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In their comment Ji *et al.* [1] correctly state that we "obtain the same result as given by the PV theory" for the self-energy of the nucleon. This means that our principal conclusion, that "The pion mass μ dependence of Σ_π is consistent with chiral perturbation theory results for small values of μ and is also linearly dependent on μ for larger values, in accord with the results of lattice QCD calculations", is correct. A consequence of the latter result is that the chiral limit is not yet numerically relevant for current QCD lattice calculations.

Ji *et al.* conclude by stating "The pseudoscalar coupling therefore cannot in general be used if one wishes to ensure consistency with the chiral properties of QCD which are respected by the pseudovector πN coupling". This conclusion is obviously correct, but no claim of general equivalence between PS and PV theory was made in our paper [2].

The conclusion of Ji *et al.* is driven by their opening statement that we argued: "form factors suppress the off-shell contact interactions". No such argument is given in our paper. They further state that our results are obtained by neglecting end-point singularities at $k^+ = 0$. This is not the case. Our results are obtained by putting the intermediate nucleon on its mass shell. With this procedure the self energy obtained with PS and PV theories is indeed the same, as is expected from the well-known equivalence theorem. The use of on-mass shell intermediate nucleons allows the use of experimentally measured πN form factors, which greatly reduces the uncertainty of the result.

Our pion-nucleon effective theory uses PV interactions without approximations, as we demonstrate. The key point is the use of the identity [3] for the nucleon propagator:

$$\frac{1}{\not p - \not k + M} = \frac{\sum_s u(p-k,s)\bar{u}(p-k,s)}{(p-k)^2 - M^2} + \frac{\gamma^+}{2(p-k)^+}, \quad (1)$$

where $u(p-k,s)$ is an on-shell Dirac light-front spinor, p is the initial momentum of the nucleon and k is the momentum of the virtual pion. Using the first term gives the same result for PS or PV interactions, and we show that the second term vanishes. This term involves the factor $\gamma^5 \not k \gamma^+ \gamma^5 \not k/(2(p^+ - k^+)$ which is evaluated as $1/(2M) k_\perp^2 p^+/(p^+ - k^+)$ in the infinite momentum frame ($p^+ \to \infty$). The integral over $k_\perp^2$ involves a well-behaved integrand if our our form factors are included, and one finds the result:

$$\Sigma \sim \int dk^+ dk^- \frac{p^+}{p^+ - k^+} \mathcal{F}(k^+ k^-). \quad (2)$$

where $\mathcal{F}(k^+ k^-)$ is a well-behaved function. The integral can be performed by using $y = k^+ k^-$ and k^+ as the integration variables. The result is that

$$\Sigma \sim \int \frac{dk^+}{k^+} \frac{p^+}{p^+ - k^+} \int dy \mathcal{F}(y) = 0.. \quad (3)$$

The principal values integration over k^+ gives 0. This is not happenstance. The Ji *et al.* finding that removing end-point singularities from the PS calculation yields the PV result is accidental.

The only possible change in our numerical results arising from the effects discussed by Ji *et al.* is on the pion distribution f_π^N. However, the work of Refs. [4, 5] shows that the effect is to increase the value of a term that has a very small contribution (when realistic values of the pion mass are used) by a factor of 4/3. The size of this effect is much, much less than the uncertainty introduced by the use of πN form factors that are not constrained by measurements.

To summarize, nothing in the Comment by Ji *et al.* changes the conclusions or numerical results of [2] in any substantive way.

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