# Ab Initio Calculations of Even Oxygen Isotopes with Chiral Two-Plus-Three-Nucleon Interactions

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# *Ab Initio* Calculations of Even Oxygen Isotopes with Chiral Two- Plus Three-Nucleon Interactions

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We formulate the In-Medium Similarity Renormalization Group (IM-SRG) for open-shell nuclei using a multi-reference formalism based on a generalized Wick theorem introduced in quantum chemistry. The resulting multi-reference IM-SRG (MR-IM-SRG) is used to perform the first *ab initio* study of all even oxygen isotopes with chiral NN and 3N Hamiltonians, from the proton to the neutron drip lines. We obtain an excellent reproduction of experimental ground-state energies with quantified uncertainties, which is validated by results from the Importance-Truncated No-Core Shell Model and the Coupled Cluster method. The agreement between conceptually different many-body approaches and experiment highlights the predictive power of current chiral two- and three-nucleon interactions, and establishes the MR-IM-SRG as a promising new tool for *ab initio* calculations of medium-mass nuclei far from shell closures.

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*Introduction.* Neutron-rich nuclei are the focus of the experimental program of current and next-generation rare isotope facilities. Emerging phenomena such as halos or neutron skins make these nuclei ideal laboratories to study nuclear interactions in delicately tuned scenarios, and motivate the use of *ab initio* many-body calculations to provide their description from first principles. Such calculations make it possible to confront modern nuclear Hamiltonians from chiral effective field theory (EFT) [1, 2] with a wealth of data beyond few-body systems.

For light nuclei, the *ab initio* No-Core Shell Model (NCSM) [3, 4] provides the capabilities for studies of isotopic chains, but for medium-mass nuclei this approach is not feasible because of its large computational effort. Many-body techniques with more modest computational scaling, such as the Coupled Cluster (CC) [5–7] or Self-Consistent Green's Function methods [8, 9], can be used to probe nuclei in the vicinity of shell closures, but are not applicable for open-shell nuclei far from shell closures. For such nuclei, a self-consistent Gor'kov formalism was developed recently [10, 11], but this approach is currently limited to second-order terms in the many-body perturbation expansion.

In this Letter, we describe the extension of the In-Medium Similarity Renormalization Group (IM-SRG) framework of Refs. [12, 13] to open-shell nuclei by means of a multi-reference formulation. We use the resulting MR-IM-SRG and two other many-body approaches, the Importance-Truncated No-Core Shell Model (IT-NCSM) and the CC method, to perform the first *ab initio* study of all even oxygen isotopes with chiral NN+3N Hamiltonians.

*Formalism.* The main tools for the derivation of the MR-IM-SRG are the generalized normal-ordering and Wick theorem by Kutzelnigg and Mukherjee [14]. We write a string of creation and annihilation operators in tensorial form,

$$A^{1\dots k}_{l\dots N} \equiv a^\dagger_1 \dots a^\dagger_k a_N \dots a_l \,, \tag{1}$$

and expand it in terms of components that are normal-ordered with respect to an arbitrary reference state $|\Phi\rangle$ [14–16]. We obtain

$$\begin{aligned} A^{1\dots k}_{l\dots N} =& :A^{1\dots k}_{l\dots N}: + \lambda^1_l :A^{23\dots k}_{mn\dots N}: - \lambda^1_m :A^{23\dots k}_{ln\dots N}: + \dots \\ &+ (\lambda^1_l\lambda^2_m - \lambda^1_m\lambda^2_l + \lambda^{12}_{lm}) :A^{3\dots k}_{n\dots N}: + \dots \,, \end{aligned} \tag{2}$$

where $:\,\cdot\,:$ indicates normal-ordering, and we have introduced irreducible one- and two-body density matrices $\lambda^{(1)}$ and $\lambda^{(2)}$:

$$\lambda^1_2 \equiv \langle\Phi|A^1_2\,|\Phi\rangle \,, \quad \lambda^{12}_{34} \equiv \langle\Phi|A^{12}_{34}\,|\Phi\rangle - \lambda^1_2\lambda^3_4 + \lambda^1_3\lambda^2_4 \,. \tag{3}$$

The particle rank of the irreducible density matrices is evident from the single-particle indices. Generally, up to $n$-body irreducible density matrices $\lambda^{(n)}$ appear in the expansion of an $n$-body operator, which are defined recursively in terms of density matrices of lower rank and encode information about $n$-body correlations in the reference state [14]. For an independent-particle state, all matrices except $\lambda^{(1)}$ vanish.

Products of normal-ordered operators can be expanded by means of a generalized Wick theorem (GWT), e.g.,

$$\begin{aligned} :A^{12}_{56}::A^{34}_{78}: =& :A^{1234}_{5678}: + \lambda^1_7 :A^{234}_{568}: - \xi^3_5 :A^{124}_{678}: + \dots \\ &+ \left(\lambda^1_7\lambda^2_8 - \lambda^1_8\lambda^2_7 + \lambda^{12}_{78}\right) :A^{34}_{56}: - \lambda^{12}_{57} :A^{34}_{68}: + \dots \,, \end{aligned} \tag{4}$$

where $\xi^1_2 \equiv \lambda^1_2 - \delta^1_2$ [16]. In addition to simple contractions containing $\lambda^{(1)}$ and $\xi^{(1)}$ which also occur in the standard Wick theorem, we obtain terms involving $\lambda^{(2)}, \dots, \lambda^{(n)}$. Each density matrix must have at least one index from each of the operators in the product — other terms vanish due to the initial normal-ordering (2) [14]. In the following, we work in natural orbitals, i.e., the eigenbasis of $\lambda^{(1)}$, where

$$\lambda^1_2 = n_1\delta^1_2 \,, \quad \xi^1_2 = -\bar{n}_1\delta^1_2 \equiv -(1-n_1)\delta^1_2 \,, \tag{5}$$

and the eigenvalues are the occupation numbers $0 \le n_a \le 1$.

We now consider the IM-SRG operator flow equation

$$\frac{d}{ds}H(s) = [\eta(s), H(s)] \,. \tag{6}$$

By integrating Eq. (6), we generate a continuous unitary transformation that decouples the ground-state of the Hamiltonian

$H(s)$ from excitations, and solve the many-body problem [12, 13]. Suppressing the flow parameter $s$ for brevity, we apply the generalized normal-ordering to $H$ and the generator $\eta$, and evaluate the commutator using the GWT to obtain the MR-IM-SRG flow equations:

$$\begin{aligned}\frac{dE}{ds} &= \sum_{ab}(n_a - n_b)\left(\eta^a_b f^b_a - f^a_b \eta^b_a\right) \\ &\quad + \frac{1}{4}\sum_{abcd}\left(\eta^{ab}_{cd}\Gamma^{cd}_{ab} - \Gamma^{ab}_{cd}\eta^{cd}_{ab}\right) n_a n_b \bar{n}_c \bar{n}_d \\ &\quad + \frac{1}{4}\sum_{abcd}\left(\frac{d}{ds}\Gamma^{ab}_{cd}\right)\lambda^{ab}_{cd}\,, \end{aligned} \tag{7}$$

$$\begin{aligned}\frac{d}{ds}f^1_2 &= \sum_a \eta^1_a f^a_2 + \sum_{ab}\eta^a_b \Gamma^{b1}_{a2}(n_a - n_b) \\ &\quad + \frac{1}{2}\sum_{abc}\eta^{1a}_{bc}\Gamma^{bc}_{2a}\left(n_a\bar{n}_b\bar{n}_c + \bar{n}_a n_b n_c\right) \\ &\quad + \frac{1}{4}\sum_{abcde}\eta^{1a}_{bc}\Gamma^{de}_{2a}\lambda^{de}_{bc} + \sum_{abcde}\eta^{1a}_{bc}\Gamma^{be}_{2d}\lambda^{ae}_{cd} \\ &\quad - \frac{1}{2}\sum_{abcde}\left(\eta^{1a}_{2b}\Gamma^{cd}_{ae}\lambda^{cd}_{be} - \eta^{1a}_{2b}\Gamma^{bc}_{de}\lambda^{ac}_{de}\right) - [\eta \leftrightarrow f, \Gamma]\,, \end{aligned} \tag{8}$$

$$\begin{aligned}\frac{d}{ds}\Gamma^{12}_{34} &= \sum_a \big(\eta^1_a\Gamma^{a2}_{34} + \eta^2_a\Gamma^{1a}_{34} - \eta^a_3\Gamma^{12}_{a4} - \eta^a_4\Gamma^{12}_{3a} \\ &\qquad - f^1_a\eta^{a2}_{34} - f^2_a\eta^{1a}_{34} + f^a_3\eta^{12}_{a4} + f^a_4\eta^{12}_{3a}\big) \\ &\quad + \frac{1}{2}\sum_{ab}\left(\eta^{12}_{ab}\Gamma^{ab}_{34} - \Gamma^{12}_{ab}\eta^{ab}_{34}\right)(1 - n_a - n_b) \\ &\quad + \sum_{ab}(n_a - n_b)\left(\left(\eta^{1a}_{3b}\Gamma^{2b}_{4a} - \Gamma^{1a}_{3b}\eta^{2b}_{4a}\right) - [1 \leftrightarrow 2]\right)\,, \end{aligned} \tag{9}$$

where $E = \langle\Phi|\, H\, |\Phi\rangle$, and the one- and two-body parts of $H$, denoted by $f$ and $\Gamma$, contain in-medium contributions from the 3N interaction because of the normal ordering [12, 13]. The symbol $[\eta \leftrightarrow f, \Gamma]$ in Eq. (8) indicates an interchange of the one- and two-body parts of $\eta$ and $H$. To close the system of flow equations (7)–(9), we truncate three-body operators [13] and a term containing $\lambda^{(3)}$ in the energy flow equation (7). We refer to this truncation as MR-IM-SRG(2).

By integrating Eqs. (7)–(9), we perform a non-perturbative resummation of the Many-Body Perturbation series [12, 13]. The flowing two-body vertex is RG-improved by Eq. (9), e.g., with contributions from generalized ladder (3rd line) and ring diagrams (4th line), which in turn generate corrections to the ground-state energy when $\Gamma$ is inserted in Eq. (7) [13].

As our default choice for the generator, we use the ansatz of White [13, 17]. The required matrix elements of the Hamiltonian, such as $\langle\Phi|\, H : A^{12}_{34} : |\Phi\rangle$, which couple the reference state to excitations, or $\langle\Phi| : A^{34}_{12} : H : A^{12}_{34} : |\Phi\rangle$, which enter the energy denominators, can be evaluated using the generalized normal ordering. This yields

$$\eta^1_2 = \frac{\bar{n}_1 n_2 f^1_2}{\bar{n}_1 f^1_1 - n_2 f^2_2 + \bar{n}_1 n_2 \Gamma^{12}_{12}} - [1 \leftrightarrow 2] + \dots\,, \tag{10}$$

$$\begin{aligned}\eta^{12}_{34} &= \frac{\bar{n}_1\bar{n}_2 n_3 n_4 \Gamma^{12}_{34}}{\bar{n}_1 f^1_1 + \bar{n}_2 f^2_2 - n_3 f^3_3 - n_4 f^4_4 + G^{12}_{34}} - [(12) \leftrightarrow (34)] \\ &\quad + \dots\,, \end{aligned} \tag{11}$$

where

$$\begin{aligned}G^{12}_{34} &= \bar{n}_1\bar{n}_2\Gamma^{12}_{12} + n_3 n_4 \Gamma^{34}_{34} \\ &\quad - \left(\bar{n}_1 n_3 \Gamma^{13}_{13} + \bar{n}_2 n_4 \Gamma^{24}_{24} + [1 \leftrightarrow 2]\right)\,. \end{aligned} \tag{12}$$

The dots in Eqs. (10) and (11) indicate terms that are linear in $\lambda^{(2)}$. Terms containing $\lambda^{(n\geq 3)}$ or nonlinear powers of $\lambda^{(2)}$ are truncated.

In cases where the flow stalls due to small energy denominators, we use Wegner's generator $\eta = [H, H^{od}]$ as a fallback, defining the one- and two-body parts of the off-diagonal Hamiltonian $H^{od}$ as

$$\begin{aligned}(f^{od})^1_2 &= \bar{n}_1 n_2 f^1_2 + [1 \leftrightarrow 2]\,, \\ (\Gamma^{od})^{12}_{34} &= \bar{n}_1\bar{n}_2 n_3 n_4 \Gamma^{12}_{34} + [(12) \leftrightarrow (34)]\,. \end{aligned} \tag{13}$$

This generator is free of numerical instabilities but less efficient because the flow equations become stiff [12, 13]. In the limit of a single Slater determinant reference state, both generators reduce to the forms used for closed-shell nuclei in [12, 13].

We obtain a reference state for each nucleus by solving the Hartree-Fock-Bogoliubov (HFB) equations, and projecting the resulting state on proton and neutron number, $|\Phi\rangle = P_N P_Z\, |\text{HFB}\rangle$ [18]. This choice allows us to enforce spherical symmetry in calculations for even nuclei [19], and greatly increases the single-particle basis sizes we can treat. The natural-orbital basis of $|\Phi\rangle$ is the usual canonical basis of the HFB vacuum, allowing us to use analytic expressions for the density matrices [20].

The MR-IM-SRG method can be extended systematically by improving the truncation scheme: One would include $3, \dots, A$-body operators when Eq. (6) is expanded in normal-ordered components, as well as additional terms involving irreducible density matrices. While the number of flow equations is the same as in the single-reference case, their complexity grows much more rapidly due to additional terms from the generalized normal ordering [12–14].

*Calculation Details.* Reference states for the MR-IM-SRG calculation are obtained by solving the HFB equations in 15 major harmonic-oscillator (HO) shells, and projecting the resulting state on good proton and neutron numbers [13, 21]. For the 3N interaction, the sum of the HO energy quantum numbers of a 3N basis state is limited by $e_1 + e_2 + e_3 \leq E_{3\text{max}} = 14$, as discussed in [13, 22]. Reducing $E_{3\text{max}}$ from 14 to 12 changes the MR-IM-SRG(2) ground-state energies for oxygen isotopes by less than 1% for the Hamiltonians used

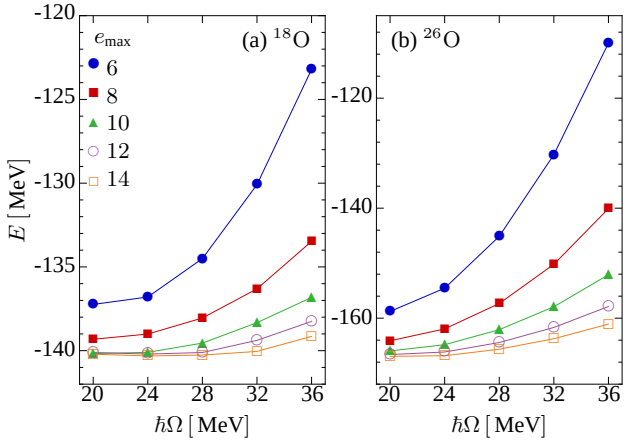


FIG. 1. (Color online) Convergence of the MR-IM-SRG(2) ground-state energies of $^{18}$O and $^{26}$O with respect to the single-particle basis size $e_{\max}$, for the NN+3N-full Hamiltonian at $\lambda_{\rm SRG} = 2.0$ fm$^{-1}$.

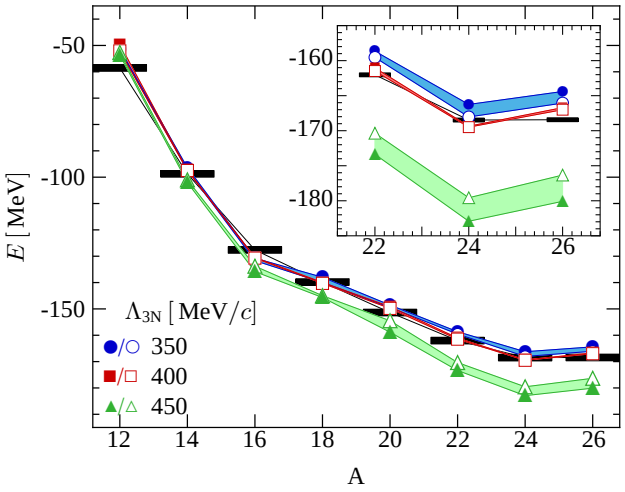


FIG. 2. (Color online) Dependence of the MR-IM-SRG(2) oxygen ground-state energies for the NN+3N-full Hamiltonian on the resolution scale and the initial cutoff $\Lambda_{3N}$. For each $\Lambda_{3N}$, the band is obtained by varying $\lambda_{\rm SRG}$ from 2.24 (open symbols) to 1.88 fm$^{-1}$ (closed symbols). Experimental values are indicated by black bars [28, 29].

in this work. The intrinsic NN+3N Hamiltonian is normal-ordered with respect to the reference state, and the residual normal-ordered 3N interaction term is discarded, leading to the normal-ordered two-body approximation (NO2B), which is found to overestimate oxygen binding energies by about 1% [13, 22].

In this Letter, we use the same nuclear Hamiltonians as in our recent IM-SRG and CC studies [13, 22, 23]: The NN interaction is the chiral N$^3$LO interaction by Entem and Machleidt, with cutoff $\Lambda_{\rm NN} = 500$ MeV/$c$ [2, 24]. Our standard three-body Hamiltonian is a local N$^2$LO 3N interaction with initial cutoff $\Lambda_{3N} = 400$ MeV/$c$. The resolution scale of the Hamiltonian is lowered to $\lambda_{\rm SRG} = 1.88, \ldots, 2.24$ fm$^{-1}$ by means of an SRG evolution in three-body space [25–27]. Hamiltonians which only contain SRG-induced 3N forces are referred to as NN+3N-induced, those also containing an initial 3N interaction as NN+3N-full.

In Fig. 1, we illustrate the convergence of the MR-IM-SRG(2) ground-state energies for $^{18}$O and $^{26}$O with respect to the single-particle basis size. At the optimal $\hbar\Omega$, the change in the ground-state energy is 0.1% when we increase the basis from $e_{\max} = 12$ to 14. This rapid convergence is representative for all Hamiltonians used in this work.

*Results.* In Fig. 2, we show MR-IM-SRG(2) ground-state energies of the even oxygen isotopes for NN+3N-full Hamiltonians with initial cutoffs $\Lambda_{3N} = 350, 400$ and 450 MeV/$c$. For the 3N low-energy constants, we use a fixed $c_D = -0.2$, and $c_E = 0.205, 0.098$, and $-0.016$, respectively, which are fit to the $^4$He binding energy in NCSM calculations [23, 27]. For the NN+3N-full Hamiltonian with $\Lambda_{3N} = 400$ MeV/$c$, we achieve an excellent reproduction of experimental data all the way to the neutron drip line at $^{24}$O [29], with deviations of 1-2%. Recent experiments place the $^{26}$O ground-state resonance at $E_x \lesssim 150$ keV above the $^{24}$O ground-state energy [30, 31]. We slightly overestimate this energy in our calculation because the HO basis expansion of our single-particle wave functions is ill-suited to the description of resonances and other continuum states. The inset in Fig. 2 illustrates that the correct drip-line systematics is independent of $\lambda_{\rm SRG}$ in the studied range and also robust against variations of the cutoff $\Lambda_{3N}$. This suggests that the long-range part of the two-pion exchange (2PE) 3N interaction, which remains unchanged as we lower $\Lambda_{3N}$, is key to obtaining the proper isotopic trends. The 2PE contribution has significant spin-orbit and tensor terms, and is therefore important for the evolution of the shell structure along the isotopic chain, as also demonstrated in other studies, e.g. [32].

Let us now discuss the effect of varying the resolution scale. As discussed in [13, 22], the $\lambda_{\rm SRG}$-dependence of our energies is the net result of omitted induced 4N interactions, the $E_{3\max}$ cut, and the MR-IM-SRG(2) truncation of the many-body expansion, while the effect of the NO2B approximation is found to be independent of $\lambda_{\rm SRG}$.

For $\Lambda_{3N} = 350$ MeV/$c$ we do not expect significant induced 4N interactions [27]. As $\lambda_{\rm SRG}$ is reduced, we capture additional repulsive 3N strength in matrix elements with $e_1 + e_2 + e_3 \leq E_{3\max}$. We also speed up the convergence of the many-body expansion and reduce the error due to the MR-IM-SRG(2) truncation, but for the resolution scales considered here, this effect is already saturated. In total, we find a slight artificial increase of the ground-state energies as we lower $\lambda_{\rm SRG}$ [13].

For our standard choice $\Lambda_{3N} = 400$ MeV/$c$, effects from omitted 4N interactions, the $E_{3\max}$ cut, and the many-body truncation cancel, and the $\lambda_{\rm SRG}$-dependence of the energies in Fig. 2 is extremely weak [13]. The omission of 4N interactions becomes the dominant source of uncertainty as we increase $\Lambda_{3N}$ to 450 MeV/$c$, resulting in an enhanced $\lambda_{\rm SRG}$-dependence of the ground-state energies of the heavier oxygen isotopes. This is consistent with the even stronger $\lambda_{\rm SRG}$-

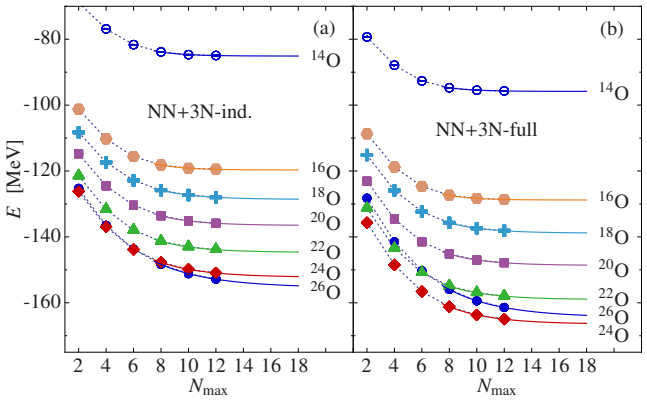


FIG. 3. (Color online) IT-NCSM ground-state energies of the even oxygen isotopes for the NN+3N-induced (a) and NN+3N-full Hamiltonians (b) at $\lambda_{\rm SRG} = 1.88$ fm$^{-1}$. Solid lines indicate the energy extrapolation based on $N_{\rm max} = 8 - 12$ data, dotted lines guide the eye for smaller $N_{\rm max}$. Uncertainties due to the importance truncation are smaller than the symbols used to represent the data. All energies are obtained at optimal $\hbar\Omega$.

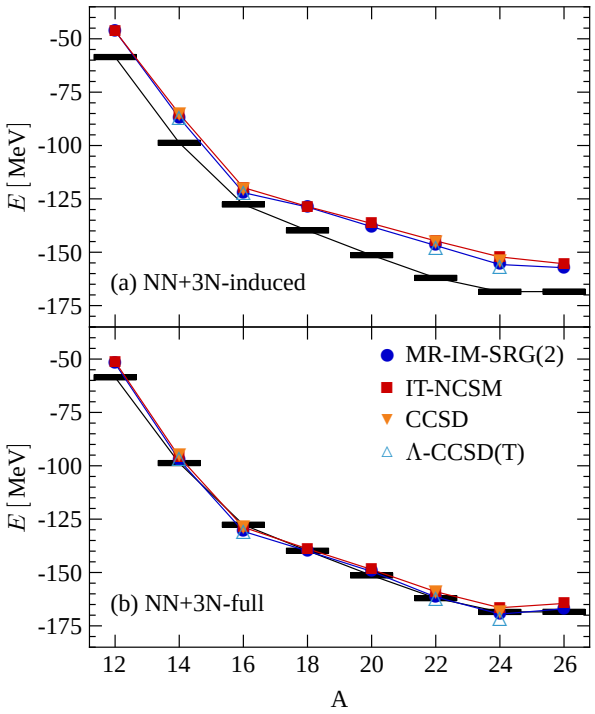


FIG. 4. (Color online) Oxygen ground-state energies for the NN+3N-induced (top) and NN+3N-full (bottom) Hamiltonian with $\Lambda_{\rm 3N} = 400$ MeV/$c$. MR-IM-SRG(2), CCSD, and Λ-CCSD(T) results are obtained at optimal $\hbar\Omega$, using 15 major oscillator shells and $E_{\rm 3max} = 14$. The IT-NCSM energies are extrapolated to infinite model space. Experimental values are indicated by black bars [28, 29].

dependence for $\Lambda_{\rm 3N} = 500$ MeV/$c$ observed in Refs. [23, 26, 27].

To assess the quality of our MR-IM-SRG(2) ground-state energies, we compare them to results from the IT-NCSM, which yields the exact NCSM results within quantified uncertainties from the importance truncation [26, 33]. In the IT-NCSM calculations, we use the full 3N interaction without NO2B approximation, and the $E_{\rm 3max}$ cut is naturally compatible with the IT-NCSM model space truncation [13]. In Fig. 3 we show the convergence of the oxygen ground-state energies for the NN+3N-induced and NN+3N-full Hamiltonians as a function of $N_{\rm max}$, along with exponential fits which extrapolate $N_{\rm max} \to \infty$ [26, 33, 34]. With the exception of $^{26}$O, all isotopes converge well, and the uncertainties of the threshold and model spaces truncations of the IT-NCSM results are typically about 1 MeV. For $^{26}$O, the rate of convergence is significantly worse, which is expected due to the resonance nature of this ground state.

The neutron-rich oxygen isotopes are the heaviest nuclei studied so far in the IT-NCSM with full $3N$ interactions. For $^{26}$O, the computation of the complete $N_{\rm max}$ sequence shown in Fig. 3 requires about 200,000 CPU hours. In contrast, a corresponding sequence of single-particle basis sizes in the MR-IM-SRG requires only about 3,000 CPU hours on a comparable system. Overall, the method scales polynomially with $\mathcal{O}(N^6)$ to larger basis sizes $N$, which makes it ideally suited for the description of medium- and heavy-mass nuclei.

In Fig. 4, we compare the MR-IM-SRG(2) and IT-NCSM ground-state energies of the oxygen isotopes, for the NN+3N-induced and NN+3N-full Hamiltonians with $\lambda_{\rm SRG} = 1.88$ fm$^{-1}$ to experiment. For the latter, the overall agreement between the two very different many-body approaches and experiment is striking: Except for slightly larger deviations in $^{12}$O and $^{26}$O, we reproduce experimental binding energies within 2-3 MeV. This is a remarkable demonstration of the predictive power of current chiral NN+3N Hamiltonians, at least for ground-state energies. For further confirmation, we perform CC calculations with singles and doubles (CCSD), as well as perturbative triples (Λ-CCSD(T)) [15, 22, 35, 36] for oxygen isotopes with sub-shell closures. Using the same Hamiltonians in NO2B approximation, the MR-IM-SRG energies are bracketed by the CC results, and similar to the Λ-CCSD(T) values, consistent with the closed-shell results discussed in [13].

For the NN+3N-induced calculation, which should be compared to calculations with the bare chiral NN interaction [6], the reproduction of experimental trends fails, and the neutron drip line is predicted at the wrong mass, because $^{26}$O is bound with respect to $^{24}$O. This illustrates the crucial importance of the chiral 3N interaction for a proper description of the structure of neutron-rich nuclei [32].

Let us now address the uncertainties of our results. The MR-IM-SRG(2) energies lie 1.5–2% below the IT-NCSM results. About 1% of this deviation is caused by the NO2B approximation. The uncertainty due to the $E_{\rm 3max}$ cut is less than 1% at low $\lambda_{\rm SRG}$. While these uncertainties exhaust the greater part of the 1.5–2% deviation between MR-IM-SRG(2) and IT-NCSM, and suggest a very small uncertainty due to the many-body truncation, we assume a more conservative many-body truncation error of 1–1.5%, and an overall uncertainty of our oxygen energies at the level of 3–3.5%, consistent with

our closed-shell IM-SRG calculations [13]. Because all irreducible many-body density matrices vanish in closed-shell nuclei, our findings indicate that the truncation of terms containing $\lambda^{(n\geq 3)}$ and non-linear powers of $\lambda^{(2)}$ is negligible compared to the truncation of induced three-body operators. A more detailed analysis of the MR-IM-SRG truncation scheme will be presented in a future publication.

*Conclusions.* We have generalized the IM-SRG approach to multi-reference states, and used the resulting MR-IM-SRG method to perform the first *ab initio* study of all even oxygen isotopes with chiral NN+3N Hamiltonians, along with the IT-NCSM and the CC method. The MR-IM-SRG results are in excellent agreement with those from the other methods, confirming its reliability, and the method's modest computational demands make it ideally suited for the description of medium- and heavy-mass open-shell nuclei far from shell closures.

Our calculated oxygen ground-state energies agree remarkably well with experimental binding energies within theoretical uncertainties of 3%. This is achieved without any readjustment of the interaction to experimental data beyond $^4$He, and therefore constitutes an impressive demonstration of the predictive power of chiral NN+3N Hamiltonians. The present work also highlights the importance of the 3N interaction for the nuclear structure of neutron-rich nuclei, as demonstrated by the robust reproduction of the oxygen drip line.

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