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Gauge Coupling Unification and Non-Equilibrium Thermal Dark Matter

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We study a new mechanism for the production of dark matter in the universe which does not rely on thermal equilibrium. Dark matter is populated from the thermal bath subsequent to inflationary reheating via a massive mediator whose mass is above the reheating scale, T_{RH}. To this end, we consider models with an extra U(1) gauge symmetry broken at some intermediate scale ($M_{int} \simeq 10^{10-12}$GeV). We show that not only does the model allow for gauge coupling unification (at a higher scale associated with grand unification) but can provide a dark matter candidate which is a Standard Model singlet but charged under the extra U(1). The intermediate scale gauge boson(s) which are predicted in several E6/SO(10) constructions can be a natural mediator between dark matter and the thermal bath. We show that the dark matter abundance, while never having achieved thermal equilibrium, is fixed shortly after the reheating epoch by the relation T_{RH}^3/M_{int}^4. As a consequence, we show that the unification of gauge couplings which determines M_{int} also fixes the reheating temperature, which can be as high as $T_{RH} \simeq 10^{11}$GeV.

I. INTRODUCTION

The Standard Model (SM) of particle physics is now more than ever motivated by the recent discovery of the Higgs boson at both the ATLAS [1] and CMS [2] detectors. The SM, however, contains many free parameters, and the gauge couplings do not unify. Among the most elegant approaches to understand some of these parameters is the idea of a grand unified theory (GUT) in which the three gauge couplings $\alpha_{1,2,3}$ originate from a single gauge coupling associated to a grand unified gauge group [3]. This idea is supported by the fact that quantum numbers of quarks and leptons in the SM nicely fill representations of a GUT symmetry, e.g., the **10** and $\bar{\mathbf{5}}$ of $SU(5)$ or **16** of $SO(10)$.

Another issue concerning the SM is the lack of a candidate to account for Dark Matter (DM) which consists of 22 % of the energy density of our universe. Stable Weakly Interacting Massive Particles (WIMPs) are among the most popular candidates for DM. In most models, such as popular supersymmetric extensions of the SM [4], the annihilation of WIMPs in thermal equilibrium in the early universe determined the relic abundance of DM.

In this letter, we will show that GUT gauge groups such as E_6 or SO(10) which contain additional U(1) gauge subgroups and are broken at an intermediate scale, can easily lead to gauge coupling unification [5] and may contain a new dark matter candidate which is charged under the extra U(1). However, unlike the standard equilibrium annihilation process, or complimentary process of freeze-in [6], we propose an alternative mechanism for producing dark matter through interactions which are mediated by the heavy gauge bosons associated with the extra U(1). While being produced from the thermal bath, these dark matter particles never reach equilibrium. We will refer to dark matter produced with this mechanism as Non-Equilibrium Thermal Dark Matter or NETDM. The final relic abundance of NETDM is obtained shortly after the inflationary reheating epoch. This mechanism is fundamentally different from other non-thermal DM production mechanisms in the literature (to our knowledge). Assuming that none of the dark matter particles are directly produced by the decays of the inflaton during reheating, we compute the production of dark matter and relate the inflationary reheat temperature to the choice of the gauge group and the intermediate scale needed for gauge coupling unification. As an added benefit, the model naturally possesses the capability of producing a baryon asymmetry through leptogenesis, although that lies beyond of the scope of this work.

The letter is organized as follows. After a summary of the unified models under consideration in section II, we show how the presence of an intermediate scale allows for the possibility of producing a dark matter candidate which respects the WMAP constraint [7] and apply it to an explicit scenario in section III. A discussion of our main result is found in section IV before concluding in section V.

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II. UNIFICATION IN SO(10) MODELS

The prototype of grand unification is based on the SU(5) gauge group. In an extension of SU(5) one can introduce SU(5) singlets as potential dark matter candidates. The simplest extension in which singlets are automatically incorporated is that of SO(10). There are, however, many ways to break SO(10) down to $SU(3) \times SU(2) \times U(1)$. This may happen in multiple stages, but here we are mainly concerned with the breaking of an additional U(1) (or SU(2)) factor at an intermediate scale M_{int}. Here, we will not go into the details of the breaking, but take some specific, well-known examples when needed. Assuming gauge coupling unification, the GUT mass scale, M_{GUT}, and the intermediate scale M_{int} can be predicted from the low–energy coupling constants with the use of the renormalisation group equation,

$$\mu \frac{d\alpha_i}{d\mu} = -b_i \alpha_i^2 \ . \tag{1}$$

The evolution of the three running coupling constants α_1, α_2 and α_3 from M_Z to the intermediate scale M_{int} is obtained from Eq.(1) using the β–functions of the Standard Model: $b_{1,2,3} = (-41/10, 19/6, 7)/2\pi$. We note that the gauge coupling, g_D, associated with $U'(1)$ is related at the GUT scale to g_1 of $U(1)_Y$ by $g_D = \sqrt{\frac{5}{3}} g_1$ and $\alpha_i = g_i^2/4\pi$. Between M_{int} and M_{GUT} (both to be determined) the running coupling constants are again obtained from Eq.(1), now using β–functions associated with the intermediate scale gauge group, which we will label $\tilde{b}_i$. The matching condition between the two different runnings at M_{int} can be written:

$$(\alpha_i^0)^{-1} + b_i t_{int} = \alpha^{-1} + \tilde{b}_i (t_{int} - t_{GUT}) \tag{2}$$

with $t_{int} = \ln(M_{int}/M_Z)$, $t_{GUT} = \ln(M_{GUT}/M_Z)$, $\alpha_i^0 = \alpha_i(M_Z)$ which is measured, and $\alpha = \alpha_i(M_{GUT})$ is the unified coupling constant at the GUT scale. This gives us a system of 3 equations, for 3 unknown parameters: α, t_{int}, t_{GUT}. Solving the Eq.(2), we obtain

$$t_{int} = \frac{1}{b_{32} - b_{21}} \left[\frac{(\alpha_3^0)^{-1} - (\alpha_2^0)^{-1}}{\tilde{b}_2 - \tilde{b}_3} - \frac{(\alpha_2^0)^{-1} - (\alpha_1^0)^{-1}}{\tilde{b}_1 - \tilde{b}_2} \right] \tag{3}$$

where $b_{ij} \equiv (b_i - b_j)/(\tilde{b}_i - \tilde{b}_j)$.

To be concrete, we will consider a specific example to derive numerical results for the case of the breaking of $SO(10)$: $SO(10) \rightarrow SU(4) \times SU(2)_L \times U(1)_R \rightarrow_{M_{int}} SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow_{M_{EW}} SU(3)_C \times U(1)_{em}$. When the intermediate symmetry is broken by a **16** of Higgs bosons, the $\tilde{b}_i$ functions are given by $\tilde{b}_{1,2,3} = (5/2, 19/6, 63/6)/2\pi$ [5], where the computation was done at 1-loop level. For this case, we obtain $M_{int} = 7.8 \times 10^{12}$ GeV and $M_{GUT} = 1.3 \times 10^{15}$ GeV using $(\alpha_{1,2,3}^0)^{-1} \simeq (59.47, 29.81, 8.45)$. The evolution of the gauge couplings for this example is shown in Fig. 1.

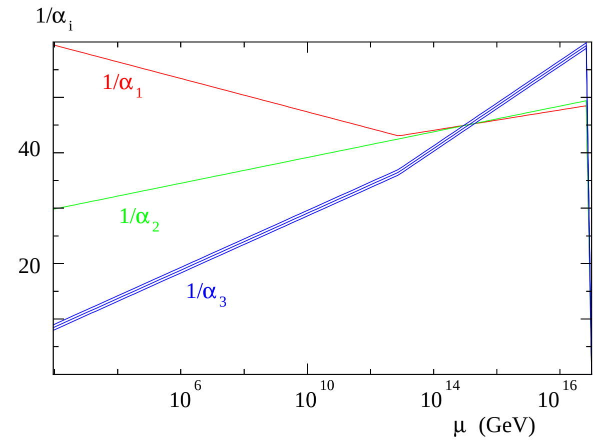

FIG. 1. Example of the running of the SM gauge couplings for $SO(10) \rightarrow SU(4) \times SU(2)_L \times U(1)_R$.

III. HEAVY Z' AND DARK MATTER

It has been shown in [8] and [9] that a stable dark matter candidate may arise in SO(10) models from an unbroken Z_2^{B-L} symmetry. If the dark matter is a fermion (scalar) it should belong to a $3(B-L)$ even (odd) representation of SO(10). For example, the **126** or **144** contains a stable component χ which is neutral under the SM, yet charged under the extra U(1). As we have seen, to explain the unification of the gauge couplings in SO(10) one needs an intermediate scale M_{int} of order 10^{10} GeV. The dark matter candidate, χ, can be produced in the early Universe through s-channel Z' exchange: $SM\ SM \rightarrow Z' \rightarrow \chi\ \chi$. Since $M_{Z'} = \frac{5}{\sqrt{3}} g_D\ M_{int}$, the exchanged particle is so heavy (above the reheating scale, as we show below) that the DM production rate is very slow, and we can neglect the self annihilation process in the Boltzmann equation. Thus while the dark matter is produced from the thermal bath, we have a non–equilibrium production mechanism for dark matter, hence NETDM.

The evolution of the yield of χ, $Y_\chi = n_\chi/\mathbf{s}$ follows

$$\frac{dY_\chi}{dx} = \sqrt{\frac{\pi}{45}} \frac{g_s}{\sqrt{g_\rho}} m_\chi M_P \frac{\langle \sigma v \rangle}{x^2} Y_{eq}^2 \tag{4}$$

where n_χ is the number density of χ and $\mathbf{s}$ the entropy of the universe, g_ρ, g_s are the effective degrees of freedom for energy density and entropy, respectively; $x = m_\chi/T$, m_χ being the dark matter mass, M_P the Planck mass and

$$\langle \sigma v \rangle n_{eq}^2 \approx \frac{\kappa^2\ T}{2048\pi^6} \int_{4m_\chi^2}^{\infty} ds d\Omega \sqrt{s - 4m_\chi^2} |\mathcal{M}|^2 K_1(\sqrt{s}/T) \ . \tag{5}$$

Here n_{eq} is the equilibrium number density of the initial state (SM) particles; and K_1 is the first order modified

Bessel function and κ the effective degrees of freedom of incoming particles.

Since the production of DM occurs mainly at $T_{RH} \gg m_\chi$, we can neglect m_χ in estimating the amplitude for production. In this case, assuming that both χ and the initial state, f, are fermions, we obtain

$$|\mathcal{M}_\chi|^2 \approx \frac{g_D^4 q_\chi^2 q_f^2 N_c^f}{(s - M_{Z'}^2)^2}\left[s^2(1+\cos^2\theta)\right] \quad (6)$$

where θ is the angle between the two outgoing DM particles, N_c^f is number of colors of the particle f, and q_i is the charge of the particle i under $U'(1)$ with a gauge coupling g_D. Here, q is an effective coupling which will ultimately depend on the specific intermediate gauge group chosen. With the approximations $m_\chi, m_f \ll \sqrt{s}$ and $M_{Z'} \gg T_{RH}$, and after integration over θ and sum over all incoming SM fermions in the thermal bath, we obtain

$$\frac{dY_\chi}{dx} = \sum_f \frac{g_D^4 q_\chi^2 q_f^2 N_c^f}{x^4}\left(\frac{45}{\pi}\right)^{3/2}\frac{1}{g_s\sqrt{g_\rho}}\frac{m_\chi^3 M_P}{M_{Z'}^4}\frac{\kappa_f^2}{2\pi^7} \quad (7)$$

Solving Eq.(7) between the reheating temperature and a temperature T gives

$$Y_\chi(T) = \sum_f q_\chi^2 q_f^2 N_c^f \left(\frac{45}{g_s\pi}\right)^{3/2}\frac{M_P}{M_{int}^4}\frac{3\,\kappa_f^2}{1250\pi^7}\left[T_{RH}^3 - T^3\right] \quad (8)$$

where we replaced the mass of the Z' by $M_{Z'} = \frac{5}{\sqrt{3}} g_D M_{int}$ and made the approximation $g_\rho = g_s$. We note that the effect of Z' decay on the abundance of χ is completely negligible due to its Boltzmann suppression in the Universe: the Z' is largely decoupled from the thermal bath already at the time of reheating.

We note several interesting features from Eq.(8). First of all, the number density of the dark matter does not depend at all on the strength of the $U'(1)$ coupling g_D but rather on the intermediate scale (that is determined by requiring gauge coupling unification as we demonstrated in the previous section). Second, the production of dark matter is mainly achieved at reheating. Thirdly, once the relic abundance is obtained, the number density per comoving frame (Y) is fixed, never having reached thermal equilibrium with the bath. And finally, upon applying the WMAP determination for the DM abundance, and assuming instantaneous reheating after inflation, we obtain a constraint on T_{RH} once the pattern of $SO(10)$ breaking is known (and thus M_{int} fixed). In a more complete model of reheating, the resulting relic density may be modified by an order of magnitude (or more) for a given reheat temperature.

Thus, given a scheme of $SO(10)$ breaking we can determine the reheating temperature very precisely from the relic abundance constraint in the Universe. From

$$Y_0 = \frac{\Omega}{m_\chi}\frac{\rho_0^{crit}}{\mathbf{s}_0} = \left(\frac{\Omega h^2}{0.1}\right)\frac{13.5}{16\pi^3}\frac{H_0^2 M_P^2}{g_s^0 T_0^3 m_\chi} \quad (9)$$

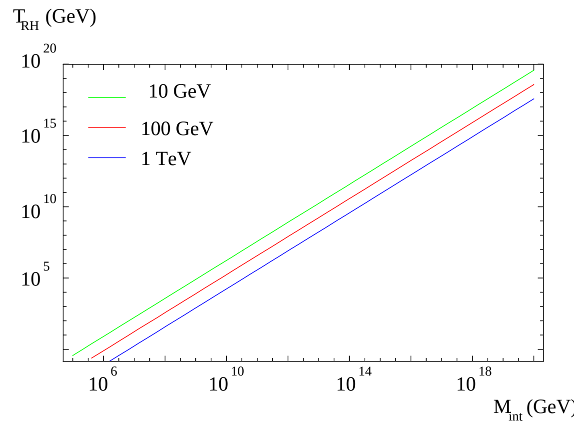

FIG. 2. Reheating temperature as function of the SO(10) breaking scale for different mass of dark matter : 10, 100 and 1000 GeV

TABLE I. Possible breaking schemes of SO(10).

	$SO(10) \to \mathcal{G}\times$ [Higgs]	M_{int}(GeV)	T_{RH}(GeV)
A	$4 \times 2_L \times 1_R$ [**16**]	$10^{12.9}$	3×10^9
A	$4 \times 2_L \times 1_R$ [**126**]	$10^{11.8}$	1×10^8
B	$4 \times 2_L \times 2_R$ [**16**]	$10^{14.4}$	3×10^{11}
B	$4 \times 2_L \times 2_R$ [**126**]	$10^{13.8}$	5×10^{10}
C	$3_C \times 2_L \times 2_R \times 1_{B-L}$ [**16**]	$10^{10.6}$	3×10^6
C	$3_C \times 2_L \times 2_R \times 1_{B-L}$ [**126**]	$10^{8.6}$	6×10^3

where H is the Hubble parameter and the index "0" corresponds to present-day values. Combining Eq.(8) and Eq.(9) we find

$$T_{RH}^3 = \frac{5625\,\pi^4}{16 q_\chi^2 \sum_f \kappa_f^2 q_f^2 N_c^f}\left(\frac{\Omega h^2}{0.1}\right)\left(\frac{g_s\pi}{45}\right)^{3/2}\frac{M_P H_0^2}{T_0^3 m_\chi g_s^0}M_{int}^4 \quad (10)$$

or

$$T_{RH} \simeq 2\times 10^8\,\text{GeV}\left(\frac{\Omega h^2}{0.1}\right)^{1/3}\left(\frac{100\text{GeV}}{m_\chi}\right)^{1/3}\left(\frac{M_{int}}{10^{12}\text{GeV}}\right)^{4/3} \quad (11)$$

where we took for illustration $q_\chi^2 \sum_f \kappa_f^2 q_f^2 N_c^f = 1$. We show in Fig.(2) the evolution of T_{RH} as function of M_{int} for different values of the dark matter mass m_χ. We can thus fix the reheating temperature predicted by different symmetry breaking patterns[1]. We summarize them in Table I, where the values of T_{RH} are given for $m_\chi = 100$ GeV. Note that fixing T_{RH} in this way is reminiscent of fixing particle masses or couplings in other dark matter matter models necessary to obtain the correct relic density. We note that although we have here and in Fig. 2

[1] We note that the value obtained for the intermediate scale in different $SO(10)$ breaking schemes is not modified by the presence of a dark matter particle which is not charged under the SM gauge group.

shown results for central values of $m_\chi = 100$ GeV, dark matter candidates are possible with mass up to the order of the reheating temperature, which, based on Eq. (11), would allow it to be as large as $10^6 - 10^7$ GeV.

Finally, we must specify the identity of the NETDM candidate in the context described above. The DM can be in the **126** or **144** representations of SO(10). There are several mechanisms to render the DM mass light [9], one of which is through a fine-tuning of the SO(10) couplings contributing with different Clebsh-Gordan coefficients (see for example, [10] and [11]) to the masses of the various **126** components. For example, for the group $\mathcal{G}_A$:

$$\overline{\mathbf{126}}(M + y_{45}\mathbf{45}_H + y_{210}\mathbf{210}_H)\mathbf{126} \qquad (12)$$

where $M \sim M_{GUT}$, and a $\mathcal{G}_A$ singlet in a linear combination of $\mathbf{210}_H$ and $\mathbf{45}_H$ has a vev at the GUT scale. m_χ is then given by a linear combination of M and the vev and can be tuned to small values, while all other particles inside the **126** live close to M_{GUT}. This is reminiscent of and no more severe than the doublet-triplet separation problem common to grand unified theories.

IV. DISCUSSION

Unfortunately, the chance of detection (direct or indirect) of NETDM with a massive mediator Z' is nearly hopeless. Indeed, the diagram for the direct detection process, measuring the elastic scattering off a nucleus, proceeds through the $t-$channel exchange of the Z' boson, and is proportional to $1/M_{Z'}^4$ yielding a negligible cross-section. In addition, due to the present low velocity of dark matter in our galaxy ($\simeq 200$ km/s), the indirect detection prospects from $s-$channel Z' annihilation $\chi\chi \to Z' \to ff$ proportional to $s^2/M_{Z'}^4$ is also negligible.

As we have seen in Eq.8, the production of dark matter occurs in the very early Universe at the epoch of reheating. A similar mechanism (though fundamentally completely different) where a dark matter candidate is produced close to the reheating time is the case of the gravitino [4, 12]. Indeed, in both cases equilibrium is never reached and the relic abundance is produced from the thermal background to attain the decoupling value Γ/H, with H the Hubble constant and $\Gamma = \langle\sigma v\rangle n_f$ the production rate. However, in the case of SO(10), the cross section decreases with the temperature like $\langle\sigma v\rangle_{Z'} \propto T^2/M_{Z'}^4$, whereas in the case of the gravitino the cross section is constant $\langle\sigma v\rangle_{3/2} \propto 1/M_P^2$ implying $Y(T) \propto T_{RH}$. Gravitinos are produced though the scattering of light particles, whereas in the present context, the dark matter is mediated by a heavy particle ($M_{Z'} > T_{RH}$) which connects the dark and observable sectors. They are physically distinct possibilities.

Finally, we note that cases B and C (in Table I) predict reheating temperatures which are larger (B) or smaller (C) than the case under consideration. Case A would also be compatible with successful thermal leptogenesis with a zero initial state abundance of right–handed neutrino [13, 14]. However in the cases B and C, the persistence of the $SU(2)_R$ symmetry would imply that the cancelation in Eq. 12 would leave behind a light $SU(2)_R$ triplet (for DM inside a **126**) or doublet (for DM inside a **144**). These would affect somewhat the beta functions for the RGE's but more importantly leave behind a test of the model. In the triplet (doublet) case, we would expect three (at least two) nearly degenerate states: one with with charge 0, being the DM candidate, and also states with electric charge ± 1 and ± 2 (or ± 1 in the doublet case).

CONCLUSION

In this work, we have shown that it is possible to produce dark matter through non–equilibrium thermal processes in the context of $SO(10)$ models which respect the WMAP constraints. Insisting on gauge coupling unification, we have demonstrated that there exists a tight link between the reheating temperature and the scheme of the $SO(10)$ breaking to the SM gauge group. Interestingly, the numerical values we obtained are quite high and very compatible with inflationary and leptogenesis-like models.

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