Superconductivity at the Onset of Spin-Density-Wave Order in a Metal

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Superconductivity at the onset of spin-density-wave order in a metal

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We revisit the issue of superconductivity at the quantum-critical point (QCP) between a 2D paramagnet and a spin-density-wave metal with ordering momentum (π,π). This problem is highly non-trivial because the system at criticality displays a non-Fermi liquid behavior and because the effective coupling constant λ for the pairing is generally of order one, even when the actual interaction is smaller than fermionic bandwidth. Previous study [M. A. Metlitski, S. Sachdev, Phys.Rev.B 82, 075128 (2010)] has found that the renormalizations of the pairing vertex are stronger than in BCS theory and hold in powers of $\log^2(1/T)$. We analyze the full gap equation and argue that summing up of the leading logarithms does not lead to a pairing instability. Yet, we show that superconductivity has no threshold and appears even if λ is set to be small, because subleading logarithmical renormalizations diverge and give rise to BCS-like result $\log 1/T_c \propto 1/\lambda$. We argue that the analogy with BCS is not accidental as at small λ superconductivity at a QCP predominantly comes from fermions which retain Fermi liquid behavior at criticality. We compute T_c for the actual $\lambda \sim O(1)$, and found that both Fermi-liquid and non-Fermi liquid fermions contribute to the pairing.

Introduction. Superconductivity at the onset of density-wave order in a metal is an issue of high current interest, with examples ranging from cuprates [1], to Fe-pnictides [2] and other correlated materials [3–5] It is widely believed that the pairing in these systems is caused by repulsive electron-electron interaction, enhanced in a particular spin or charge channel, which becomes critical at the quantum-critical point (QCP). The pairing problem at QCP is highly non-trivial in $D \leq 3$, as scattering by a critical collective mode destroys Fermi liquid (FL) behavior above T_c (Ref. [6, 9]). This is particularly relevant for systems near uniform density-wave instability (e.g., a ferromagnetic or a nematic one). In this case, FL behavior is lost on the whole Fermi-surface (FS), and superconductivity can be viewed as a pairing of incoherent fermions which exchange quanta of gapless collective bosons [7–11]. The pairing of incoherent fermions is qualitatively different from BCS/Eliashberg pairing of coherent fermions in a FL because in the incoherent case the pairing in $D < 3$ occurs only if the interaction exceeds a certain threshold [8, 12, 13]. For $D = 3$ there is no threshold, but at small coupling constant λ, $\log \Lambda/T_c = 1/\sqrt{\lambda}$ rather than $1/\lambda$ (Ref. [15]), in close analogy to T_c in color superconductivity (CSC) of quarks mediated by the exchange of gluons [16]

The non-FL behavior at criticality is less pronounced for systems near density-wave order at a finite momentum, because only fermions near particular points along the FS (hot spots) lose FL behavior at criticality. Still, fermions from hot regions mostly contribute to the pairing, and early studies of superconductivity at the onset of (π,π) spin-density-wave (SDW) order [8, 9] placed the pairing problem into the same universality class as for QCP with $q = 0$. The 2D problem has been recently re-analyzed [17] by Metlitski and Sachdev (MS). They argued that it is important to include into the consideration the momentum dependence of the self-energy along the FS, neglected in earlier studies. Using the full form of $\Sigma(\omega_m, \mathbf{k})$ for $\mathbf{k}$ on the FS, they found that the one-loop renormalization of the pairing vertex is larger than previously thought – it is $\log^2$ instead of log, and that the enhancement comes from fermions somewhat away from hot spots, for which $\Sigma(\omega_m, \mathbf{k})$ has a FL form at the smallest frequencies. The $\log^2$ behavior in the perturbation theory holds for CSC, and MS result raises the question whether the pairing problem at a 2D SDW QCP is in the same universality class as CSC. The related issues raised by MS work are: (i) is the problem analogous to the pairing at a 2D SDW QCP a FL phenomenon, or non-FL physics is essential, (ii) what sets the scale of T_c, and (iii) is T_c non-zero only if the coupling λ exceed a finite threshold, as it happens if one approximates $\Sigma(\omega_m, \mathbf{k})$ by $\Sigma(\omega_m)$ at a hot spot, or T_c is non-zero even at smallest λ, like in CSC?

In this letter, we address these issues. We first show that the analogy with CSC does not extend beyond one-loop order, and in our case the summation of $\log^2$ terms in the Cooper channel does not give rise to a pairing instability. However, that subleading log terms do give rise to a pairing instability, and at weak coupling yield $\log \Lambda/T_c \propto 1/\lambda$, like in BCS theory. We show that the analogy with BCS formula is not accidental because the pairing at small λ predominantly comes from fermions for which fermionic self-energy has a FL form. We then analyze the physical case $\lambda = O(1)$ and argue that in this case fermions from both FL and non-FL regimes contribute to the pairing and that $T_c \approx 0.04\omega_0$, where ω_0 is the frequency at which $\Sigma(\omega_m)$ at a hot spot becomes equal to ω_m. The numerical prefactor agrees with the slope of T_c obtained by solving the gap equation numerically along the full FS [21].

The model. We follow earlier works[8, 9, 17] and analyze the pairing near an antiferromagnetic QCP within the semi-phenomenological spin-fermion model. The model assumes that antiferromagnetic correlations develop already at high energies, of order bandwidth, and mediate interactions between low-energy fermions. The static part of the spin-fluctuation propagator is treated as a phenomenological input from high-energy physics, but the the dynamical Landau damping part is self-consistently obtained within the model as it comes entirely from low-energy fermions [8, 9, 17]. In the Supplementary material we review justifications for the spin-fermion model and compare spin-fermion approach with the RG-based approaches [22–25] which treat superconductivity, magnetism, and specific charge density-wave orders on equal footings.

We assume, like in [8, 9, 17], that fermions have $N \gg 1$ flavors and that collective spin excitations are peaked at $\mathbf{Q} = (\pi, \pi)$, and focus on the hot regions on the FS, i.e., on momenta near $\mathbf{k}_F$, for which $\mathbf{k}_F + \mathbf{Q}$ is also near the FS. The Lagrangian of the model is given by [3, 9, 17]

$$\begin{aligned} \mathcal{S} &= -\int_k^\Lambda G_0^{-1}(k)\, \psi^\dagger_{k,\alpha}\psi_{k,\alpha} + \frac{1}{2}\int_q^\Lambda \chi_0^{-1}(q)\ \mathbf{S}_q \cdot \mathbf{S}_{-q} \\ &\quad + g\int_{k,q}^\Lambda \psi^\dagger_{k+q,\alpha}\sigma_{\alpha\beta}\psi_{k,\beta}\cdot \mathbf{S}_{-q}. \end{aligned} \tag{1}$$

where $\int_k^\Lambda$ stands for the integral over $d-$dimensional $\mathbf{k}$ (up to some upper cutoff Λ) and the sum over fermionic and bosonic Matsubara frequencies, $G_0(k) = G_0(\omega_m, \mathbf{k}) = 1/(i\omega_m - \mathbf{v}_{F,\mathbf{k}}(\mathbf{k}-\mathbf{k}_F))$ is the bare fermion propagator, and $\chi_0(q) = \chi_0(\Omega_m, \mathbf{q}) = \chi_0/(\mathbf{q}^2 + \xi^{-2})$ is the static propagator of collective bosons, in which ξ^{-1} measures a distance to a QCP and $\mathbf{q}$ is measured with respect to $\mathbf{Q}$. We set $\xi^{-1} = 0$ below. The fermion-boson coupling g and χ_0 appear in theory only in combination $\bar{g} = g^2\chi_0$ and we will use $\bar{g}$ below. The Fermi velocities at hot spots separated by $\mathbf{Q}$ can be expressed as $\mathbf{v}_{F,1} = (v_x, v_y)$ and $\mathbf{v}_{F,2} = (-v_x, v_y)$, where x axis is along $\mathbf{Q}$. We will also use $\alpha = v_y/v_x$ and $v_F = (v_x^2 + v_y^2)^{1/2}$. The model of Eq. (1) can be equivalently viewed as a four-patch model for fermions near hot spots at $\pm\mathbf{k}_F$ and $\pm(\mathbf{k}_F + \mathbf{Q})$ (Ref. [17, 20]). The hot spot model is obviously justified only when the interaction $\bar{g}$ is smaller than E_F.

The fermion-boson coupling gives rise to fermionic and bosonic self-energies. In the normal state, bosonic self-energy accounts for Landau damping of spin excitations, while fermionic self-energy accounts for the mass renormalization and a finite lifetime of a fermion. At one-loop level, self-consistent normal-state analysis yields [9, 17, 18]

$$\chi(\Omega_m, \mathbf{q}) = \frac{\chi_0}{\mathbf{q}^2 + |\Omega_m|\gamma} \tag{2}$$

$$\Sigma(\omega_m, k_\parallel) = \frac{3\bar{g}}{4\pi v_F}\ \frac{2\omega_m}{\sqrt{\gamma|\omega_m| + \left(\frac{2k_\parallel\alpha}{1+\alpha^2}\right)^2} + \left|\frac{2k_\parallel\alpha}{1+\alpha^2}\right|} \tag{3}$$

where $\gamma = 2N\bar{g}/(\pi v_x v_y)$ and $k_\parallel$ is a deviation from a hot spot along the FS. The bosonic propagator $\chi(\Omega_m, \mathbf{q})$ describes Landau-overdamped spin fluctuations. The fermionic self-energy has a non-FL form right at a hot spot – $\Sigma(\omega_m, 0) = (|\omega_m|\omega_0)^{1/2} sign\omega_m$, where $\omega_0 = (9\bar{g}/(16\pi N))(2v_x v_y/v_F^2)$. Away from a hot spot, $\Sigma(\omega_m, k_\parallel)$ retains a FL form at the smallest ω_m and scales as $\Sigma(\omega_m, k_\parallel) \propto \omega_m/|k_\parallel|$.

We use Eqs. 2 and 3 as inputs for the pairing problem and neglect higher order terms in the loop expansion. Most of higher-order terms are small in $1/N$, but some terms with $n \geq 4$ loops do not contain $1/N$ (Refs. [17, 19, 20]). The terms without $1/N$ include, in particular, feedback effects from pairing fluctuations on the fermionic and bosonic propagators. We verified that these feedback effects preserve the forms of χ and Σ, and we just assume that they do not substantially modify the prefactors.

The pairing vertex We add to the action the anomalous term $\Phi_0(k)\psi_{k,\alpha}(i\sigma^y)_{\alpha\beta}\psi_{-k,\beta}$ and use Eq. (1) to renormalize it into the full $\Phi(k)$. At T_c, the pairing susceptibility $\chi_{pp}(k) = \Phi(k)/\Phi_0$ must diverge for all k. The bare Φ_0 can be set constant within a patch, but has to change signs between patches separated by $\mathbf{Q}$ (the pairing symmetry at the onset of SDW order is $d-$wave [26]). The one-loop renormalization of $\Phi(k)$ at $k = (\omega \sim T, 0)$ was obtained by MS:

$$\Phi(\omega \sim T, 0) = \Phi_0(1 + \frac{\lambda}{2\pi}\log^2\Lambda/T), \quad \lambda = \frac{2\alpha}{(1+\alpha^2)}, \tag{4}$$

where Λ is the smaller of ω_0 and $\alpha^4 E_F^2/\omega_0$. Notice that neither the coupling constant $\bar{g}$ not $1/N$ appear in (4), the only parameter is the ratio of the velocities α, which is a geometrical property of the FS. For a cuprate-like FS, $\alpha \sim 1$, i.e., the pairing coupling constant $\lambda = O(1)$. To understand the physics of the pairing at the QCP, we find that it is instructive to formally replace λ by $\varepsilon\lambda$ and first analyze the pairing in the "weak coupling" case $\varepsilon \ll 1$.

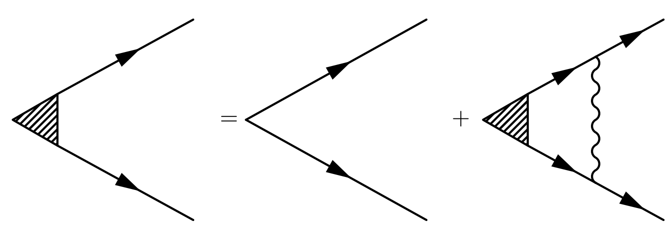

FIG. 1: Diagrammatic representation for the pairing vertex. The shaded triangle is the full Φ_k, the unshaded vertex is the bare Φ_0, solid lines are full fermionic propagators, and the wavy line is the Landau-overdamped spin propagator. The pairing vertex contains $i\sigma^y_{\alpha,\beta}$, the vertices where wavy and solid lines meet contain $\sigma_{\gamma\delta}$.

Let's first see where $\log^2$ renormalization comes from. The one-loop diagram for Φ contains two fermionic propagators $G(k)$ and $G(-k)$ and one bosonic $\chi(k)$ (Fig.1).

Large N allows one to restrict $\chi(\Omega_m, \mathbf{k})$ to momenta connecting points at the FS and integrate over momenta transverse to the FS in the fermionic propagators only. Because Σ does not depend on this momentum, the integration is straightforward, and yields, to logarithmic accuracy $\int GG\chi \propto \int dk_\parallel \int_T d\Omega_m (\chi(\Omega_m, k_\parallel)/|\Omega_m + \Sigma(\Omega_m, k_\parallel))|$. At $k_\parallel^2 > \gamma\Omega_m$ and $|k_\parallel| < k_F\bar{g}/v_F$, $1/|\Omega_m + \Sigma(\Omega_m, k_\parallel)|$ scales as $|k_\parallel/\Omega_m|$ and $\chi(\Omega_m, k_\parallel) \propto 1/k_\parallel^2$. Integrating over $k_\parallel$ we obtain $\int_{\gamma|\Omega_m|} dk_\parallel^2/k_\parallel^2 \propto \log|\Omega_m|$, and the remaining integral over frequency yields $\int GG\chi \propto \int_T (d\Omega_m/|\Omega_m|) \log|\Omega_m| \propto \log^2 T$. We see that the $\log^2 T$ dependence originates from extra logarithm from $k-$integration. This extra logarithm is in turn the consequence of $\Omega_m/k_\parallel$ form of self-energy $\Sigma(\Omega_m, k_\parallel)$ at $k_\parallel^2 > \gamma\Omega_m$. As $\Sigma \propto \omega$ is the property of a FL, the $\log^2 T$ renormalization comes from fermions which preserve a FL behavior at a QCP. We further see that the one-loop renormalization can be interpreted as coming from the process in which fermions are exchanging quanta of an effective local $\log\Omega$ interaction. The same process determines one-loop renormalization of Φ in CSC.

The $\log^2$ analysis can be extended beyond leading order. We assume that $\lambda = 2\varepsilon\alpha/(1+\alpha^2)$ is small (because we set ε to be small), but $\lambda \log^2 T = O(1)$, and sum up ladder series of $\lambda \log^2 T$ terms, neglecting smaller powers of logarithms at each order of loop expansion. Performing the calculations (see Supplementary material for details), we find that the analogy with CSC does not extend beyond leading order: for CSC the summation of $\lambda \log^2 T$ terms yields $\Phi = \Phi_0/\cos[(2\lambda \log^2 T)^{1/2}]$ (Ref.[15]), and the system develops a pairing instability at $|\log T_c| = \pi/2\sqrt{2\lambda}$ (Ref. [16]). In our case, perturbation series yield $\Phi = \Phi_0 e^{\lambda/2\pi \log^2 T}$, i.e., the pairing susceptibility increases with decreasing T, but never diverges. Because the summation of the leading logarithms does not lead to a finite T_c, one has to go beyond the leading logarithmical approximation and analyze the full equation for $\Phi(k)$ at $\Phi_0 = 0$ in order to understand whether or not T_c is finite at a QCP. This is what we do next.

Full gap equation. Within our approximation, the full linearized equation for the anomalous vertex is obtained by summing up ladder diagrams and keeping the self-energy in the fermionic propagator. Integrating the r.h.s. of this equation over momenta transverse to the FS, we obtain

$$\Phi(\omega_m, k_\parallel) = \frac{3\bar{g}}{2v_F} T \sum_{m'} \int \frac{dk_\parallel'}{2\pi} \frac{\Phi(\omega_{m'}, k_\parallel')}{|\omega_{m'} + \Sigma(\omega_{m'}, k_\parallel')|} \times \frac{1}{k_\parallel^2 + k_\parallel'^2 - 2\mu k_\parallel k_\parallel' + \gamma|\omega_m - \omega_{m'}|} \quad (5)$$

where $\mu = (1-\alpha^2)/(1+\alpha^2)$. The temperature at which the solution exists is T_c. The overall factor $3\bar{g}/(2v_F)$ is eliminated by rescaling and get replaced by λ, which, we recall, we treat as a small parameter. One can verify that typical $k_\parallel^2$ are larger than typical $\gamma\omega_m$, and that the vertex $\Phi(\omega_m, k_\parallel)$ has a stronger dependence on $k_\parallel$ than on frequency. In this situation, one can approximate $\Phi(\omega_m, k_\parallel)$ by $\Phi(k_\parallel)$, explicitly sum up over frequency and reduce (5) to 1D integral equation.

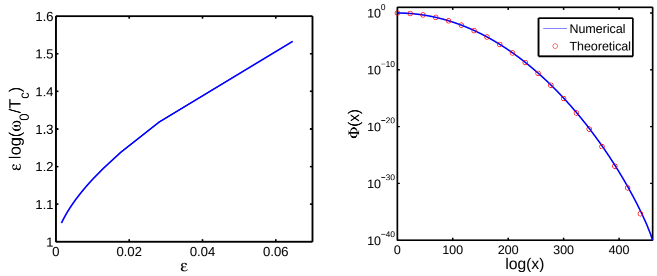

FIG. 2: Numerical solution of Eq. (6) at small ε. (a) The transition temperature. When ε decreases, $\varepsilon \log\omega_0/T_c$ approaches 1, as in Eq. (8). (b) The eigenfunction $\Phi(y)$, where $y = k_\parallel^2/(\pi T\gamma)$. Solid and dashed lines are numerical and analytical solutions of Eq. 6, respectively. The two are very close, except for the largest $y \sim \omega_0/T$, when the cutoff becomes relevant.

For simplicity, we first consider the case when $\alpha = 1$, i.e $\lambda = \varepsilon$. Introducing $\bar{T} = \pi T/\omega_0$ and $x = k_\parallel^2/(\gamma\omega_0\bar{T})$, we obtain from (5)

$$\Phi(y) = \frac{\varepsilon}{\pi}\int_1 \frac{dx}{x+y}\frac{\log x}{2\sqrt{x\bar{T}}+1}\Phi(x) \quad (6)$$

The term in the denominator with $\sqrt{x\bar{T}}$ is a soft upper cutoff.

The r.h.s. of (6) contains $\log^2$ contributions from the range $x \gg y$, but, as we just found, they do not lead to a pairing instability. We therefore focus on the contribution from $x \sim y$. Because the kernel is logarithmical, we search for $\Phi(x)$ in the form $\Phi(x) = \exp[-f(p(x))]$, where $p(x) = \varepsilon \log x$. Substituting this into (6), we find that the form is reproduced at $1 \ll x \ll 1/\bar{T}$, when soft cutoff can be omitted. The self-consistency condition yields (see Supplementary material)

$$f(z) = \frac{1}{\pi\varepsilon}\left(z \arcsin z + \sqrt{1-z^2} - 1\right). \quad (7)$$

At small ε, the soft cutoff can be replaced by the boundary condition that $df(z)/dz$ must be at a maximum at $z = \varepsilon|\log\bar{T}|$. This condition sets

$$T_c \sim \omega_0 e^{-1/\varepsilon}. \quad (8)$$

To verify this reasoning, we solved Eq. (6) numerically and found very good agreement with analytical results (see Fig.2).

We next analyze the gap equation at $\alpha \neq 1$ Using the same logic as before we find (see Supplementary material for details) that Eq. (8) does not change, i.e., to logarithmical accuracy, T_c/ω_0 does not depend on the angle between Fermi velocities at $\mathbf{k}_F$ and $\mathbf{k}+\mathbf{Q}$. We verified the independence of T_c/ω_0 on α by solving Eq. (6) numerically for different α.

We see from (8) that T_c is non-zero already at infinitesimally small coupling, like in BCS theory. The analogy is not accidental as the pairing predominantly comes from momenta away from hot spots, for which $x \sim y \sim \bar{T}$, i.e., $k_\parallel \sim k_\perp \sim (\gamma\omega_0)^{1/2}$. Because $T_c \ll \omega_0$, typical $\gamma\omega \geq \gamma T_c$ are much smaller than $\gamma\omega_0$, hence fermionic self-energy for $k_\parallel \sim (\gamma\omega_0)^{1/2}$ has the FL form $\Sigma(\omega_m, k_\parallel) \propto \omega_m/|k_\parallel|$. Furthermore, for $x \sim y$ in (6), the integration over x does not give rise to an additional logarithm besides $\log x$, which is a Cooper logarithm. The instability at T_c is then a conventional Cooper instability of a FL with a weak and non-singular attractive coupling ε. In other words, for small ε, the pairing at a SDW QCP is entirely a FL phenomenon.

Although Eq. (8) looks like BCS formula, the problem we are solving is not a weak-coupling pairing by a static attractive interaction. We emphasize in this regard that a non-zero T_c at small ε is the consequence of the dependence of the self-energy on the momenta along the FS. Earlier works [8, 9] neglected this momentum dependence and approximated the self-energy by its non-FL form $\Sigma(\omega) = \omega_m(\omega_0/|\omega_m|)^{1/2}$ at a hot spot. These studies found a different result: T_c at an AFM QCP becomes non-zero only if ε exceeds a certain threshold, like in the pairing problem at a QCP with $q = 0$ (Refs.[13, 27]). Specifically, for $\Sigma = \Sigma(\omega_m)$, the anomalous vertex Φ also depends only on frequency, and Eq. (5) reduces to 1D integral equation in frequency rather than in momentum:

$$\Phi(\omega_m) = \frac{\pi\varepsilon T}{2} \sum_{m' \neq m} \frac{\Phi(\omega_{m'})}{\sqrt{|\omega_{m'}|} Z_{\omega_{m'}} \sqrt{|\omega_m - \omega_{m'}|}}. \quad (9)$$

where $Z_{\omega_{m'}} = 1 + \sqrt{|\omega_{m'}|/\omega_0}$. This equation has been solved for arbitrary ε [8], and the result is that T_c becomes non-zero only when ε exceeds a critical value $\varepsilon_c = 0.22$. Near critical coupling $T_c \sim \omega_0 e^{-3.41/(\varepsilon-\varepsilon_c)^{1/2}}$, and for $\varepsilon = 1$, $T_c = 0.17\omega_0$.

T_c at moderate coupling. For the actual physical case $\varepsilon = 1$ we solved Eq. 5 numerically and found that the behavior of $T_c(\alpha)$ is very similar to that at small ε. Namely, T_c scales with ω_0 and the prefactor is essentially independent on α as long as $\alpha \gg \bar{g}/E_F$. We obtained

$$T_c \approx 0.04\omega_0. \quad (10)$$

For $\varepsilon = 1$, typical $(\alpha k_\parallel)^2 \sim \gamma\omega_0$ and typical $\gamma\omega \sim \gamma T_c$ are now comparable, i.e., for $\varepsilon = 1$ the pairing comes from fermions whose self-energy is in a grey area between a FL and a non-FL. We checked this by solving for T_c using the two limiting forms of the self-energy in Eq. (2) – the non-Fl $\Sigma(\omega_m)$ right at a hot spot (this gives $T_c \sim 0.17\omega_0$) and the FL form $\Sigma(\omega_m, k_\parallel) \propto \omega_m/k_\parallel$ (this gives $T_c = 0.005\omega_0$). The actual T_c given by Eq. (10) is in between the two limits. We also verified (see Supplementary material) that in the extreme case of strong nesting, when α exceeds $(\bar{g}/E_F)$, the momentum dependence of the self-energy becomes irrelevant for all $k_\parallel$ along the FS, and T_c crosses over to $T_c \sim 0.17\omega_0$.

The linearized gap equation has been previously solved numerically along the full FS, without restriction to hot spots [21]. In notations of Ref. [21], $T_c = (v_F/a)f(u)$, where dimensionless $u = 4\omega_0 a/(3v_F)$. Eq. (10) implies that $f(u) = 0.03u$ at small u. This agrees well with the numerical solution in [21]. At larger $u \geq 1/2$, $f(u)$ saturates at around $0.015 - 0.02$ (Refs. 21, 28), and at larger u decreases as $1/u$ because of Mott physics.

Conclusions. In this paper we analyzed the equation for superconducting T_c at the onset of SDW order in a 2D metal. We demonstrated that the leading perturbation correction to the bare pairing vertex contains $\log^2 T$, but the series of $\log^2 T$ renormalizations do not give rise to the pairing instability. Yet, T_c is finite, even when coupling λ is artificially set to be small, because of subleading, $\log T$ terms. We showed that for physical $\lambda = O(1)$, the pairing at a QCP comes from fermions with both FL and non-FL forms of the self-energy. The overall scale of T_c is set by the interaction ($\omega_0 \sim \bar{g}$), as long as the interaction is smaller than the Fermi energy, and the prefactor is essentially independent on the details of the geometry of the FS.

The issue which requires a further study is how robust these results are with respect to feedback effects from pairing fluctuations on the fermionic and bosonic propagators. These feedbacks are quite relevant in the RG-based studies [22–25]. In the spin-fermion model, the corrections from the pairing channel come from diagrams with $n \geq 4$ loops and are not small in $1/N$. These corrections preserve the Landau-overdamped form of the bosonic propagator and the $\omega/k_\parallel$ form of the fermionic self-energy, but may contribute additional logarithm $\log k_\parallel^2/(\gamma|\omega|)$ to Σ (see Ref. [20] and Supplementary material). The argument of the logarithm is, however, of order one for typical $k_\parallel$ and ω in the calculations of T_c, hence we expect that the feedbacks from the pairing channel will at most change the prefactor for T_c but do not change our two main conclusions that (i) T_c scales with ω_0, and (ii) in the physical case the pairing involves fermions with both FL and non-FL forms of the self-energy. It is very likely that the same conclusions can be reached within RG-based approaches as the results of the RG analysis are generally comparable to those obtained in the spin-fermion model [25].

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