



This is the accepted manuscript made available via CHORUS. The article has been published as:

# Gyrokinetic Theory of Turbulent Acceleration of Parallel Rotation in Tokamak Plasmas

Lu Wang and P. H. Diamond

Phys. Rev. Lett. **110**, 265006 — Published 27 June 2013

DOI: [10.1103/PhysRevLett.110.265006](https://doi.org/10.1103/PhysRevLett.110.265006)

# Gyrokinetic theory of turbulent acceleration of parallel rotation in tokamak plasmas

Lu Wang<sup>1,2,\*</sup> and P. H. Diamond<sup>2,3</sup>

<sup>1</sup>CEEE, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China

<sup>2</sup>WCI Center for Fusion Theory, NFRI, Gwahangno 113, Yuseong-gu, Daejeon 305-333, Korea

<sup>3</sup>CMTFO and CASS, University of California, San Diego, La Jolla, CA 92093-0424, USA

(Dated: May 15, 2013)

A mechanism for turbulent acceleration of parallel rotation is discovered using gyrokinetic theory. This new turbulent acceleration term cannot be written as a divergence of parallel Reynolds stress. Therefore, turbulent acceleration acts as a *local* source/sink of parallel rotation. The physics of turbulent acceleration is intrinsically different from the Reynolds stress. For symmetry breaking by positive intensity gradient, a positive turbulent acceleration, i.e., co-current rotation is predicted. The turbulent acceleration is independent of mean rotation and mean rotation gradient, and so constitutes a *new* candidate for the origin of spontaneous rotation. A quasilinear estimate for ion temperature gradient turbulence shows that the turbulent acceleration of parallel rotation is explicitly linked to the ion temperature gradient scale length and temperature ratio  $T_{i0}/T_{e0}$ . Methods for testing the effects of turbulent parallel acceleration by gyrokinetic simulation and experiment are proposed.

Problems of spontaneous flow generation, spin-up and formation of differential rotation are ubiquitous in physics. Examples of these include, but are not limited to: the origin of the solar differential rotation[1], the formation of the atmospheric jet stream[2], the mechanism of spin up of a stratified fluid in a container[3], the formation of the solar tachocline[4], and the origin of intrinsic rotation in tokamak plasmas[5]. Many instances of spontaneous flow generation occur in a state of eddy or wave turbulence, and thus qualify as problems in the self-organization of flow patterns in turbulence[6]. This type of problem has considerable overlap with the classic paradigm of the turbulent magnetic dynamo[7]. Theoretical approaches to the question of flow self-organization are usually based on mean field theory methods. Examples include the anisotropic kinetic Alpha effect[8] and the closely related Lambda effect[9], both derived from mean field hydrodynamics.

Spontaneous (or, intrinsic) plasma rotation is an example of a spin-up process and is of great interest in magnetic fusion[10]. Plasma rotation is thought to play an important role in stabilizing macroscopic magnetohydrodynamic (MHD) instabilities, such as resistive wall modes[11] and in reducing or regulating microturbulence and the associated losses. Intrinsic rotation is particularly important for ITER, which cannot be adequately penetrated by conventional neutral beam injection (NBI), and so cannot achieve sufficient NBI-driven rotation. Realization of this has driven intensive research in spontaneous rotation in the MFE community in recent years[12, 13]. The associated theoretical research has also focused on mean field approaches to calculating the parallel rotation profile by Reynolds stress modelling, specialized to the complex geometry of tokamak plasmas[14]. Interestingly, just as the solar differential rotation is thought to arise from heat flux driven convective turbulence in a rotating system, spontaneous toka-

mak rotation is thought to arise from heat flux driven drift wave turbulence in a helical magnetic field[15].

In this paper, we propose a new mechanism for the origin of spontaneous rotation in tokamaks. This mechanism is *turbulent acceleration*, and arises from the partially acoustic character of drift-ion temperature gradient (ITG) turbulence. This mechanism does not arise from a Reynolds stress or from momentum *transport*, and thus has no antecedent in previous work on the mean field theory of rotation or flow generation.

Momentum transport can influence plasma rotation. The general parallel momentum transport equation per ion mass can be written as

$$\frac{\partial \langle nU_{\parallel} \rangle}{\partial t} + \nabla \cdot \Gamma_{r,\parallel} = M_{\parallel}, \quad (1)$$

where  $\Gamma_{r,\parallel}$  is the radial flux of parallel momentum, and  $M_{\parallel}$  is the turbulent momentum source/sink. The basic form of the momentum flux is given by  $\Gamma_{\parallel} = \langle n \rangle \langle \tilde{v}_r \tilde{U}_{\parallel} \rangle + \langle \tilde{v}_r \tilde{n} \rangle \langle U_{\parallel} \rangle + \langle \tilde{v}_r \tilde{n} \tilde{U}_{\parallel} \rangle$ [16]. Here, the parallel Reynolds stress is  $\Pi_{r,\parallel} = \langle \tilde{v}_r \tilde{U}_{\parallel} \rangle$ , including diffusion, velocity pinch[17] and residual stress,  $\Pi_{r,\parallel}^{Res}$ , which has been intensively investigated[14]. In particular, the residual stress is one component of the parallel Reynolds stress which is not proportional to either flow or flow gradient. It is thought to be the origin of intrinsic torque, since it contributes a term which is proportional to  $\nabla \cdot \Pi_{r,\parallel}^{Res}$  to the parallel mean flow equation. This term contributes an intrinsic torque  $-\nabla \cdot \Pi_{r,\parallel}^{Res}$ , which can accelerate the plasma.

Although the most natural quantity for theoretical study is toroidal angular momentum density, the quantity measured and estimated from experimental observation is the toroidal ion velocity  $U_{\phi}$ . The magnitude of  $U_{\phi}$  can be approximated by  $U_{\parallel}$  for tokamaks, since the toroidal field is much larger than the poloidal field. To explicitly link theoretical results to experimental observations, it is more convenient to investigate the evolution

equation of parallel velocity rather than that of parallel momentum density. In general, the mean parallel velocity evolution equation can be written as

$$\frac{\partial \langle U_{\parallel} \rangle}{\partial t} + \nabla \cdot \Pi_{r,\parallel} = a_{\parallel}, \quad (2)$$

where  $\Pi_{r,\parallel}$  is the parallel Reynolds stress, and  $a_{\parallel}$  is the turbulent acceleration. In this work, we identify a turbulent acceleration term which cannot be written as a divergence of a Reynolds stress. It enters the RHS of the parallel rotation equation, and follows from gyrokinetic theory. The turbulent acceleration significantly affects parallel rotation, but its physics is fundamentally different from that of the residual stress. The residual stress is one component of the parallel Reynolds stress, so it enters into the rotation equation via its divergence, while turbulent acceleration is a local source/sink. We note that this difference is somewhat analogous to the difference between the turbulent energy pinch[18] (one component of the heat flux) and turbulent heating[19]. Just as the pinch and the turbulent heating, the turbulent acceleration and the turbulent residual stress also co-exist and *both* are relevant to spontaneous parallel rotation. Therefore, investigation of the parallel turbulent acceleration is meaningful as well as potentially important.

We start from the nonlinear electrostatic gyrokinetic equation, in the continuity form[20]

$$\frac{\partial}{\partial t} (FB^*) + \nabla \cdot \left( \frac{d\mathbf{R}}{dt} FB^* \right) + \frac{\partial}{\partial v_{\parallel}} \left( \frac{dv_{\parallel}}{dt} FB^* \right) = 0, \quad (3)$$

for which gyrocenter equations of motion are

$$\frac{d\mathbf{R}}{dt} = v_{\parallel} \hat{\mathbf{b}} + \frac{c}{eB^*} \hat{\mathbf{b}} \times \left( e \nabla \langle \delta \phi \rangle + \mu \nabla B + m_i v_{\parallel}^2 \hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}} \right), \quad (4)$$

and

$$\frac{dv_{\parallel}}{dt} = - \frac{\mathbf{B}^*}{m_i B^*} \cdot (e \nabla \langle \delta \phi \rangle + \mu \nabla B). \quad (5)$$

Here,  $F = F(\mathbf{R}, \mu, v_{\parallel}, t)$  is the gyrocenter distribution function,  $\mu$  is the gyrocenter magnetic moment,  $\mathbf{B}^* = \mathbf{B} + v_{\parallel} \nabla \times \hat{\mathbf{b}}$ ,  $B^* = \hat{\mathbf{b}} \cdot \mathbf{B}^*$  is the Jacobian of the transformation from the particle phase space to the gyrocenter phase space, and  $\langle \langle \dots \rangle \rangle$  denotes gyroaveraging.

By taking the moments of the nonlinear gyrokinetic equation, we obtain the equation for gyrocenter density,  $n \equiv (2\pi/m_i) \int d\mu dv_{\parallel} FB^*$ ,

$$\frac{\partial}{\partial t} n + \nabla \cdot \left[ \left( U_{\parallel} \hat{\mathbf{b}} + \mathbf{v}_{E \times B} + \mathbf{v}_{d\kappa} + \mathbf{v}_{d\nabla} \right) n \right] = 0, \quad (6)$$

and the equation for gyrocenter parallel momentum per ion mass,  $nU_{\parallel} \equiv (2\pi/m_i) \int d\mu dv_{\parallel} FB^* v_{\parallel}$ ,

$$\begin{aligned} & \frac{\partial}{\partial t} (nU_{\parallel}) + \nabla \cdot \left[ \frac{P_i}{m_i} \hat{\mathbf{b}} + (\mathbf{v}_{E \times B} + 3\mathbf{v}_{d\kappa} + \mathbf{v}_{d\nabla}) nU_{\parallel} \right] \\ &= - \left[ \frac{e}{m_i} \hat{\mathbf{b}} \cdot \nabla \delta \phi + \frac{c}{B} \hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}) \cdot \nabla \delta \phi U_{\parallel} \right] n. \end{aligned} \quad (7)$$

Here, a long wavelength approximation  $k_{\perp}^2 \rho_i^2 \ll 1$  is used,  $P_i = 2\pi \int d\mu dv_{\parallel} FB^* (v_{\parallel} - U_{\parallel})^2 = (2\pi/m_i) \int d\mu dv_{\parallel} FB^* \mu B$  is the ion pressure,  $\mathbf{v}_{E \times B} = c \hat{\mathbf{b}} \times \nabla \delta \phi / B$  is the fluctuating  $\mathbf{E} \times \mathbf{B}$  drift velocity,  $\mathbf{v}_{d\kappa} = cT_i / (eB) \hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}})$  is the magnetic curvature drift velocity, and  $\mathbf{v}_{d\nabla} = cT_i / (eB^2) \hat{\mathbf{b}} \times \nabla B$  is the magnetic gradient drift velocity. By summing Eq. (7) over all species and using the quasi-neutrality equation, a total momentum conservation equation can be obtained. We do not present it in this work, since gyrokinetic momentum conservation has already been discussed in detail in several recent works[21, 22]. On one hand, the off-diagonal component of the electric part of the Maxwell stress tensor,  $\Pi_{r,\parallel}^E \propto \tilde{E}_r \tilde{E}_{\parallel}$ , coming from the polarization density[21], can easily be recovered. It plays a similar role to the usual residual stress, which also enters the total momentum equation, via its divergence. On the other hand, the time variation of toroidal momentum density due to  $\mathbf{E} \times \mathbf{B}$  drifts [22] should be absent here, because the parallel component of  $\mathbf{E} \times \mathbf{B}$  vanishes. The terms on the RHS of Eq. (7) are consistent with the turbulent toroidal -rather than parallel- momentum source in Ref.[23]. This comes from the parallel electric field, along with the effective magnetic field  $\mathbf{B}^*/B^*$ . It should be noted that spontaneous rotation does *not* contradict momentum conservation. The first is mainly carried by ions, while the second follows from the sum over the momenta of *all* species and the field momentum.

To obtain a more experimentally relevant quantity, the focus of this work is parallel *ion rotation velocity*, but not parallel momentum conservation or the ion parallel momentum. We subtract Eq. (6) from Eq. (7), and so obtain the ion parallel flow velocity equation

$$\begin{aligned} & \frac{\partial}{\partial t} U_{\parallel} + \nabla \cdot [(\mathbf{v}_{E \times B} + 4\mathbf{v}_{d\nabla}) U_{\parallel}] \\ &= - \left[ 2\mathbf{v}_{d\nabla} \cdot \frac{\nabla n}{n} - \frac{e}{T_i} \mathbf{v}_{d\nabla} \cdot \nabla \delta \phi - 2\mathbf{v}_{d\nabla} \cdot \frac{\nabla T_i}{T_i} \right] U_{\parallel} \\ & \quad - \frac{1}{m_i} \hat{\mathbf{b}} \cdot \left( e \nabla \delta \phi + \frac{1}{n} \nabla P_i \right). \end{aligned} \quad (8)$$

In low- $\beta$  plasmas,  $\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}) \simeq (1/B) \hat{\mathbf{b}} \times \nabla B$ , so the magnetic curvature drift can be approximated as the magnetic gradient drift, i.e.,  $\mathbf{v}_{d\kappa} \simeq \mathbf{v}_{d\nabla}$ . Note that the drift velocities are compressible in toroidal geometry.  $\nabla \cdot \mathbf{v}_{E \times B} \simeq 2(e/T_i) \mathbf{v}_{d\nabla} \cdot \nabla \delta \phi$  and  $\nabla \cdot \mathbf{v}_{d\nabla} = \mathbf{v}_{d\nabla} \cdot (\nabla T_i / T_i)$  are used when deriving the preceding equation. The parallel (toroidal) velocity evolution equation was also studied in [24, 25] from reduced MHD equations and Braginskii fluid equations, respectively. However, ion pressure gradient along the toroidal direction was ignored in [25]. Both these works focused on magnetic geometric effects on rotation, which is different from the emphasis of our work. In particular, the key effect of our study is not a

toroidal effect, but rather is related to parallel pressure gradients (i.e. acoustics).

The mean parallel velocity equation can be derived by taking a flux surface average of Eq. (8), i.e.,

$$\frac{\partial}{\partial t} \langle U_{\parallel} \rangle + \nabla \cdot \Pi_{r,\parallel} = a_{\parallel}. \quad (9)$$

As mentioned before,  $\Pi_{r,\parallel}$  is the parallel Reynolds stress, which has been intensively studied in previous work. In this work, we focus on the RHS of Eq. (9),  $a_{\parallel}$ , the parallel turbulent acceleration. It can be written as

$$a_{\parallel} = \frac{1}{m_i n_0} \left\langle \delta n \hat{\mathbf{b}} \cdot \nabla \delta T_i \right\rangle - 2 \left\langle \frac{\delta T_i}{T_i} \mathbf{v}_{d\nabla} \cdot \nabla \frac{\delta n}{n_0} \right\rangle \langle U_{\parallel} \rangle + \left\langle \delta U_{\parallel} \mathbf{v}_{d\nabla} \cdot \nabla \left( \frac{e\delta\phi}{T_i} - 2 \frac{\delta n}{n_0} - 2 \frac{\delta T_i}{T_i} \right) \right\rangle. \quad (10)$$

Note that this turbulent acceleration term *cannot* be written as a divergence of a parallel Reynolds stress! It plays the role of local source/sink of parallel rotation, and so is significant for parallel rotation. In particular, the first term in the turbulent acceleration is related to gyrocenter density fluctuations and ion temperature fluctuations, but is independent of the parallel velocity. Therefore, it can provide a net drive for spontaneous parallel rotation without any external momentum input. However, the physics is fundamentally different from the intrinsic torque induced by residual stress which enters the parallel rotation equation via the term  $\sim \nabla \cdot \Pi_{r,\parallel}^{Res}$ . We also note that the first term results from the parallel ion pressure gradient, and so is related to ion acoustic dynamics. The origin of this turbulent acceleration is different from that of the turbulent momentum source in Ref. [23], which resulted from the toroidal electric field. The first term always exists, whether the geometry is toroidal or cylindrical. This is different from the toroidal effects discussed in [24, 25]. However, the other two terms come from the correction to the parallel direction along the background magnetic field, due to toroidal geometry effects. These are subdominant to the first term. Thus, we focus only on the ion acoustic related turbulent acceleration in the following discussion.

We present a quasilinear estimation of the parallel acceleration,  $a_{\parallel} \approx v_{thi}^2 \left\langle \delta n \hat{\mathbf{b}} \cdot \nabla \delta \hat{T}_i \right\rangle$ , with  $v_{thi} = \sqrt{T_{i0}/m_i}$  is the ion thermal velocity,  $\delta \hat{n} = \delta n/n_0$ , and  $\delta \hat{T}_i = \delta T_i/T_{i0}$ . For simplicity, an adiabatic electron response is assumed, i.e.,  $\delta \hat{n} = e\delta\phi/T_{e0} = \tau\delta\hat{\phi}$ , with  $\tau = T_{i0}/T_{e0}$  and  $\delta\hat{\phi} = e\delta\phi/T_{i0}$ . Therefore, the temperature ratio dependence will be introduced in the quasilinear expression of the turbulent acceleration. The ion temperature fluctuations can be obtained by linearizing the ion temperature evolution equation[16] as follows

$$-i \left( \omega_k - \frac{14}{3} \omega_{di,k} + i|\Delta\omega_k| \right) \delta \hat{T}_i = i\omega_{*Ti} \delta \hat{\phi}_k - i \frac{4}{3} \omega_{di,k} \left( \delta \hat{n}_k + \delta \hat{\phi}_k \right), \quad (11)$$

where  $\omega_{*Ti} = -k_{\theta} \rho_i v_{thi}/L_{Ti}$  is the ion diamagnetic drift frequency, with  $L_{Ti} = -(\partial \ln T_i / \partial r)^{-1}$  as the ion temperature gradient scale length,  $\omega_{di} \approx -k_{\theta} \rho_i v_{thi}/R_0$  is the ion magnetic drift frequency,  $\Delta\omega_k$  is the  $\mathbf{E} \times \mathbf{B}$  nonlinearity-induced self-decorrelation rate, and the absolute value of  $\Delta\omega_k$  is required by causality. Combining the density fluctuations and the ion temperature fluctuations, the turbulent acceleration can be written as

$$a_{\parallel} \approx \tau v_{thi}^3 \frac{\rho_i}{L_{Ti}} \sum_k (\Re \tau_{ck}) \left[ 1 - \frac{4}{3} (1 + \tau) \frac{L_{Ti}}{R_0} \right] k_{\parallel} k_{\theta} \langle |\delta \hat{\phi}_k|^2 \rangle, \quad (12)$$

where  $\tau_{ck} = [-i(\omega_k - \frac{14}{3}\omega_{di,k} + i|\Delta\omega_k|)]^{-1}$  is inverse of the ion propagator, and  $\Re$  means real part. Since  $L_{Ti}$  is small in comparison with the major radius  $R_0$ , the second term in Eq. (12) is subdominant. In the following, we keep only the first term, which comes from the ion diamagnetic drift.

Note that nonzero turbulent acceleration also requires parallel symmetry breaking as does the residual stress in the parallel Reynolds stress. Various cases of parallel symmetry breaking mechanisms for the residual stress, such as  $\mathbf{E} \times \mathbf{B}$  shear[26], intensity gradient[27] etc, have been studied. In toroidal geometry,  $k_{\theta} = m/r$  and  $k_{\parallel} = k_{\theta} x \hat{s}/(qR_0)$ , with  $\hat{s}$  is the magnetic shear,  $x = r_{m,n} - r$ , and  $r_{m,n}$  is the radial location of the resonant surface. Proceeding as in the study of the residual stress caused by intensity gradient, i.e.,  $I_k(x) = |\phi_k|^2(x) = I_k(0) + x(\partial I_k/\partial x)$ [27], it follows that the turbulent acceleration can be written as

$$a_{\parallel} \approx \tau v_{thi}^3 \frac{\rho_i}{L_{Ti}} \frac{\hat{s}}{qR_0} \sum_k (\Re \tau_{ck}) k_{\theta}^2 x^2 \frac{\partial I_k}{\partial x}. \quad (13)$$

For  $\hat{s} > 0$ , a positive (negative) intensity gradient results in a positive (negative) parallel turbulent acceleration, and a co-current (counter-current) rotation is thus driven by this effect.

To elucidate the magnitude of the turbulent acceleration, we compare it to the divergence of the residual stress  $\nabla \cdot \Pi_{\parallel}^{Res}$  and to the divergence of the diffusive velocity flux  $\nabla \cdot (\chi_{\parallel} \nabla \langle U_{\parallel} \rangle)$ . We claim there are the appropriate comparisons, as the effects are all dimensionally similar, and scale as the rate of change of velocity. For symmetry breaking by intensity gradient, the residual stress is given by  $\Pi_{\parallel}^{Res} = v_{thi}^3 \frac{\hat{s}}{qR_0} \sum_k (\Re \tau_{ck}) k_{\theta}^2 \rho_i x^2 \frac{\partial I_k}{\partial x}$  [27]. Thus, the ratio of the turbulent acceleration to the divergence of the residual stress is

$$a_{\parallel} / \nabla \cdot \Pi_{\parallel}^{Res} \sim \tau L / L_{Ti}. \quad (14)$$

Here,  $L$  is the length scale of variation of the residual stress, which can vary between  $L_{Ti}$  and  $L_I$ , with  $L_I = ((\partial I_k/\partial x)/I_k)^{-1}$  being the intensity gradient scale length. We see that the two contributions to the intrinsic torque are roughly comparable, depending upon  $\tau$  and

$L/L_{T_i}$ . Comparing  $a_{\parallel}$  to the turbulent diffusive velocity decay rate  $\chi_{\parallel}\langle U_{\parallel}\rangle/L_v^2$  (corresponding to diffusive velocity confinement rate  $\langle U_{\parallel}\rangle/\tau_{\parallel}$ , where  $1/\tau_{\parallel} \sim \chi_{\parallel}/L_v^2$ ) gives:

$$\frac{a_{\parallel}L_v^2}{\chi_{\parallel}\langle U_{\parallel}\rangle} = \frac{L_v^2\Delta^2}{\rho_i L_{T_i} L_s L_I} \tau \frac{v_{thi}}{\langle U_{\parallel}\rangle}. \quad (15)$$

Here,  $L_v$  is the scale length of the parallel flow gradient,  $L_s$  is the magnetic shear scale length, and  $\chi_{\parallel} = \sum_k (\Re \tau_{ck}) k_{\theta}^2 \rho_i^2 v_{thi}^2 I_k$  is the turbulent diffusivity of parallel flow. For the particular case  $v_{thi}/\langle U_{\parallel}\rangle \sim a/\rho_i$  and  $L_v \sim L_{T_i} \sim a$ , Eq. (15) reduces to  $(a_{\parallel}L_v^2)/(\chi_{\parallel}\langle U_{\parallel}\rangle) \sim \tau(\Delta^2/L_s L_I)(a^2/\rho_i^2)$ . This ratio is roughly order of unity. We have shown that the turbulent acceleration is qualitatively different from, but is quantitatively comparable to, the divergence of the residual stress and the divergence of the diffusive flux. Therefore, it is necessary to include the turbulent acceleration for the study of parallel rotation.

One important question concerning turbulent acceleration is how the theory can be tested by numerical simulations. The crux of this issue is that while the *total* local intrinsic torque density  $\tau_I$  can be measured directly [28], -by numerical cancellation experiments [29] or other means, -it is not so clear how to distinguish residual stress and acceleration contributions, since  $\tau_I = -\nabla \cdot \Pi_{\parallel}^{Res}(r) + a_{\parallel}(r)$ . To this end, we propose a comparison between integrated intrinsic torque, as measure for the cases of (i) vanishing fluctuations on the boundary (i.e.,  $\delta\phi(\pm a) = 0$ ) and (ii) finite boundary fluctuations ( $\delta\phi(\pm a) \neq 0$ ). This choice follows from the observation that the radially integrated intrinsic torque is given by

$$T_I = \int_{-a}^a \tau_I(r) dr = -\Pi_{\parallel}^{Res}(r)|_{-a}^a + \int_{-a}^a a_{\parallel}(r) dr. \quad (16)$$

Note that  $T_I \neq 0$  is required for a *net* spin-up. For the case of vanishing turbulence on the boundary,  $T_I = \int_{-a}^a a_{\parallel}(r) dr$ , so a finite value can result *only* from turbulent acceleration, and thus  $T_I$  constitutes a direct measure of the integrated turbulent acceleration. However, for a corresponding case with  $\delta\phi(\pm a) \neq 0$ , but other quantities (i.e., parameters, profiles, etc) the same,  $T_I$  has contributions from *both* the residual stress on the boundary and the radially integrated  $a_{\parallel}$ . Subtracting the results for  $\int_{-a}^a \tau_I(r) dr$  for the two cases could suggest a trend which reveals the residual stress contribution. Note that  $a_{\parallel}$  and  $\Pi_{\parallel}^{Res}$  both depend upon the same spectral cross-correlator and the same symmetry breaking mechanism. This comparison should separate at least the radially integrated contributions to the intrinsic torque from  $\Pi_{\parallel}^{Res}$ . Since it is a comparison of radially integrated quantities, it should not be very sensitive to turbulence spreading near the boundary and related phenomena. Finally, truth in advertising compels us to say that this test will elucidate only the radially integrated  $a_{\parallel}$ , but not the

*local profile* of  $a_{\parallel}$ . Further consideration is required to address that.

Of course, a second important question is how to measure  $a_{\parallel}$  and thus test the theory in a *physical* experiment. To this end, we note that the presence of  $a_{\parallel}$  necessarily breaks the condition of zero total velocity flux (i.e. Reynolds stress) in a steady state of intrinsic rotation. Thus,  $\partial\langle U_{\parallel}\rangle/\partial t = -\partial_r\langle \tilde{v}_r \tilde{U}_{\parallel}\rangle + a_{\parallel}$  and stationarity imply

$$\langle \tilde{v}_r \tilde{U}_{\parallel}\rangle = \int_0^r dr' a_{\parallel}(r'), \quad (17)$$

so that a finite value of  $\langle \tilde{v}_r \tilde{U}_{\parallel}\rangle(r)$  implies  $\int_0^r dr' a_{\parallel}(r') \neq 0$ , i.e., a finite value of the radially integrated turbulent acceleration. The parallel Reynolds stress  $\langle \tilde{v}_r \tilde{U}_{\parallel}\rangle$  could be measured directly by a number of means, such as Mach probes (at the edge) [30] or beam emission spectroscopy (BES) velocimetry [31] using an image in the  $r$ -parallel plane. We note that this would be a challenging new application of BES, which so far has been used only for velocimetry measurement of  $\langle \tilde{v}_r \tilde{v}_{\theta}\rangle$  [32]. Given the daunting prospect of measuring  $\langle \tilde{v}_r \tilde{U}_{\parallel}\rangle$ , we offer a second, purely macroscopic test. Using the condition of stationarity and the total velocity balance condition, we have the jump condition  $\nabla\langle U_{\parallel}\rangle|_r^{r+\Delta r} = \left(\frac{\Pi_{\parallel}^{Res}}{\chi_{\parallel}} + \frac{V_{pinch}\langle U_{\parallel}\rangle}{\chi_{\parallel}} - \frac{\int_0^r dr' a_{\parallel}(r')}{\chi_{\parallel}}\right)_r^{r+\Delta r}$ . For smoothly varying profiles and  $\chi_{\parallel}$ ,  $V_{pinch}$ , and  $\Pi_{\parallel}^{Res}$ , it follows that  $\nabla\langle U_{\parallel}\rangle|_r^{r+\Delta r} \cong -\int_r^{r+\Delta r} dr' a_{\parallel}(r')/\chi_{\parallel} \sim -\Delta r a_{\parallel}/\chi_{\parallel}$ . Here,  $\chi_{\parallel}$  could be approximated by  $\chi_i$ , determined independently. To assure smoothness,  $\Delta r$  must be smaller than the scale of possible variation of  $\Pi_{\parallel}^{Res}$ ,  $V_{pinch}$ , etc. In practice, this surely is satisfied if  $\Delta r < L_I$ , a typical mesoscale. Taking  $L_I \sim (\rho_i L_{T_i})^{1/2}$ , for  $\rho_i \sim 0.1cm$ ,  $L_{T_i} \sim 50cm$ , one can have  $\Delta r \lesssim 2.24cm$ . Thus, present day high resolution beam blip charge exchange recombination spectroscopy (CXRS) measurements [33] should have sufficient accuracy for this. Note that the key point here is that the fastest varying contribution to  $\nabla\langle U_{\parallel}\rangle$  on mesoscales comes from  $\int_0^r dr' a_{\parallel}(r')$ . Observe that for sufficiently small  $\Delta r$ , this method gives an effectively *local* measurement of  $a_{\parallel}$ !

In summary, we discovered a turbulent acceleration term in the parallel rotation equation by a calculation base on the gyrokinetic equation. The turbulent acceleration cannot be written as a divergence of the parallel Reynolds stress, which is similar to the turbulent momentum source found independently in Ref.[23]. It has different physics from the residual stress, which enters the rotation equation as a divergence. In particular, the fact that the residual stress contributes a divergence term to the rotation equation means that its effect on *net* rotation enters via its value at the *edge*. In contrast, parallel acceleration can be distributed throughout the entire cross



section and is not particularly edge sensitive. A new candidate mechanism for the origin of spontaneous rotation is thus revealed. We proposed a method for testing the effects of turbulent parallel acceleration by gyrokinetic simulation based on the difference between the turbulent acceleration and residual stress. We also proposed a direct experimental test to determine the relative contributions from  $\Pi_{\parallel}^{Res}$  and  $a_{\parallel}$ .

We are grateful to X. Garbet, T. S. Hahm, L. Chen, J. M. Kwon, X. W. Hu and Z. X. Lu for useful discussions. This work was supported by the MOST of China, under Contract Nos. 2013GB112002 and 2011GB109001, the NSFC Grant Nos. 10990214 and 10935004, the MEST of Korea via the WCI project 2009-001, the U.S. DOE Grant No. DE-FG02-04ER54738.

\*luwang@hust.edu.cn

- 
- [1] G. Rüdiger, *Differential Rotation and Stellar Convection: Sun and Solar-Type Stars* (Gordon and Breach, New York, 1989).
  - [2] M. P. Baldwin et al., *Science* **315**, 467 (2007).
  - [3] J. Pedlosky et al., *J. Fluid Mech.* **28**, 463 (1967).
  - [4] D. W. Hughes et al., *The Solar Tachocline* (Cambridge Univ. Press, Cambridge, 2007).
  - [5] J. E. Rice et al., *Nucl. Fusion* **47**, 1618 (2007).
  - [6] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge Univ. Press, Cambridge, 1995).
  - [7] H. K. Moffatt, *Magnetic Field Generation in an Electrically Conducting Fluid* (Cambridge Univ. Press, Cambridge, 1978).
  - [8] U. Frish et al., *Physica D* **26**, 382 (1987).
  - [9] L. L. Kitchatinov et al., *Astron. Astrophys.* **292**, 125 (1994).
  - [10] M. Kikuchi et al., *Rev. Mod. Phys.* **84**, 1807 (2012).
  - [11] A. Bondeson et al., *Phys. Rev. Lett.* **72**, 2709 (1994); R. Betti et al., *Phys. Rev. Lett.* **74**, 2949 (1995).
  - [12] M. Yoshida et al., *Phys. Rev. Lett.* **100**, 105002 (2008).
  - [13] K. Ida et al., *Nucl. Fusion* **50**, 064007 (2010).
  - [14] P. H. Diamond et al., *Nucl. Fusion*, in press (2013).
  - [15] Y. Kosuga et al., *Phys. Plasmas* **17**, 102313 (2010).
  - [16] P. H. Diamond et al., *Nucl. Fusion* **49**, 045002 (2009).
  - [17] T. S. Hahm et al., *Phys. Plasmas* **14**, 072302 (2007); A. G. Peeters et al., *Phys. Rev. Lett.* **98**, 265003 (2007).
  - [18] L. Wang et al., *Nucl. Fusion* **51**, 083006 (2011).
  - [19] L. Zhao et al., *Phys. Plasmas* **19**, 082309 (2012).
  - [20] T. S. Hahm, *Phys. Fluids* **31**, 2670 (1988).
  - [21] C. J. McDevitt et al., *Phys. Plasmas* **16**, 052302 (2009).
  - [22] B. D. Scott et al., *Phys. Plasmas* **17**, 112302 (2010); A. J. Brizard et al., *Phys. Plasmas* **18**, 082307 (2011); J. Abiteboul et al., *Phys. Plasmas* **18**, 082503 (2011).
  - [23] X. Garbet et al., submitted to PoP (2013).
  - [24] A. I. Smolyakov et al., *Nucl. Fusion* **49**, 125001 (2009).
  - [25] J. Weiland et al., *Nucl. Fusion* **49**, 065033 (2009).
  - [26] O. D. Gurcan et al., *Phys. Plasmas* **14**, 042306 (2007).
  - [27] O. D. Gurcan et al., *Phys. Plasmas* **17**, 112309 (2010).
  - [28] S. Ku et al., *Nucl. Fusion* **52**, 063013 (2012); W. X. Wang et al., *Phys. Plasmas* **17**, 072511 (2010); J. M. Kwon et al., *Nucl. Fusion* **52**, 013004 (2012).
  - [29] W. M. Solomon et al., *Plasma Phys. Control. Fusion* **49**, B313 (2007).
  - [30] S. H. Müller et al., *Phys. Rev. Lett.* **106**, 115001 (2011).
  - [31] G. R. McKee et al., *Phys. Plasmas* **10**, 1712 (2003).
  - [32] Z. Yan et al., "The Dynamics of Turbulence and Flow During the L-H Transition", Bulletin of the American Physical Society, 54th Annual Meeting of the APS Division of Plasma Physics (2012) Vol. 57, No. 12, NO4.00007.
  - [33] J. L. Luxon, *Nucl. Fusion* **42**, 614 (2002).