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## Measurement of Radiative Proton Capture on <sup>18</sup>F and Implications for Oxygen-Neon Novae

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The rate of the  ${}^{18}F(p,\gamma){}^{19}Ne$  reaction affects the final abundance of the  $\gamma$ -ray observable radioisotope  ${}^{18}F$ , produced in novae. However, no successful measurement of this reaction exists and the rate used is calculated from incomplete information on the contributing resonances. Of the two resonances thought to play a significant role, one has a radiative width estimated from the assumed analogue state in the mirror nucleus,  ${}^{19}F$ . The second does not have an analogue state assignment at all, resulting in an arbitrary radiative width being assumed. Here we report the first successful direct measurement of the  ${}^{18}F(p,\gamma){}^{19}Ne$  reaction. The strength of the 665 keV resonance ( $E_x=7.076$  MeV) is found to be over an order of magnitude weaker than currently assumed in nova models. Reaction rate calculations show that this resonance therefore plays no significant role in the destruction of  ${}^{18}F$  at any astrophysical energy.

The observation of  $\gamma$ -rays and X-rays from radioisotope decay by satellite missions is a powerful tool for providing information on the astrophysical processes occurring during the lives and deaths of stars. In 2002, ESA launched the INTEGRAL satellite with the goal of mapping out  $\gamma$ -ray emission across the whole sky. NASA has also launched the SWIFT  $\gamma$ -ray burst telescope in 2004 and both are capable of making  $\gamma$ -ray spectroscopic measurements of stellar objects. Observations from these missions can provide data on the abundance of particular isotopes synthesized in such environments and thus put constraints on the astrophysical models. For example, the recent INTEGRAL observation of hard X-rays from Supernova 1987A [1] allowed the amount of  $^{44}$ Ti produced during the core collapse supernova explosion to be derived and compared to that predicted by different models.

In the case of nova explosions one radioisotope of interest is <sup>18</sup>F [2]. This isotope is thought to be the major source of line emission  $\gamma$ -rays after the nova outburst at 511 keV, as a result of positron annihilation following its  $\beta^+$  decay. The relatively long half life of <sup>18</sup>F (t<sub>1/2</sub>=110 mins) means that a significant number of positrons are emitted shortly after the expanding nova envelope becomes transparent to  $\gamma$ -rays. If the reaction rates for the production and destruction of <sup>18</sup>F are sufficiently well known then observations of this  $\gamma$ -ray emission can provide constraints on physical conditions inside novae, leading to improvements in the astrophysical models. This could, for instance, help address the discrepancy between the observed ejected mass and that predicted by current models. Observation of 511 keV  $\gamma$ -ray line emission, however, requires the difficult task of a-posteriori searches of data from wide field instruments as the peak optical brightness occurs days after the positron annihilation flash [3, 4]. Of the two types of white dwarf stars responsible for novae, carbon-oxygen (CO) and oxygenneon (ONe), the latter result in hotter novae that eject more  $^{18}$ F, and are thus of greater interest for satellite observations.

Although there is still some uncertainty in the production rate at nova temperatures of between 0.1 and 0.4GK [5, 6] (and references therein), the main uncertainty in the final abundance of  $^{18}$ F depends on its destruction rate via the  $(p,\gamma)$  and  $(p,\alpha)$  pathways. The <sup>18</sup>F $(p,\alpha)$ <sup>15</sup>O reaction is estimated to be a few thousand times faster [7] and thus it has been the focus of many studies [8-10] (and references therein). By contrast, little effort has gone into probing the properties of the  ${}^{18}F(p,\gamma){}^{19}Ne$  reaction based on the much lower estimated cross section and limited beam intensity available. However, a sensitivity study by Iliadis et al. [11] indicates that, for ONe novae, a factor of 10 increase or decrease in the  $(p,\gamma)$  rate changes the abundance of  $^{18}$ F by a factor of 2.5 or 0.9 respectively, significantly affecting the potential number of novae detectable via satellite missions. Given the recent improvements in knowledge of the  $(p,\alpha)$  rate, it is imperative that effort is now made to reduce the uncertainty in the  $(p,\gamma)$  reaction rate.

The  ${}^{18}F(p,\gamma){}^{19}Ne$  reaction proceeds through proton capture into states in  ${}^{19}Ne$  in an excitation range which is not yet fully described. Not all expected states have

been observed in <sup>19</sup>Ne but it was previously thought that the  $E_{cm}$  ( $E_x$ )=330 keV (6.741 MeV) resonance was the main contributor to the reaction rate in novae, together with the 665 keV (7.076 MeV) resonance at the higher temperatures reached in ONe nova [7, 12, 13]. There is also a non-resonant direct capture contribution that becomes influential at lower novae temperatures and is considered in [7, 13, 17]. A very recent study of <sup>19</sup>Ne levels [14] suggests new energies and spin assignments for states. However, due to their proximity to the proton threshold there is no impact on the <sup>18</sup>F(p, $\gamma$ )<sup>19</sup>Ne rate at energies relevant to ONe novae.

Although the  $\alpha$  and proton partial widths ( $\Gamma_{\alpha}$  and  $\Gamma_{p}$ ) are well known for the 665 keV resonance [15] the radiative width  $\Gamma_{\gamma}$  remains unmeasured and the lack of an analogue state assignment [16] means no reliable estimate can be made. An experimental upper limit does exist, set by Rehm et al [17], but does not constrain the reaction rate contribution of this state at nova temperatures. An arbitrary value of 1 eV [15] is therefore currently assumed in the literature, taken from surrounding states in <sup>19</sup>F.

Similarly the  $\Gamma_{\gamma}$  of the 330 keV resonance has not been experimentally determined and a value of  $5.0\pm2.6$  eV is used based on an assumed analogue assignment [15]. This analogue assignment was determined from comparison of the population of states in  $^{19}\mathrm{F}$  and  $^{19}\mathrm{Ne}$  through a mirror reaction study by Utku et al. [7]. The quoted  $\Gamma_{\alpha}$  (effectively the total width) of  $5.2\pm3.7$  keV is also based on this analogue assignment. By contrast, the  $\Gamma_p$  has been determined experimentally to be  $2.22\pm0.69$  eV, from a direct measurement of the  ${}^{18}F(p,\alpha){}^{15}O$  cross section [18]. A  $\Gamma_p$  of 7.3±0.6 eV was extracted from proton transfer data [10] but here the population of the 330 keV resonance could have been contaminated by nearby states. In summary, key parameters of both these resonances are experimentally unconstrained and thus the  ${}^{18}F(p,\gamma){}^{19}Ne$ reaction rate, and its impact on <sup>18</sup>F abundance, must be regarded as very uncertain.

In this Letter we report on the first successful direct measurement of the <sup>18</sup>F(p, $\gamma$ )<sup>19</sup>Ne reaction, at any energy. This work was performed using the DRAGON recoil separator, at the ISAC radioactive beam facility located at the TRIUMF laboratory in Vancouver, Canada. The <sup>18</sup>F beam was produced using a 500 MeV proton beam incident on a silicon carbide target. Mass 18 products were extracted, ionized in a FEBIAD ion source [19] and filtered using a high resolution mass separator with a mass resolving power of 10,000 [20]. An average <sup>18</sup>F<sup>4+</sup> beam intensity of  $1.74 \times 10^6$  ions/sec was delivered to the experiment.

DRAGON (Detector of Recoils and Gammas Of Nuclear reactions) is a specialized facility designed to study radiative proton and  $\alpha$  capture reactions at sub Coulomb barrier energies. It consists of three main sections, 1) a differentially pumped windowless gas target chamber surrounded by a high efficiency BGO  $\gamma$  array; 2) a high suppression two-stage electromagnetic separator; and 3) a heavy ion detector system. The heavy ion detection sys-

tem, located downstream of the final focus of the separator, consists of two micro-channel plate detectors (MCP, used for measuring recoil time of flight) and a multianode ionization chamber (IC, used for measuring recoil energy loss). For a more detailed discussion of the facility the reader is referred to [21, 22].

A laboratory beam energy  $(E_{^{18}F}^{lab})$  of 12.9 MeV was chosen to place the  $E_{cm}$ =665 keV resonance near the centre of the target and the recoil separator was tuned to accept ions with a charge state of 6<sup>+</sup>. The maximum calculated recoil cone angle of 11±1 mrad was well within DRAGON's acceptance limits of ±20 mrad [21]. <sup>18</sup>O contamination was also present in the beam and a <sup>18</sup>O run at  $E_{lab}$ =12.9 MeV was taken in order to account for any possible <sup>18</sup>O(p, $\gamma$ )<sup>19</sup>F events that could have contributed to background in the signal region.

Extracting the reaction cross section from the number of reaction events measured requires a knowledge of the integrated beam exposure, number density of target nuclei and the overall detection efficiency. The number density of target nuclei is deduced from the central gas pressure, of around 7.5 Torr. This value was chosen to ensure as much of the resonance was contained within the target region as possible without stressing the recirculation pumps. Energy loss was determined by using the first dipole magnet of the separator to measure the beam energy with and without gas in the target. The gas pressure was monitored throughout the experiment and remained within the range 7.3 to 7.6 Torr.

The number of incident beam particles is usually determined by normalizing the measured rate of target nuclei, elastically scattered by the beam into two silicon detectors (Sb) mounted inside the target assembly, to Faraday Cup (FC) readings from just before and after the target [23]. However due to the low beam intensity, the FCs could not provide a reliable current measurement. An alternative approach was employed using a Monte Carlo simulation of the gas target, together with elastic cross section data from [8] and SRIM energy loss calculations. This enabled the number of incident beam particles to be determined from the number of events detected in the silicon detector (Sb0) inside the target which was angled at 30 degrees to the beamline.

The systematic errors in the simulation were estimated to produce an uncertainty of  $\pm 18\%$  in the calculated beam intensity. This value was determined by comparing results from the simulation to the normal DRAGON procedure for both the pure <sup>18</sup>O data and a later <sup>26</sup>Al(p, $\gamma$ )<sup>27</sup>Si experiment, both of which had precise FC cup data available. Comparisons of the experimental data points in [8] with the corresponding R-matrix fit were also used to determine uncertainty. Using this simulation, the ratio  $\frac{N_{beam}}{N_{Sb}}$  was found, allowing the total integrated beam intensity on target to be calculated from the silicon detector data, giving a value of  $1.01\pm0.18 \times 10^{12}$  <sup>18</sup>F ions.

The overall detection efficiency depends on the fraction of recoils in the charge state for which the separator is set, the transmission efficiency through the separator and the efficiency of the final detection elements (MCP and IC). The charge state distribution for <sup>20</sup>Ne was measured at the relevant ion speed after the main run and the 6<sup>+</sup> charge state fraction was found to be  $23.7\pm1.1\%$ . Heavy ion transmission through the grid-supported foils of the MCP system was measured to be  $76.9\pm0.6\%$  [24]. Of the beam ions transmitted and detected in the IC,  $99.363\pm0.006\%$  had an associated MCP TOF count. The detection efficiency of the IC was essentially 100%.



FIG. 1. (colour online) Energy loss vs. energy loss plot obtained from the first two anodes in the IC. The attenuated beam run is shown in black and two well-separated loci are clearly visible signifying the presence of both <sup>18</sup>F and <sup>18</sup>O. Circles (triangles), both red online, correspond to observed <sup>19</sup>Ne (<sup>18</sup>F) events when the separator was tuned to recoils. Squares (blue online) correspond to <sup>19</sup>F recoils during the separate <sup>18</sup>O beam run.

With <sup>18</sup>F beam, two <sup>19</sup>Ne recoil events were recorded after just under a week of continuous beamtime (Fig. 1). At the 95% confidence level we deduce a result of  $2.0^{+4.8}_{-1.7}$ for these recoil events, using the profile likelihood technique outlined in [25]. This technique involves characterizing the likelihood function for our data which includes the parameters of interest as well as nuisance variables. For this analysis the signal and background were treated as Poisson distributions while the beam normalization and detector efficiencies were Gaussian. The profile likelihood (given as  $\lambda$ ) can then be calculated giving the likelihood of observing the given parameter of interest as a function of that parameter only (i.e. in the absence of the nuisance ones). The minimum of the function  $-2 {\rm log} \lambda$ converges to a  $\chi^2$  distribution [26]. A 95% confidence interval is extracted by taking the limits where the distribution increases by 3.84 from the minimum. The result is not statistically consistent with zero at this confidence level. Consequently we find a resonance strength  $(\omega\gamma)$  of  $19^{+45}_{-16}$  meV and a  $\Gamma_{\gamma}$  of  $72^{+172}_{-61}$  meV which are both a factor of 14 smaller than the previously assigned values.

Three potential sources of background were identified: <sup>18</sup>F beam particles that managed to pass through the recoil separator (referred to as leaky beam), <sup>18</sup>O leaky beam contaminants and <sup>19</sup>F recoil events from <sup>18</sup>O contamination. Using attenuated beam a  $\Delta E$  plot from the IC allowed the locus of such background events to be de-

Source of uncertainty	Uncertainty
Beam Normalization	18%
Charge state distribution	4.6%
Target stopping power	5.4%
MCP efficiency	0.79%

TABLE I. List of systematic experimental uncertainties

termined (Fig. 1). Three events appear in the IC region corresponding to <sup>18</sup>F leaky beam, which is as expected when the total beam intensity and DRAGON's typical suppression factor are taken into account. However the uncertainty in ion energy loss in the IC meant that these could also potentially have been recoil events. This issue was resolved by looking at the MCP data which gave TOF information on each particle. Attenuated beam runs provide a very precise expected TOF region where leaky beam particles would appear. The three suspected leaky beam events all have TOF values consistent with the <sup>18</sup>F region, to within  $2\sigma$ , and the two recoil candidates resided in a region with minimal background. The three events observed in the  ${}^{18}$ F region resulted in an expected background of just  $3 \times 10^{-5}$  events per channel where the recoil candidates were observed (see Fig. 2). When calculating the background in the MCP signal region a gate was placed on this spectrum such that only events that appeared above channel 174 in IC0 were included. This value corresponds to the highest channel in which we observed  ${}^{18}O(p,\gamma){}^{19}F$  events (see Fig. 3). The second peak in the attenuated beam data is most likely an electronic artifact. It accounts for just 0.33% of the real peak events however and any recoils that experience this same effect would still be distinguishable from leaky beam events in this spectrum and in Fig. 1.

The level of isobaric contamination from <sup>18</sup>O was measured, at regular intervals, by sending attenuated beam directly into the IC and measuring the ratio of peaks due to <sup>18</sup>O and <sup>18</sup>F (see Fig. 1). The <sup>18</sup>O:<sup>18</sup>F ratio was observed to decrease from 1:20 to 1:260 throughout the experiment, as the residual gas in the ion source diminished. The position of the <sup>18</sup>O and <sup>18</sup>F peaks in the IC spectrum was reproduced each time, and the position of the <sup>18</sup>O peak was in good agreement with the peak from the <sup>18</sup>O beam runs.

The IC spectrum from the <sup>18</sup>O run is shown in Fig. 3 together with the position of the two <sup>19</sup>Ne recoil events. Analysis of the silicon detector data together with the isobaric contamination ratio for these runs allowed the expected number of background events from <sup>18</sup>O(p, $\gamma$ )<sup>19</sup>F to be calculated. Only 0.40±0.08 background <sup>19</sup>F events were expected in total and a negligible fraction of them were predicted to appear above channel 174 during the <sup>18</sup>F(p, $\gamma$ )<sup>19</sup>Ne run.

There were also events in the lower energy region of the IC spectrum corresponding to particles scattered inside the recoil separator, changing charge state and losing momentum, that were able to reach the IC via an extremely erratic path. As they appear so far away from the leaky



FIG. 2. MCP time of flight spectrum (colour online) showing where  ${}^{18}$ F was observed during the attenuated beam run (blue) and events when tuned to  ${}^{19}$ Ne recoils (red). Leaky beam during the recoil run is clearly identified as it appears in the  ${}^{18}$ F peak.



FIG. 3. Energy loss plot from the IC's first anode showing the position of  ${}^{18}O(p,\gamma){}^{19}F$  (solid, blue online) and  ${}^{19}Ne$  events (shaded, red online).

beam and recoil loci they do not contribute to the region of interest however.

Neither of the potential <sup>19</sup>Ne recoils had a coincident  $\gamma$ -event detected in the BGO's which is often used as an additional source of background suppression. Lack of <sup>19</sup>Ne level structure data also made predicting the BGO efficiency difficult. A GEANT simulation was used with gamma cascades assumed from <sup>19</sup>F in an excitation region similar to that populated by the observed 665 keV resonance. Reasonable variations were considered and an efficiency range of 54 $\rightarrow$ 78% resulted. As only two events in the recoil region were observed, the lack of  $\gamma$ -heavy ion coincidence events is not inconsistent with the observed singles yield. All these checks give high confidence that the two events observed are <sup>19</sup>Ne recoils in a region of negligible background.

The reduction of a factor of 14 in the reaction yield below that used in previous models is rather dramatic. Using this new resonance strength the reaction rate due to the 665 keV resonance was recalculated (Fig. 4). The contributions from individual resonances were calculated in the context of an R-Matrix framework, including the external capture component and interference between resonances. Parameters were taken from [10, 16] and the the 665 keV resonance was given a positive phase in order to maximize its potential influence on the total reaction rate. The upper graph has the 665 keV resonant contribution using the previous upper limit from Rehm et al [17]. Although the 330 keV resonance ( $E_x=6.741$  MeV) does dominate, only the  $\Gamma_p$  of the  $E_x=6.741$  MeV state



FIG. 4. Fractional resonant and direct capture contributions to the total  ${}^{18}$ F(p, $\gamma$ ) ${}^{19}$ Ne reaction rate at ONe nova peak temperatures (colour online). The upper graph shows the 665 keV rate (solid, red) using the experimental upper limit taken from Rehm et al [17] whilst the lower graph has the rate using the current work. The direct capture contribution (dashed, green) and the 330 keV resonance (fine dashed, blue) contribution was computed using data from [7, 27] & [10, 16, 18] respectively. Note that at lower temperatures the resonance at 38 keV together with a sub threshold state at -122 keV account for the remaining contribution to the total rate [28].

has been experimentally determined [18] and the model calculations are based on  $\Gamma_{\gamma}$  and  $\Gamma_{\alpha}$  from an assumed analogue assignment [7]. Another state with the same  $J^{\pi}$  (but different width) does lie nearby so this assignment is by no means definitive meaning that the 665 keV resonance could have played a major role in this temperature range (0.1 $\rightarrow$ 0.4 GK). It can be clearly seen that, compared to the 330 keV resonance and direct capture component, only now can it be definitively shown that this resonance makes no significant contribution to the reaction rate at any temperature relevant to ONe novae [11].

In summary the <sup>18</sup>F(p, $\gamma$ )<sup>19</sup>Ne resonance strength has been measured at the 665 keV resonance in inverse kinematics using the recoil mass separator DRAGON. Two <sup>19</sup>Ne recoil events were detected and identified with high confidence, resulting in a  $\omega\gamma$  that is a factor of 14 smaller than the previous assignment. As a consequence this resonance has now, for the first time, been shown experimentally to play a negligible role in the destruction of <sup>18</sup>F at temperatures associated with ONe novae and thus does not influence the <sup>18</sup>F abundance after the resulting outburst. It is therefore crucial that either a direct measurement of the 330 keV resonance, or an indirect determination of the  $\Gamma_{\gamma}$  and  $\Gamma$  of the associated state, are made if future <sup>18</sup>F abundance observations are to be fully exploited.

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