarrow 0.6$	1.88	3.133	0.468	0.665	0.273	0.59	0.01
$\downarrow 0.6$	2.49	2.762	0.598	0.570	0.313	0.57	0.12
0	1.22	4.191	0.179	0.899	0.075	0.01	0.06
0	1.88	3.005	0.468	0.637	0.284	0.08	0.24
0	2.49	2.749	0.598	0.574	0.320	0.10	0.23
$\uparrow 0.6$	1.22	3.678	0.179	0.894	0.065	0.60	0.59
$\uparrow 0.6$	1.88	2.886	0.468	0.618	0.284	0.59	0.45
$\uparrow 0.6$	2.49	2.704	0.598	0.600	0.320	0.57	0.33
$\uparrow 0.85$	1.22	3.053	0.179	0.875	0.053	0.84	0.80
$\uparrow 0.85$	2.49	2.643	0.598	0.557	0.292	0.82	0.39

TABLE I. Fractional kinetic energy radiated and absorbed for representative simulations with aligned spins ($\uparrow$), antialigned spins ($\downarrow$) or nonrotating BHs ('0'). We also list spin estimates $\chi_{\rm i}$ and $\chi_{\rm s}$ before and after the encounter. χ_i is measured at a time $\sim 20\ M$ after the beginning of the simulation. Small deviations from the initial data parameter $\chi = \pm 0.85, \pm 0.6, 0$ can presumably be attributed to the BHs absorbing an increasing amount of junk radiation as γ increases.

counter consisting of two revolutions (the second close encounter is visible as a small "bump" at $t/M \sim 550$), followed by a merger signal with relatively low amplitude. Note that the binary radiates and partially absorbs much of the system's kinetic energy during the first encounter, therefore subsequent encounters occur at low velocity and radiate much less. We display this behaviour in the right panel of Fig. 1, where we plot the trajectory of one BH for the configuration $b/M = 2.755$ and represent snapshots (labeled '1' to '9') of the BH at time intervals $\Delta t = 10\ M$ by circles with radius equal to the irreducible mass $M_{\rm irr}$. From snapshots '2' to '4' we observe a rapid increase in the "size" of the black hole; successive snapshots are located closer to each other, showing that the BH has slowed down as a result of the interaction.

In order to determine $b_{\rm scat}$ as a function of spin and boost we need to distinguish between merging and scattering collisions. Mergers are easily identified by finding a common AH. We identify an encounter as a scattering case when the following criteria are met: (i) no common AH is found; (ii) the Kretschmann scalar (defined in terms of the Riemann tensor as $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$) at the origin approaches zero at late times within numerical uncertainties; and (iii) the coordinate trajectories of the BHs, Eq. (14) in Ref. [20], separate out to values comparable to their initial distance.

The scattering thresholds obtained in this way are plotted in the upper panel of Fig. 2. Errors in $b_{\rm scat}$ come from numerical truncation error and discrete sampling of the parameter space, estimated as follows. In the most challenging case ($\gamma = 2.49$) our standard-resolution runs with grid spacing Δx yield $b_{\rm scat}/M = 2.760$ for the antialigned case with $\chi = 0.6$. By running simulations at two higher resolutions $0.9\ \Delta x$ and $0.8\ \Delta x$, we find $b_{\rm scat}/M = 2.741$ and 2.731 respectively, corresponding to about fourth-order convergence, a Richardson-extrapolated value $b_{\rm scat}/M = 2.713$, and therefore a numerical uncertainty of 0.047. The error due to discretization of the parameter space ($\sim 1.6 \times 10^{-3}$) is negligible in comparison, so we adopt $\delta b_{\rm scat} \approx 0.05$ as a conservative error estimate. Fig. 2 shows that the scattering threshold is spin-independent in the limit $\gamma \to \infty$. Our nonspinning simulations are consistent with the results obtained by Shibata et al. [17] at lower boosts and within ~ 50 % of the shock-wave analysis in Table II of [30], that suggests $b_{\rm scat}/M \gtrsim 1.68$ in the ultrarelativistic limit.

IV. Maximum radiation. As pointed out in [16, 31], the total energy $E_{\rm rad}/M$ radiated in grazing BH collisions increases steeply as the impact parameter approaches b^* or $b_{\rm scat}$. This is true also for spinning binaries. For $\gamma =$ (1.22, 1.42, 1.88, 2.49), respectively, we find the maxima in $E_{\rm rad}/M$ plotted in the lower panel of Fig. 2. For reference, the initial fraction of total energy in the form of kinetic energy $K/M = (\gamma - 1)/\gamma$ is $(17.9, 29.4, 46.8, 59.8)\%$, and for $\chi = 0.6$ we have $(4.2, 3.6, 2.6, 1.7)\%$ in initial spin energy.

For a subset of scattering runs where we monitored the apparent horizon as a function of time, in Table I we list estimates of radiated energies and spins before/after the first encounter. These numbers reveal two striking features: (i) the maximum radiated energy varies mildly with spin at any given γ, and (ii) for small boosts the maximum radiation is comparable to the initial kinetic energy; however as γ increases the ratio drops, down to $\sim 60\%$ for $\gamma = 2.49$. This observation prompts two questions. Where has the remaining kinetic energy gone? Why does the deficit increase with boost?

V. Absorption. The answer to these questions is found in the apparent horizon dynamics of the individual holes before and after the first encounter. We have analyzed the data in detail for a set of binary configurations where the individual holes separate sufficiently after first encounter to warrant application of the isolated horizon limit. Specifically, we measure the equatorial circumference $C_{\rm e} = 4\pi M$ and the irreducible mass $M_{\rm irr}$ of each BH before and after the encounter. The inset of the right panel of Fig. 1 shows the variation of these quantities with time in a typical simulation: absorption occurs over a short timescale $\approx 10M$. Since the apparent horizon area $A_{\rm AH} = 16\pi M_{\rm irr}^2 = [C_{\rm e}^2/(2\pi)](1+\sqrt{1-\chi^2})$, in this way we can estimate the rest mass and spin of each hole before ($M_{\rm i}$, $\chi_{\rm i}$) and after ($M_{\rm s}$, $\chi_{\rm s}$) the first encounter. We define the absorbed energy $E_{\rm abs} = 2(M_s - M_i)$. The results in Table I show that the sum $(E_{\rm rad} + E_{\rm abs})/M$ accounts for most of the total available kinetic energy in the system, and therefore the system is no longer kinetic-energy dominated after the encounter. A fit of the data yields $E_{\rm rad}/K = 0.46(1+1.4/\gamma^2)$ and $E_{\rm abs}/K = 0.55(1-1/\gamma)$, suggesting that radiation and absorption contribute about equally in the ultrarelativistic limit, and therefore that absorption sets an upper bound on the maximum energy that can be radiated.

The fact that absorption and emission are comparable in the ultrarelativistic limit is supported by point-particle calculations in BH perturbation theory. For example, Misner et al. [32] studied the radiation from ultrarelativistic particles in circular orbits near the Schwarzschild light ring, i.e. at $r = 3M(1+\epsilon)$. Using a scalar-field model they found that 50% of the radiation is absorbed and 50% is radiated as $\epsilon \to 0$. We verified by an explicit calculation ignoring self-force effects that the same conclusion applies to *gravitational* perturbations of Schwarzschild BHs (cf. [33]). A recent analysis including self-force effects finds that 42% of the energy should be absorbed by nonrotating BHs as $\epsilon \to 0$ (cf. Fig. 4 in [19]).

Rather than considering particles near the light ring, we can model our problem using particles plunging ultrarelativistically into (for simplicity) a Schwarzschild BH. Davis et al. [34] first computed the energy absorbed when a particle of mass m falls *from rest* into a Schwarzschild BH of mass $M_{\rm BH}$. Remarkably, they found that the total absorbed energy (summed over all multipoles $\ell \geq 2$) diverges. This is due to the fact that most of the absorption occurs near the horizon, so we must go beyond the point-particle approximation and introduce a physical cutoff at $\ell_{\max} \approx \pi M_{\rm BH}/2m$ to take into account the finite size of the infalling particle. For comparable-mass encounters it is reasonable to truncate the sum at $\ell = 2$. By adapting the BH perturbation theory code of [35], we extended the calculation of [34] to generic particle energies $p_0 = E/m$. Our calculation shows that the radiated (absorbed) energy is $E^{\rm PP}_{\rm rad,abs} = k_{\rm rad,abs}(p_0 m)^2/M_{\rm BH}$, with $k_{\rm rad} = (1.04\times10^{-2}, 3.52\times10^{-2}, 0.119, 0.262)$ and $k_{\rm abs} = (0.304, 0.310, 0.384, 0.445)$ for $p_0 = (1, 1.5, 3, 100)$, respectively. So, again, in the ultra-relativistic limit the point-particle model predicts a roughly comparable amount of emission and absorption. As a consequence of this significant energy absorption, in the large-γ limit close scattering encounters between two arbitrarily small (in rest mass) BHs can result in two, slowly moving BHs with rest mass increased by a factor of order γ.

Another remarkable implication of Table I is that high-energy scattering encounters can significantly modify the spin magnitude. For example, when $\gamma = 2.5$ the absolute value of the BH spin decreases from ~ 0.6 to ~ 0.3 in the aligned case, from ~ 0.6 to ~ 0.1 in the antialigned case, and we measure a post-scattering spin $\chi \sim 0.2$ for initially nonspinning encounters. These changes in dimensionless spin parameter correspond to roughly the *same* total angular momentum being transferred to the BHs during the interaction, independent of the initial spin.

VI. Ultrarelativistic extrapolation. We estimate the maximum radiated energy when $\gamma \to \infty$ as follows. According to Ref. [15], head-on collisions ($b = 0$) radiate at most $E_{\rm rad}/M \sim 0.14$ in this limit. For each γ, the increase in radiation induced by fine-tuning near threshold can be characterized by the ratio $\mathcal{R} \equiv E_{\rm rad}(b=0)/E^{\max}_{\rm rad}$. For our largest γ we find $\mathcal{R} \sim 0.2$, and a fit to our data yields $\mathcal{R}(\gamma) = 0.34(1-1/\gamma)$. Using this fit in combination with results from Ref. [15] we find $E^{\max}_{\rm rad}/M \approx 0.14/0.34 \sim 0.41$ as $\gamma \to \infty$. A more conservative upper bound on $E^{\max}_{\rm rad}$ can be obtained noting that $\mathcal{R}$ increases with γ, but using the last data point in our simulations as a lower limit on $\mathcal{R}$: this yields a limit $E^{\max}_{\rm rad}/M \lesssim 0.14/0.2 = 0.7$ as $\gamma \to \infty$. These results are consistent (within the errors) with our discussion of Table I, which indicates that radiation in high-energy encounters accounts for roughly 0.46 of the available energy. Our simulations thus settle the long-standing question of whether it is possible to release all of the CM energy as GWs in high-energy BH collisions: the answer is no.

Two crucial assumptions underlie the study of BH production from trans-Planckian particle collisions. The first assumption, that BHs are indeed produced in the collision, is on a firmer footing due to the results of [11–13], where the hoop conjecture was found to be valid even in highly dynamical situations. The present study addresses the second crucial assumption, i.e., that the internal structure of the colliding bodies is irrelevant at high energies. Furthermore our simulations provide strong evidence that, because of absorption, the maximum radiation produced in ultrarelativistic encounters in four dimensions cannot exceed $\approx 50\%$ of the CM energy.

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