Qin et al. Reply:

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Reply to "Comment on "Woltjer-Taylor State without Taylor's Conjecture: Plasma Relaxation at all Wavelengths""

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We welcome the Comment [1] and energetic criticism by Chen and Fan of our Letter [2]. Nevertheless, we disagree with their main conclusions.

Firstly, the counter-example put forward does not support the claim of the Comment. For this solution, $\dot{x} = 0$ for all x as $t \to \infty$, contradicting the property that $\dot{\Delta} = 0$ if and only if $\Delta = 0$ as proved in [2]. The smoothness of functions will ensure that if $\dot{\Delta}$ is very small, then Δ is also, *i.e.*, the system is very close to the Woltjer-Taylor (WT) state.

The main result of the Comment is based on the equation $d\boldsymbol{B}_{\boldsymbol{k}}/dt \approx -k^2 \boldsymbol{B}_{\boldsymbol{k}}/\sigma\mu_0$ (in SI units). This equation is flawed, stating that each wavelength decays independently and completely ignoring all coupling between different scales due to the nonlinear dynamics of the plasma. Indeed, this is simply a diffusion equation for $\boldsymbol{B}$, showing that all effects due to magnetic field advection, the $\boldsymbol{j} \times \boldsymbol{B}$ force, and nonlinear convection have been removed. The Comment makes the incorrect claim that this equation is related to Eq. (27) of our Letter [2]. In actuality, all that is implied by Eq. (27) of [2] is that the total magnetic energy variation is primarily due to resistive dissipation. There is no stipulation that plasma dynamics at each individual wavelength must be dominated by resistive decay, and indeed this will not be true in all but the most simple cases due to the strongly nonlinear behavior of the plasma.

To emphasize these nonlinear characteristics and to further rule out the possibility of Δ decaying at the same rate as H^2, we would like to refine the physical picture presented in [2]. Relaxation of the plasma towards the Woltjer-Taylor state can be divided into three phases, as illustrated in Fig. 1. Phase I is characterized by the dominance of large-scale nonlinear interactions. Since the resistivity is assumed to be small, plasma dynamics should be close to ideal and the magnetic energy fluctuation spectrum will broaden. In this phase, the rate of change of Q, W, and H will be comparable, *i.e.*, $\left|\dot{Q}/Q\right| \sim \left|\dot{W}/W\right| \sim \left|\dot{H}/H\right|$. As the plasma evolves, the cascade to shorter wavelength and widening of the spectrum will increase the decay rate of W faster than that of Q for almost any spectrum of finite width. Since $\dot{Q}/Q \approx -c^2\eta W/2\pi Q$ and W is decreasing faster than Q, the decay rate of Q must then decrease in time, and at the end of phase I it is reasonable to expect $\left|\dot{W}/W\right| \gg \left|\dot{Q}/Q\right|$. From the Cauchy-Schwartz inequality $\left(\dot{H}\right)^2 \leq \dot{W}\dot{Q}$, we have $\left|\dot{W}/W\right|^2 \gg \left|\dot{W}\dot{Q}/WQ\right| \geq \left(\dot{H}\right)^2/WQ$. Assuming $H^2 \sim QW$, we expect that at the beginning of phase II $\left|\partial_t(QW)/QW\right| \sim \left|\dot{W}/W\right| \gg \left|\dot{H}/H\right|$, *i.e.*, $\partial_t\left(\Delta/H^2\right) \ll 0$. This indicates that QW will decrease at a faster rate than H^2 in phase II, and the gap between QW and H^2 will close up faster than H^2 decays. Towards the end of phase II, Δ will become very small and the system approaches the Woltjer-Taylor state.

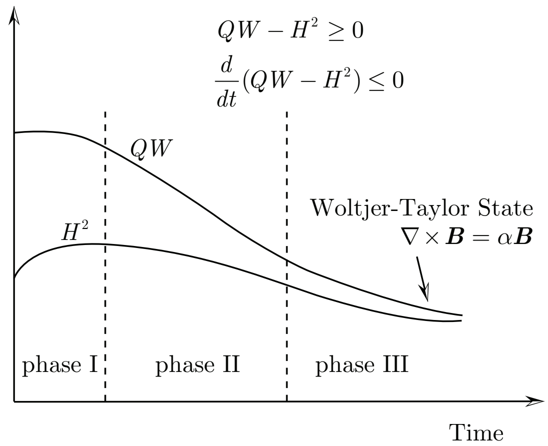

Figure 1. Three phases of the evolution toward the Woltjer-Taylor state.

In addition, as a result of approaching the WT state, the decay rates of Q, W, and H again become comparable. As the decay process enters phase III, the system can only continue to evolve towards the WT state because as we have proved, Δ must decrease with time. In particular, the system cannot reverse the outcome of its fast evolution accumulated in phase II.

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[1] J. H. Chen and H. Y. Fan, Physical Review Letters **110**, in press (2013).

[2] H. Qin, W. Liu, H. Li, and J. Squire, Phy. Rev. Lett. **109**, 235001 (2012).