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Phys. Rev. Lett. **111**, 021802 — Published 11 July 2013

DOI: 10.1103/PhysRevLett.111.021802

Strong Coupling and Bounds on the Spin-2 Mass in 'Massive Gravity'

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(Dated: June 21, 2013)

The dRGT theory of a single massive spin-2 field has a cutoff much below its Planck scale, because the extra modes from the massive spin-2 multiplet involve higher derivative self-interactions, controlled by a scale convoluted from its mass. Generically, these correct the propagator by environmental effects. The resulting effective cutoff depends on the environmental parameters and the spin-2 'graviton' mass. Requiring the theory to be perturbative down to $\mathcal{O}(1)$ mm, we derive bounds on the mass, corresponding to $\gtrsim \mathcal{O}(1)$ meV for the generic case, assuming the coupling to be given by the standard Newton's constant, and somewhat weaker bounds in cases with fine-tuning. Thus the theory of a single massive spin-2 can really only be viewed as a theory describing the full nonlinear propagation of a massive spin-2 field on a fixed background, and not as an approximation to GR.

What is the range of the gravitational force? Must it be infinite, or could it be finite, by virtue of a graviton mass-induced Yukawa suppression, like in massive gauge theories? This question has been looming about a long time, since the pioneering work by Pauli and Fierz [1], the subsequent exploration by Boulware and Deser [2], and its recent followup [3]. The problem one encounters is that since mass breaks the residual gauge symmetries of gravity, there are six new propagating degrees of freedom. Generically, one is a ghost. While [1] exorcised the ghost away in the linearized limit, it seemed unavoidable in the full theory [2, 3]. On the other hand, discovery of cosmic acceleration [4] and the dearth of its theoretical explanations, save the landscape paradigm [5] (many less satisfactory dark energies are reviewed in [6]) fueled speculations that changing gravity away from General Relativity (GR) may account for dark energy [7]. Hence the question: *can the graviton have a mass?* becomes more than just a mere theoretical curiosity.

Construction of classically consistent massive spin-2 theory has been difficult (for a review see [8]). Linearised Pauli-Fierz (PF) theory [1] suffers from the vDVZ discontinuity [9], and its linearized perturbation theory is unreliable. This can be improved by non-linear interactions implementing the Vainshtein mechanism [10, 11]. However, typical non-linear completions have the sixth mode Boulware-Deser ghost [2], in addition to the five massive helicities of Poincare-invariant spin-2 theory. Many of these issues are related to the dynamics of the helicity-0 component of the massive multiplet, hereon denoted π.

Very recently, it has been shown that many problems can be avoided in a specific non-linear completion of PF theory, known as dRGT [12, 13]. Classically, the dRGT model does not propagate the troublesome sixth mode [14]. Hence, it gives a consistent classical system with massive spin-2 that can be used as a straw-man for phenomenological purposes, with a definable perturbative expansion at any order of truncation of the theory. However, the full taxonomy of the background solutions on which to expand still does not exist (for some problems, see [15, 16]). Nevertheless this theory has been advocated to be a massive gravity that can approximate GR at phenomenologically tested scales. Note that these theories have been argued to be challenged by issues of causality [24].

In this *Letter* we will address the strong coupling limits on the spin-2 mass in dRGT. Since the theory has a UV cutoff much below the Planck scale, if one wishes to use it to approximate GR one must require it to be perturbatively well behaved at least between the distances scales of $\mathcal{O}(1)$ mm, and the present Hubble scale, the range where we have more or less found gravity to be weak. Even more generally, if one abandons the desire to use dRGT as a replacement for GR, the question of its perturbative validity remains. We will show that demanding perturbativity places a bound on the mass of the graviton, which is directly related to the UV cutoff. We include environmental effects which affect the short distance cutoff due to the higher derivative self interactions of the π field.

To analyse short distance properties, we can ignore the Yukawa suppressions, and focus on the dynamics of the Stuckelberg field π, being guided by the Goldstone equivalence theorem from massive gauge theory. In massive spin-2 the (low) UV cutoff is (still) higher than the spin-2 mass, and so there is a (broad) regime of scales where this approximation is valid. We also ignore the dynamics of the helicity-1 modes, assuming Lorentz invariance, so that they decouple at the lowest order. We then compute the effective action in the background fields of a source, given by the dRGT spin-2/matter coupling. In case of the Earth as the source, compatibility with the tabletop experiments that probe gravity below mm [17] implies that making the strong coupling scale high enough pulls up the spin-2 mass much above the current Hubble scale. In general, our formulas apply to any ghost-free Lorentz invariant spin-2 setup that couples to some stress energy tensor of lower spin matter, with numerical values determined by the strength of the coupling and the amplitude of a source.

Framework. The dRGT theory of spin-2 with mass m is described by the following action [12, 13]

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\mathcal{U}(g,H)\right] + S_m[g_{\mu\nu};\Psi_n]\,, \tag{1}$$

with spin-2 potential U and coupling κ, usually taken to be the standard Newtonian coupling $\kappa \sim \sqrt{16\pi G} \sim \sqrt{2}/M_{Pl}$. S_m is the action for matter fields, Ψ_n, minimally coupled to the 'metric' $g_{\mu\nu}$. The tensor $H_{\mu\nu}$ is related to the metric as $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} = H_{\mu\nu} + \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$ where the four Stuckelberg fields ϕ^a transform as scalars and $\eta_{ab} = \mathrm{diag}(-1,1,1,1)$. The potential can be expressed using $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu}$, so that we have $\mathcal{U}(g,H) = -4\sum_{n\geq 2}\alpha_n\mathcal{K}^{[\mu_1}_{\mu_1}\ldots\mathcal{K}^{\mu_n]}_{\mu_n}$; $[\ldots]$ denotes antisymmetrization, *without* the factor of $1/n!$; helicity-0 field π is extracted using $\phi^a = x^a - \eta^{a\mu}\partial_\mu\pi$.

Let us work in the decoupling limit [12]: $m,\kappa \to 0$, $T_{\mu\nu}\to\infty$ with $\Lambda_3 = (m^2/\kappa)^{1/3}$ and $\kappa T_{\mu\nu}$ held fixed, where $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}$ is the stress-energy tensor of the source. The effective Lagrangian in this limit is [12]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}h_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta}h_{\alpha\beta} + \frac{3}{2}\pi\Box\pi \\ &- \frac{u}{\Lambda_3^3}\pi\pi^{[\mu}_\mu\pi^{\nu]}_\nu + \frac{1}{4\Lambda_3^6}(u^2-4v)\pi\pi^{[\mu}_\mu\pi^\nu_\nu\pi^{\alpha]}_\alpha \\ &+ \frac{3v}{\Lambda_3^6}\left(h_{\mu\nu} - \frac{1}{3}h^\gamma_\gamma\eta_{\mu\nu}\right)\pi^{\mu[\nu}\pi^\alpha_\alpha\pi^{\beta]}_\beta + \frac{uv}{\Lambda_3^9}\pi\pi\pi^{[\mu}_\mu\pi^\nu_\nu\pi^\alpha_\alpha\pi^{\beta]}_\beta \\ &+ \frac{\kappa}{2}h^{\mu\nu}T_{\mu\nu} + \frac{\kappa}{2}\pi T^\alpha_\alpha + \frac{\kappa u}{2\Lambda_3^3}\partial^\mu\pi\partial^\nu\pi T_{\mu\nu}\,, \end{aligned}\tag{2}$$

where $u = -(1+3\alpha_3)$, $v = -\frac{1}{2}(\alpha_3 + 4\alpha_4)$, $\pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$ and indices are raised/lowered with the fiducial Minkowski metric. The operator $\mathcal{E}^{\mu\nu\alpha\beta}$ is related to the linearised Einstein tensor[1]. Note that we have performed the following field redefinitions: $\pi \to \pi/\Lambda_3^3$, $h_{\mu\nu} \to h_{\mu\nu} + \pi\eta_{\mu\nu} + \frac{u}{\Lambda_3^3}\partial_\mu\pi\partial_\nu\pi$, the latter to diagonalise the action up to cubic order. It is impossible to fully diagonalise the theory in an explicitly local way.

Clearly, the interactions in (2) become strongly coupled at the scale Λ_3. For example, in DGP for a graviton whose mass lies at the current Hubble scale, $m \sim H_0 \sim 10^{-33}$ eV, in vacuum this occurs at distances $\lesssim 1000$ km [18]. However, in the presence of a source with background fields, the quadratic Lagrangian in (2) will be renormalized by the contributions from the higher dimension operators in (2) evaluated on the background. To compute them, we model the source's background field with the spherically symmetric static solutions found in [19]. In the decoupling limit this background solution is given by $h_{\mu\nu} = \bar{h}_{\mu\nu}$, $\pi = \bar{\pi}$ where, writing the fiducial Minkowski metric in spherical coordinates with radius ρ and time t, one finds that $\bar{h}_{tt}(\rho) = \int^\rho dz\frac{\bar{h}_{\rho\rho}(z)}{z}$, $\bar{h}_{\rho\rho}(\rho) = \frac{\kappa M}{8\pi\rho} + 2v\rho^2\Lambda_3^3Q^3$, and that $Q = \frac{\bar{\pi}'(\rho)}{\Lambda_3^3\rho}$ satisfies

$$3Q - 6uQ^2 + 2(u^2-4v)Q^3 - 6vQ^2\left(\frac{\bar{h}_{\rho\rho}}{\Lambda_3^3\rho^2}\right) = \frac{1}{4\pi}\left(\frac{\rho_V}{\rho}\right)^3. \tag{3}$$

Here M is the mass of the source and $\rho_V = \frac{(\kappa M)^{\frac{1}{3}}}{\Lambda_3}$ its Vainshtein radius. If we require that the Vainshtein shielding is efficient, such that that $|\bar{\pi}| \ll |\bar{h}_{\mu\nu}|$ for $\rho \ll \rho_V$, a usual assumption in dRGT, for generic values $|u|,|v| \sim \mathcal{O}(1)$, we must require $v < 0$ [19], in which case we obtain $Q \sim \mathcal{O}(1)$ and $|\bar{h}_{\mu\nu}| \sim \mathcal{O}\left(\frac{\kappa M}{8\pi\rho}\right)$, implying $|\bar{\pi}| \ll |\bar{h}_{\mu\nu}|$, as desired. We see similar behavior when $|u| \ll 1$ and $|v| \sim \mathcal{O}(1)$. For $|u| \sim \mathcal{O}(1)$ and $|v| \ll 1$ we have $Q \sim \mathcal{O}(\frac{\rho_V}{\rho})$ and $|\bar{h}_{\mu\nu}| \sim \mathcal{O}(\frac{\kappa M}{8\pi\rho})$, so again, the Vainshtein mechanism is successful. Eq. (3) quantifies the statement $v \ll 1$. It must obey $vQ^2 < 1$ (in this limit it implies $v < (\rho/\rho_V)^2 \sim 10^{-22}$ for a Hubble mass spin-2 with standard Newtonian coupling in Earth's environment). So this region of phase space is an extremely narrow sliver around the u-axis.

Working in the strict decoupling limit of Lagrangian (2) would appear to disguise any connection with some of the original parameters of the theory. However, here we simply use this Lagrangian as describing the leading order interaction of the full theory, omitting irrelevant operators suppressed by scales between $\Lambda_3 = (m^2/\kappa)^{1/3}$ and the Planck scale. In fact, one can easily check using the exact results of [19] that these operators are subleading, and remain subdominant for the processes considered here.

Effective theory. Let us now determine the effective theory in the background field of the source. This means, we perturb about the background solution, setting $h_{\mu\nu} = \bar{h}_{\mu\nu} + \chi_{\mu\nu}$, $\pi = \bar{\pi} + \varphi$ and $T_{\mu\nu} = \bar{T}_{\mu\nu} + \tau_{\mu\nu}$. Working with the Lagrangian (2) and defining $[r,s]_{\mu\nu} = r\varphi_{\mu[\nu}\varphi^{\mu_2}_{\mu_2}\ldots\varphi^{\mu_r}_{\mu_r}\bar{\pi}^{\nu_1}_{\nu_1}\ldots\bar{\pi}^{\nu_s}_{\nu_s]} + s\bar{\pi}_{\mu[\nu}\varphi^{\mu_1}_{\mu_1}\ldots\varphi^{\mu_r}_{\mu_r}\bar{\pi}^{\nu_2}_{\nu_2}\ldots\bar{\pi}^{\nu_s}_{\nu_s]}$ and $[r,s] = \varphi^{\mu_1}_{[\mu_1}\ldots\varphi^{\mu_r}_{\mu_r}\bar{\pi}^{\nu_1}_{\nu_1}\ldots\bar{\pi}^{\nu_s}_{\nu_s]}$, we obtain

$$\begin{aligned}\delta\mathcal{L} = &-\frac{1}{2}\chi_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta}\chi_{\alpha\beta} + \frac{1}{2}\varphi\mathcal{K}\varphi \\ &+ \frac{3v}{\Lambda_3^6}\chi^{\mu\nu}([1,2]_{\mu\nu} - \eta_{\mu\nu}[1,2]) + \delta\mathcal{L}_{int} + \delta\mathcal{L}_m\,,\end{aligned}\tag{4}$$

where

$$\begin{aligned}\varphi\mathcal{K}\varphi = &\frac{6v}{\Lambda_3^6}\bar{h}^{\mu\nu}([2,1]_{\mu\nu} - \eta_{\mu\nu}[2,1]) + \varphi\Big[3\Box\varphi - \frac{6u}{\Lambda_3^3}[1,1] \\ &+ \frac{3(u^2-4v)}{\Lambda_3^6}[1,2] + \frac{20uv}{\Lambda_3^9}[1,3] - \frac{\kappa u}{\Lambda_3^3}\bar{T}^{\mu\nu}\varphi_{\mu\nu}\Big]\,,\end{aligned}\tag{5}$$

[1] $\mathcal{E}^{\mu\nu\alpha\beta}h_{\alpha\beta} = \delta G^{\mu\nu} = -\frac{1}{2}\Box\left(h^{\mu\nu} - \frac{1}{2}h^\alpha_\alpha\eta^{\mu\nu}\right) + \ldots$

$$\delta\mathcal{L}_{int} = -\frac{u}{\Lambda_3^3}\varphi[2,0] + \frac{1}{4\Lambda_3^6}(u^2-4v)\varphi(4[2,1]+[3,0]) + \frac{uv}{\Lambda_3^9}\varphi(10[2,2]+5[3,1]+[4,0]) + \frac{v}{\Lambda_3^6}\left(\bar{h}^{\mu\nu} - \frac{1}{3}\bar{h}^\alpha_\alpha\eta^{\mu\nu}\right)[3,0]_{\mu\nu} + \frac{v}{\Lambda_3^6}\left(\chi^{\mu\nu} - \frac{1}{3}\chi^\alpha_\alpha\eta^{\mu\nu}\right)(3[2,1]_{\mu\nu} + [3,0]_{\mu\nu}), \quad (6)$$

$$\delta\mathcal{L}_m = \frac{\kappa}{2}\chi^{\mu\nu}\tau_{\mu\nu} + \frac{\kappa}{2}\varphi\tau^\alpha_\alpha + \frac{\kappa u}{\Lambda_3^3}\varphi\bar{\pi}^{\mu\nu}\tau_{\mu\nu} - \frac{\kappa u}{2\Lambda_3^3}\varphi\varphi^{\mu\nu}\tau_{\mu\nu}. \quad (7)$$

Next, we diagonalize the bilinears by means of the non-local field redefinition $\chi_{\mu\nu} = \tilde{\chi}_{\mu\nu} + 3vA_{\mu\nu}$, where $A_{\mu\nu} = -\frac{2}{\Lambda_3^6}\Box^{-1}\left([1,2]_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}[1,2]\right)$. First, we obtain:

$$\delta\mathcal{L} = -\frac{1}{2}\tilde{\chi}_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta}\tilde{\chi}_{\alpha\beta} + \frac{1}{2}\varphi\mathcal{K}\varphi + \frac{9v^2}{\Lambda_3^6}A^{\mu\nu}\left([1,2]_{\mu\nu} - \eta_{\mu\nu}[1,2]\right) + \delta\mathcal{L}_{int} + \delta\mathcal{L}_m, \quad (8)$$

We will neglect the non-local contribution to the scalar propagator. The nonlocal terms are systematically smaller from the contributions $\propto \bar{h}_{\mu\nu}$ inside the Vainshtein radius, as can be directly checked. We will also neglect a new coupling to matter $\frac{3}{2}v\kappa A^{\mu\nu}\tau_{\mu\nu} \lesssim \frac{\kappa}{2}\varphi\tau^\alpha_\alpha$ introduced by the change of variables since $r, \rho \lesssim \rho_V$. However, we emphasize this truncation may be unsuitable for studying this theory in the far infra-red $p \lesssim m$, but that is not the regime we are interested in here.

The strong coupling scale To find the effective strong coupling scale in the source's background, $\Lambda_\oplus$, note that the linearised fluctuations are described by

$$\delta\mathcal{L}_{kin} = -\frac{1}{2}\tilde{\chi}_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta}\tilde{\chi}_{\alpha\beta} - \frac{1}{2}\varphi(\xi\partial_t^2 - P^{ij}\partial_i\partial_j)\varphi, \quad (9)$$

where

$$\xi = 3 - \frac{6u}{\Lambda_3^3}[0,1] + \frac{3}{\Lambda_3^6}(u^2-4v)[0,2] + \frac{20uv}{\Lambda_3^9}[0,3] + \frac{6v}{\Lambda_3^6}\left[(\hat{h}^t_t\bar{\pi}^{[i}_i)^{j]}_j + (\bar{\pi}^k_i\hat{h}^{[i}_k)^{j]}_j + (\bar{\pi}^{[i}_i\hat{h}^{j]}_k)^k_j\right],$$
$$P^{ij}k_ik_j = 3|\underline{k}|^2 - \frac{6u}{\Lambda_3^3}D_{[1,1]}(\underline{k}) + \frac{3}{\Lambda_3^6}(u^2-4v)D_{[1,2]}(\underline{k}) + \frac{20uv}{\Lambda_3^9}D_{[1,3]}(\underline{k}) + \frac{6v}{\Lambda_3^6}\left[(\hat{h}^{[i}_j\bar{\pi}^k_k)^{l]}_l\underline{k}_i\underline{k}^j + (\bar{\pi}^l_i\hat{h}^{[i}_l)^j_j\underline{k}^{k]}\underline{k}_k + \underline{k}^{[i}\underline{k}_i(\bar{\pi}^j_j\hat{h}^{k]}_l)^l_k\right], \quad (10)$$

and $D_{[1,s]}(\underline{k}) = \underline{k}^i\underline{k}_{[i}\bar{\pi}^{j_1}_{j_1}\dots\bar{\pi}^{j_s}_{j_s]}$, $\hat{h}_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{3}\bar{h}^\alpha_\alpha\eta_{\mu\nu}$. Here we are assuming that the variation of the background is small compared to the variation of the fluctuations. We also neglect the non-local contributions and for simplicity set $\bar{T}_{\mu\nu} = 0$ outside the source[2]. The interactions, schematically, are

$$\mathcal{I} = \frac{f(u,v)(\bar{h})^a(\tilde{\chi})^b\varphi^c(\Box^{-1})^d[\alpha,\beta]}{\Lambda_3^{a+b+c-2d+3\alpha+3\beta-4}}. \quad (11)$$

Now, to extract the strong coupling scale(s), we first canonically normalise the kinetic term. Noting $\xi, P \sim \mathcal{O}(1) + u\mathcal{O}\left(\frac{\partial\partial\bar{\pi}}{\Lambda_3^3}\right) + (u^2-4v)\mathcal{O}\left(\frac{\partial\partial\bar{\pi}}{\Lambda_3^3}\right)^2 + uv\mathcal{O}\left(\frac{\partial\partial\bar{\pi}}{\Lambda_3^3}\right)^3 + v\mathcal{O}\left(\frac{\bar{h}\partial\partial\bar{\pi}}{\rho^2\Lambda_3^6}\right)$, when $|u|, |v| \sim \mathcal{O}(1)$, the last term dominates inside the Vainshtein radius and so $\xi \sim P \sim \left(\frac{\rho_V}{\rho}\right)^3 \gg 1$. The same is true when $|u| \ll 1, |v| \sim \mathcal{O}(1)$. In contrast, when $|u| \sim \mathcal{O}(1)$, $|v| \ll 1$, $\bar{\pi} \propto \rho$ inside the Vainshtein radius, and so $\bar{\pi}''(\rho) \ll \bar{\pi}'(\rho)/\rho$ [20], which leads to a hierarchy of eigenvalues for P. In particular, we find one very large eigenvalue $P_1 \sim \xi \sim Q^2 \sim \left(\frac{\rho_V}{\rho}\right)^2$, and two smaller eigenvalues $P_2 \sim P_3 \sim Q \sim \frac{\rho_V}{\rho}$

Thus, for $|u|, |v| \sim \mathcal{O}(1)$ and $|u| \ll 1, |v| \sim \mathcal{O}(1)$, the canonical scalar field is $\hat{\varphi} \sim \left(\frac{\rho_V}{\rho}\right)^{3/2}\varphi$. Using $\bar{h} \sim \mathcal{O}\left(\frac{\kappa M}{8\pi\rho}\right)$, and $\partial\partial\pi \sim \Lambda_3^3 Q \sim \Lambda_3^3$, an interaction

$$\mathcal{I} = \frac{f(u,v)\left(\frac{\kappa M}{8\pi\rho}\right)^a\left(\frac{\rho_V}{\rho}\right)^{-\frac{3}{2}(c+\alpha)}(\tilde{\chi})^b\hat{\varphi}^c(\Box^{-1})^d(\partial\partial\hat{\varphi})^\alpha}{\Lambda_3^{a+b+c-2d+3\alpha-4}} \quad (12)$$

becomes strong at the scale

$$\Lambda_\mathcal{I} \sim \Lambda_3\left[\frac{(8\pi)^a(\rho\Lambda_3)^{a-\frac{3}{2}(c+\alpha)}}{f(u,v)(\kappa M)^{a-\frac{1}{2}(c+\alpha)}}\right]^{\frac{1}{b+c-2d+3\alpha-4}}. \quad (13)$$

The theory clearly becomes nonperturbative at the lowest such scale coming from any interactions present. It turns out that the lowest strong coupling scale arises from $\frac{v}{\Lambda_3^6}\left(\bar{h}^{\mu\nu} - \frac{1}{3}\bar{h}^\alpha_\alpha\eta^{\mu\nu}\right)[3,0]_{\mu\nu}$, so inserting $a = 1,\ b = c = d = 0,\ \alpha = 3$ into equation (13) we find the energy-momentum cut-off (in units of inverse kilometres)

$$\Lambda_{\text{cut-off}} \sim \frac{1}{\text{km}}\left(\frac{m}{H_0}\right)^{1/5}\left(\frac{\rho_\oplus}{\rho}\right)^{7/10}\left(\frac{M}{M_\oplus}\right)^{1/10} \quad (14)$$

where $\rho_\oplus \sim 6000$ km and $M_\oplus \sim 6\times10^{24}$ kg are the radius and mass of the earth respectively. The Hubble scale, H_0, is simply inserted to set units for the graviton mass. If we apply (14) to Earth's background to check how dRGT compares to GR, we find that

$$\Lambda_\oplus \sim \frac{1}{\text{km}}\left(\frac{m}{H_0}\right)^{1/5}. \quad (15)$$

[2] For explanation, see [22], which also addresses the concerns raised in [23].

This bound is very strong, even though the interaction involves Earth's weak Newtonian potential, $\bar{h} \propto \Phi_N$ because by locality and Lorentz symmetry there is a power of M_{Pl} in the coupling to compensate one of the extra powers of Λ_3 in the denominator (the other two are compensated by the momenta). So the coupling is enhanced by the ratio $M_{Pl}/\Lambda_3 \simeq (M_{Pl}/m)^{2/3}$ which is $\sim 10^{40}$ for Hubble mass gravitons. To conform with tabletop experiments [17], in dRGT one must suppress $\Lambda_\oplus$ by a factor of 10^6 to $\mathcal{O}(1)$ mm. This pushes the graviton mass up by $\gtrsim 30$ orders of magnitude, to $m \gtrsim 10^{-3}$ eV. So heavy a spin-2 field would experience Yukawa suppression at distances longer than a millimeter, failing to conform with GR at all currently tested scales [17]. Note, that the precise form of the background metric is irrelevant here. The only things needed are the form the interactions in (2) and the scaling with the source's potential which they encode. Clearly, if the spin-2 is not required to approximate GR the bounds are weakened.

One might hope to find better behavior in the special limits $|u| \sim \mathcal{O}(1)$, $|v| \ll 1$ when the helicity-2/scalar mixing is absent when $v = 0$. However, while the mass bounds are weaker, they are still significant. First, recall that now there is a hierarchy of eigenvalues for P. Using an orthogonal coordinate transformation to diagonalize P, the kinetic term for the scalar is given by

$$\delta\mathcal{L}_{kin} \supset -\frac{1}{2}\varphi(\xi\partial_t^2 - P_1\partial_1^2 - P_2\partial_2^2 - P_3\partial_3^2)\varphi\,. \quad (16)$$

We canonically normalise it by stretching two of the space directions, $(\hat{t}, \hat{x}_1, \hat{x}_2, \hat{x}_3) = \left(t, x_1, x_2\sqrt{\frac{\rho_V}{\rho}}, x_3\sqrt{\frac{\rho_V}{\rho}}\right)$, and defining $\hat{\varphi} \sim \varphi\sqrt{\frac{\rho_V}{\rho}}$, which yields the interactions

$$d^4x\frac{\varphi[\alpha,\beta]}{\Lambda_3^{3(\alpha+\beta-1)}} \sim d^4x\left(\frac{\rho_V}{\rho}\right)^{\beta-\frac{1}{2}(1+\alpha)}\frac{\hat{\varphi}(\partial\partial\hat{\varphi})^\alpha}{\Lambda_3^{3(\alpha-1)}}\,, \quad (17)$$

where we have used the fact that $\partial\partial\pi \sim \Lambda_3^3 Q \sim \Lambda_3^3\left(\frac{\rho_V}{\rho}\right)$ for the fine-tuned scenario. We focus on the processes involving spatial momentum transfer, mediated by

$$d^4x\frac{u^2-4v}{\Lambda_3^6}\varphi[2,1] \supset d^4\hat{x}\left[\frac{\mathcal{O}(1)}{\Lambda_3^3}\left(\frac{\rho_V}{\rho}\right)^{-\frac{1}{2}}\hat{\varphi}\hat{\partial}_1^2\hat{\varphi}\hat{\partial}_\perp^2\hat{\varphi} + \frac{\mathcal{O}(1)}{\Lambda_3^3}\left(\frac{\rho_V}{\rho}\right)^{\frac{1}{2}}\hat{\varphi}(\hat{\partial}_\perp^2\hat{\varphi})^2\right] \quad (18)$$

where as in [21], we are using the stretched transverse coordinates, denoted by $\perp = 2, 3$. Then the analysis as in [21] of the $2 \to 2$ scattering amplitude shows that the theory is strongly coupled at momenta above

$$\Lambda_{\text{cut-off}} \sim \frac{1}{20\text{km}}\left(\frac{m}{H_0}\right)^{5/9}\left(\frac{\rho_\oplus^3 M}{\rho^3 M_\oplus}\right)^{1/18}\left(\frac{\sqrt{16\pi G}}{\kappa}\right)^{-1/18} \quad (19)$$

with G Newton's constant, and where we have taken extra care to include the appropriate recaling when switching back from stretched to physical coordinates[3]. Comparing dRGT to GR, on Earth with standard Newtonian coupling $\kappa \sim \sqrt{16\pi G}$, yields

$$\Lambda_\oplus^{(k)} \sim \frac{1}{20\text{km}}\left(\frac{m}{H_0}\right)^{5/9}\,. \quad (20)$$

A spin-2 with Hubble mass suffers from breakdown of predictability due to quantum effects on scales of tens of kilometres. This is stronger than the bound in DGP on Earth [18] due to the coupling enhancement from the quartic galileon-like piece in (2) evaluated on the background, of order $\sim \pi'/(\Lambda_3^3\rho) \sim \rho_V/\rho \sim 10^{11}$. This term also dominates in the eigenvalues of the background matrix P, setting up the hierarchy which we find. Thus, to push the strong coupling scale in this limit down to the a millimeter we must require $m \gtrsim 10^{-15}$ eV. This places the Vainshtein radius of the Sun at $\lesssim 10^4$ km, well inside the orbit of Mercury, and also implying that outside of the Solar System the full potential has Yukawa suppression, in contrast with GR.

Summary We have shown that massive spin-2 dRGT theory has strong coupling problems leading to loss of predictivity at very low scales. This requires the mass of a spin-2 field with GR couplings to be much higher than the present Hubble scale. Hence the theory does not approximate GR well at cosmological scales.

Acknowledgments: We thank G. D'Amico, A. Geraci and G. Tasinato for discussions. NK thanks the School of Physics and Astronomy, Univ. of Nottingham, and YITP, Kyoto, Japan (YITP workshop YITP-T-12-03), for hospitality in the course of this work. CB was funded by a Univ. of Nottingham Anne McLaren fellowship. NK is supported by the DOE Grant DE-FG03-91ER40674. AP was funded by a Royal Society URF.

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