Comment on “Coherent Electron Cooling”

G. Stupakov and M. S. Zolotorev

Phys. Rev. Lett. **110**, 269503 — Published 26 June 2013

DOI: 10.1103/PhysRevLett.110.269503

Comment on "Coherent Electron Cooling" by Vladimir N. Litvinenko and Yaroslav S. Derbenev, PRL 102, 114801 (2009)*

G. Stupakov
SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA
M. S. Zolotorev
Center for Beam Physics, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

In Ref. [1] the authors put forward a concept of coherent electron cooling of hadrons. At the core of the concept lies the following idea: a density perturbation induced by an hadron in a co-propagating relativistic electron beam is amplified by several orders of magnitude in a free electron laser (FEL). After the FEL the electron beam is merged again with the hadron one and the amplified electric field in the electron beam acts back on each hadron resulting, after many repetitions, in cooling of the hadron beam. The efficiency of the process is critically determined by the amplification factor of the longitudinal electric field induced by the hadron in the electron beam. The authors claim that this amplification is equal to the FEL gain factor. In this comment we show that it is actually considerably smaller than the (conventionally defined) FEL gain with the smallness parameter to be the relative bandwidth σ_ω/ω_0 of the FEL amplifier.

In our analysis we use a standard one-dimensional linear FEL theory which gives a reasonably good approximation for typical parameters of modern FELs, (see, e.g., [2, 3]). We assume a helical undulator with the undulator parameter K, the undulator period $\lambda_u = 2\pi/k_u$ and length l_u. An electron beam with a localized line density perturbation $\delta n_0(z) = Z\delta(z)$ induced by an hadron (δn_0 has dimension of inverse length, z is the longitudinal coordinate inside the bunch) enters the FEL.

We expand $\delta n_0(z)$ into Fourier integral and then use the linear FEL theory [3] to propagate each harmonic from the beginning to the end assuming a high-gain regime of the FEL. Making the inverse Fourier transformation at the exit we find the final density in the beam:

$$\delta n(z,\tau) = \frac{k_0 Z\sqrt{\rho}}{\sqrt{2\pi\tau}} e^{ik_0 z + (\sqrt{3}+i)\rho\tau + i\pi/12} e^{-\frac{\rho(\tau-3k_0 z)^2}{(\sqrt{3}-i)\tau}}, \quad (1)$$

where $\tau = k_u l_u$, $k_0 = \omega_0/c = 2\gamma^2 k_u/(1+K^2)$ corresponds to the fundamental FEL wavelength and ρ is the standard FEL parameter defined by

$$(2\rho)^3 = \frac{2\lambda_u}{\gamma k_0 S}\frac{K^2}{1+K^2}\frac{I}{I_A}, \quad (2)$$

with γ the electron beam Lorentz factor, S the electron beam transverse area, I the electron beam current and $I_A = mc^3/e \approx 17$ kA the Alfven current. Introducing the standard *power* gain length L_g for the FEL, $L_g^{-1} = 2\sqrt{3}\rho k_u$, we replace $\rho\tau$ by $l_u/2\sqrt{3}L_g$. Note that in a high-gain FEL $l_u \gg L_g$. It follows from (1) that the maximal value of $|\delta n|$ is $\max|\delta n| = 3^{1/4}\pi^{-1/2}k_0 Z\rho(L_g/l_u)^{-1/2}e^{l_u/2L_g}$. The longitudinal electric field $\delta E_\parallel(z,\tau)$ generated by the density perturbation $\delta n(z,\tau)$ in the beam is $\delta E_\parallel = 4\pi e\delta n/k_0 S$, and

$$\max|\delta E_\parallel(z,\tau)| = 4\cdot 3^{1/4}\sqrt{\pi}\rho\frac{Ze}{S}\sqrt{\frac{L_g}{l_u}}e^{l_u/2L_g}. \quad (3)$$

The initial electric field of the localized perturbation in the 1D model is $E_0 = 2\pi Ze/S$. Hence we can write the result (3) as $\max|\delta E_\parallel| = GE_0$, where the *amplification factor* G is

$$G = 2\frac{3^{1/4}}{\sqrt{\pi}}\rho\sqrt{\frac{L_g}{l_u}}e^{l_u/2L_g}. \quad (4)$$

The factor G can be expressed through the standard (amplitude) *FEL amplification factor* G_0. The latter is usually defined as a ratio of the final (exit) amplitude of a sinusoidal density perturbation at the fundamental wavelength $2\pi/k_0$ to its initial value; as it follows from the linear FEL theory, in high-gain regime, $G_0 = \frac{1}{3}e^{l_u/2L_g}$. We see that the amplification factor of the longitudinal field (4) is *much smaller* than G_0:

$$G = 2\frac{3^{5/4}}{\sqrt{\pi}}\rho\sqrt{\frac{L_g}{l_u}}G_0 \approx \frac{\sigma_\omega}{\omega_0}G_0, \quad (5)$$

in contrast to the statement in [1] where G is identified with G_0. The formula on the right hand side of (5) is more general then (4) and is valid even in 3D FEL case. Given that the parameter ρ is of order of 10^{-3} in a typical modern FEL, the amplification of the longitudinal field is likely to be two or three orders of magnitude smaller than G_0.

As discussed in [1], the maximally achievable FEL gain is limited by FEL saturation. The saturation length l^{sat} can be estimated from the linear FEL theory if one equates the FEL power exponentially growing from shot noise in the electron beam to the FEL power in saturation which is approximately equal to $\rho\gamma mc^2 I/e$ (see [2, 3]). Using such an estimate and the parameters quoted in [1] for a hypothetical FEL for an LHC cooler: $\lambda_0 = 10$ nm, $I = 100$ A, $\gamma = 7.6\times 10^3$ and assuming the beam area $S = 150\mu m \times 150\mu m$, we found $\rho = 8.7\times 10^{-4}$ and the saturation length $l^{\mathrm{sat}} = 18.3L_g$. Assuming $l_u = l^{\mathrm{sat}}$, Eq. (4) gives $G = 2.8$ which is more than two orders short of the value $G = 500$ assumed by the authors of [1].

[1] V. N. Litvinenko and Y. S. Derbenev, Phys. Rev. Lett. **102**, 114801 (2009).
[2] P. Schmuser, M. Dohlus, and J. Rossbach, *Ultraviolet and soft x-ray free-electron lasers* (Springer, 2008).
[3] S. Krinsky, AIP Conference Proceedings **648**, 23 (2002).

* Work supported by the U.S. Department of Energy under contracts No. DE-AC02-76SF00515 and DE-AC02-05CH11231.