Acausality of Massive Gravity

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Acausality of Massive Gravity

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We show that the ghost-free, 5 degree of freedom, Wess–Zumino (1970) massive gravity model is acausal. By analyzing its characteristics, we demonstrate that shock wave solutions exhibit superluminal behavior. Ironically, this pathology arises from the very constraint that removes the Boulware-Deser ghost mode.

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INTRODUCTION

Over four decades ago, Isham, Salam and Strathdee proposed a 2-tensor "f-g" theory [1] by adding to the Einstein action that of a second vierbein, $f_\mu{}^m$, plus a non-derivative coupling term, leaving a single common coordinate invariance. Of particular interest is the limit of non-dynamical (say flat) f, giving a finite range to the gravitons due to the coupling "mass" term. It was rapidly shown [2] however, that unlike their linearized massive spin 2 Fierz-Pauli (FP) limits, these models suffered from a ghost problem: generic non-linearities reinstate a 6th degree of freedom (DoF), beyond the linearized $2s+1=5$ DoF, one of which is necessarily ghost-like. A final twist, also from that time, was Wess and Zumino's [3] discovery of a distinguished set of f-g mass terms of which at least one is immune from this disease, keeping 5 DoF. Because [3] was only published without detail in lecture notes, it remained unknown. Separately, other analyses showed that the linearized theory's matter coupling seemed to suffer a "vDVZ" discontinuity [4], as well as a failure of the Birkhoff theorem [5]. Hence, the subject remained moribund until the recent (independent) rediscovery [6] of the results [3] plus two new f-g models. This exhumation has, unsurprisingly, generated an immense industry (see the recent survey [7]). Our purpose is to re-inter f-g. We will show that the first, 5 DoF, Wess-Zumino, model is acausal. Our methods also show that of the two remaining such models [6], one is definitely acausal and the other likely so [8] [23]. Paradoxically, acausalities arise precisely because of the very constraint that removes the ghost. Note that there is no conflict between acausality and ghostlessness, as witness the old "charged" higher ($s>1$) spin interactions with Maxwell and gravity, say those of $s=(3/2,2)$ [13–15], that are also invalidated only by acausality.

Our results will be obtained by using the method of characteristics, analyzing the constraints' shock wave discontinuities, in particular that of the "5th" scalar one that results from combining the trace and double divergence of the field equations, just as is done in the linear FP model, to find a derivative-free constraint.

THE MODEL AND THE FIFTH CONSTRAINT

Our concrete 5 DoF model is

$$\mathcal{G}_{\mu\nu} := G_{\mu\nu} + m^2\big(f_{\mu\nu} - g_{\mu\nu} f\big) = 0\,, \tag{1}$$

where all indices are moved by the dynamical metric $g_{\mu\nu}$ and its associated vierbein $e_\mu{}^m$; in particular $f_{\mu\nu}$ is the fixed background vierbein $f_\mu{}^m$ times $e_{\nu m}$, and is manifestly symmetric on-shell. Vanishing of its antisymmetric part yields 6 conditions. Taking the reference $f_\mu{}^m$ field as the flat bein is a popular choice but is not physically required; in fact, our results, both for acausality and the absence of the sixth ghost mode, depend neither on f being flat nor the dimensionality of spacetime. The parameter m^2 reduces to the FP mass in the weak e-field limit. Next, we proceed as in the FP development and seek 5 constraints to reduce the *a priori* 10 metric DoF (now that coordinate invariance is lost due to the preferred background). The single derivative, 4-vector, constraint is obviously (by the Bianchi identity) the covariant g-divergence of Equation (1),

$$0 = \mathcal{C}_\nu := \nabla^\mu \mathcal{G}_{\mu\nu} = m^2\big(\nabla . f_\nu - \nabla_\nu f\big)\,.$$

The scalar constraint results from taking the (covariantized) FP combination

$$0 = \mathcal{C} := \nabla_\mu\big(\ell^{\mu\nu}\nabla . \mathcal{G}_\nu\big) + \frac{m^2}{2}\,\mathcal{G} \tag{2}$$

with $\ell^{\mu\nu} := \ell^\mu{}_m e^{\nu m}$, where $\ell^\mu{}_m$ is the inverse of the background vierbein $f_\mu{}^m$. The proof that $\mathcal{C}$ is indeed a constraint, *i.e.*, devoid of second derivatives, is simple: following [16], we observe that the (torsion-free) Levi–Civita spin connection $\omega(e)_\mu{}^m{}_n$ corresponding to the vierbeine $e_\mu{}^m$ will in general be torsion-full if employed as the spin connection for the non-dynamical vierbeine $f_\mu{}^m$. The difference between this connection and the Levi–Civita spin connection $\omega(f)_\mu{}^m{}_n$ of $f_\mu{}^m$ yields the contorsion tensor

$$K_\mu{}^m{}_n := \omega(e)_\mu{}^m{}_n - \omega(f)_\mu{}^m{}_n\,.$$

It measures the failure of parallelograms of the dynamical metric to close with respect to the background metric (and *vice versa*). For reasons that will later become apparent, it is important to emphasize that flatness of the background metric does *not* ensure vanishing contorsion. In these terms, the vector constraint reads

$$0 = \mathcal{C}_\mu = m^2 K_\nu{}^{\nu\rho} f_{\mu\rho}\,.$$

In particular, this means that metric derivatives enter the vector constraint only through the trace of the spin connection $\omega(e)$. However the leading (second) derivative terms of the scalar curvature R are proportional to $\partial_\mu \omega(e)_\nu{}^{\nu\mu}$. Hence the linear combination of the divergence of the vector constraint and the trace of the equation of motion quoted in Equation (2) yields the remaining scalar constraint $\mathcal{C} = 0$. This ensures that the model does not propagate spurious ghost degrees of freedom and thus evades the generic difficulties associated with massive gravity theories [2].

For our purposes an explicit evaluation of the scalar constraint $\mathcal{C}$ is needed. For that we first express the scalar curvature in terms of the contorsion

$$\begin{aligned} R \;&=\; 2\,\nabla_\mu K_\nu{}^{\nu\mu} - K_{\mu\nu\rho}K^{\nu\rho\mu} - K_\mu{}^{\mu\rho}K_\nu{}^\nu{}_\rho \\ &+\; e^\nu{}_m e^\mu{}_n R(f)_{\mu\nu}{}^{mn}\,, \end{aligned}$$

where $R(f)$ is the Riemann tensor corresponding to the vierbeine $f_\mu{}^m$. Observing that the last term on the first line of the above display is the square of the vector constraint $-\frac{1}{m^4}\,\mathcal{C}_\mu \ell^{\mu\nu}\ell_{\nu\rho}\mathcal{C}^\rho$, we have the modified constraint

$$\begin{aligned} 0 &= \mathcal{C} - \frac{1}{2m^2}\,(\mathcal{C}.\ell_\nu)^2 \\ &= -\frac{3m^4}{2}\,f - \frac{m^2}{2}\,e^\nu{}_m e^\mu{}_n R(f)_{\mu\nu}{}^{mn} + \frac{m^2}{2}\,K_{\mu\nu\rho}K^{\nu\rho\mu}\,. \end{aligned}$$

The first term is the familiar FP-trace and the second one vanishes for flat $f_\mu{}^m$. We will see in the next Section that the third term has dire consequences for the causality of the model. It does vanish for special solutions whose contorsion obeys $K_{\mu\nu\rho} - K_{\rho\nu\mu} = 0$; however, imposing this condition as an additional constraint would remove further field theoretical degrees of freedom of the model, an obviously unacceptable tradeoff.

ACAUSALITY

To study the causality of the model we study its characteristics by employing the method first introduced in a field theoretical context in [13, 17]. This allows us to determine the maximum speed of propagation by studying a shock whose second derivatives are discontinuous across its wavefront. Since the model is second order in derivatives, we assume that the dynamical metric $g_{\mu\nu}$ and its first derivatives are continuous across the hypersurface spanned by the shock's wavefront by Σ. The inert $f_\mu{}^m$ background is of course continuous. Note that we are studying causality with respect to the dynamical metric g, not the background, this being a putative theory of the metric field. (Actually, our conclusions are equally valid with respect to the background.) Then g, being smooth across Σ, defines local light-cones which allows us to decide whether the shock wavefront corresponds to superluminal propagation.

To start with, we denote the leading discontinuity in the metric across Σ by square brackets

$$\big[\partial_\alpha\partial_\beta g_{\mu\nu}\big]_\Sigma = \xi_\alpha\xi_\beta\gamma_{\mu\nu}\,,$$

where ξ_μ is a vector normal to the characteristic and $\gamma_{\mu\nu}$ is some non-vanishing symmetric tensor defined on the characteristic surface. Propagation is acausal whenever the field equations admit characteristics with timelike normal ξ_μ, *i.e.*,

$$\xi^\mu g_{\mu\nu}\xi^\nu < 0\,;$$

it can be analyzed by studying the field equations and any combinations of field equations and their derivatives that are of degree two or less in derivatives on $g_{\mu\nu}$ and so have a well-defined discontinuity across Σ. This, of course, amounts to studying the discontinuity of $\mathcal{G}_{\mu\nu}$ and the constraints $\mathcal{C}_\mu$ and $\mathcal{C}$ across Σ.

Firstly we consider the anti-symmetric part of the equation of motion $\mathcal{G}_{\mu\nu}$ implying $f_{\mu\nu} = f_{\nu\mu}$. For this we must compute the discontinuity of the vierbeine. Since these depend algebraically on the metric we have

$$\big[\partial_\alpha\partial_\beta e_\mu{}^m\big]_\Sigma = \xi_\alpha\xi_\beta\mathcal{E}_\mu{}^m\,,$$

where $\mathcal{E}_\mu{}^m$ is some tensor defined on the characteristic surface. Computing the discontinuity of the relation $e_\mu{}^m\eta_{mn}e_\nu{}^n = g_{\mu\nu}$ gives $\xi_\alpha\xi_\beta\big(\mathcal{E}_{\mu\nu} + \mathcal{E}_{\nu\mu}\big) = \xi_\alpha\xi_\beta\gamma_{\mu\nu}$. At this point, we proceed by contradiction by taking ξ_μ timelike. Without loss of generality, we may therefore set

$$\xi^\mu g_{\mu\nu}\xi^\nu = -1\,,$$

and thus learn

$$\mathcal{E}_{\mu\nu} + \mathcal{E}_{\nu\mu} = \gamma_{\mu\nu}\,.$$

A similar computation based on the symmetry of $f_{\mu\nu}$ gives

$$f_\mu{}^\rho\mathcal{E}_{\nu\rho} = f_\nu{}^\rho\mathcal{E}_{\mu\rho}\,. \tag{3}$$

Next we compute the leading discontinuity in the field equation $\mathcal{G}_{\mu\nu}$ and in turn its trace $\mathcal{G}$. Since this amounts to studying the second derivative terms in these equations, the result coincides with that of the FP theory computed long ago in [14, 15] (save that indices are raised and lowered with the metric $g_{\mu\nu}$):

$$\xi^2\gamma_{\mu\nu} - \xi_\mu\xi.\gamma_\nu - \xi_\nu\xi.\gamma_\mu + \xi_\mu\xi_\nu\,\gamma \;=\; 0\,, \qquad (4)$$
$$\xi^2\gamma - \xi.\xi.\gamma \;=\; 0\,.$$

It is clearly useful to decompose our variables with respect to the (unit) timelike vector ξ_μ. In particular, for a vector, symmetric tensor and antisymmetric tensor we have respectively

$$V_\mu := V^\perp_\mu - \xi_\mu\xi.V\,,$$
$$S_{\mu\nu} := S^\perp_{\mu\nu} - \xi_\mu S^\perp_\nu - \xi_\nu S^\perp_\mu + \xi_\mu\xi_\nu\,\xi.\xi.S\,,\ \big(S_\mu := \xi.S_\mu\big),$$
$$A_{\mu\nu} := A^\perp_{\mu\nu} + \xi_\mu A^\perp_\nu - \xi_\nu A^\perp_\mu\,, \qquad \big(A^\perp_\mu := A_{\mu\nu}\xi^\nu\big).$$

In this language, Equation (4) implies that $\gamma^\perp_{\mu\nu} = 0$ so

$$\gamma_{\mu\nu} = -\xi_\mu\gamma^\perp_\nu - \xi_\nu\gamma^\perp_\mu + \xi_\mu\xi_\nu\,\xi.\xi.\gamma\,. \qquad (5)$$

The next task is to compute the discontinuity in the vector constraint:

$$\big[\xi^\alpha\partial_\alpha\mathcal{C}_\mu\big]_\Sigma \;=\; m^2\,\xi^\alpha\big[\partial_\alpha\omega(e)_\rho{}^{\rho\sigma}\big]_\Sigma\,f_{\mu\sigma}$$
$$\;=\; -m^2\big(\mathcal{E}_\nu{}^\nu\xi^\sigma - \mathcal{E}^{\sigma\nu}\xi_\nu\big)\,f_{\mu\sigma}\,.$$

Since $f_{\mu\nu}$ is assumed invertible, by decomposing

$$2\mathcal{E}_{\mu\nu} = \gamma_{\mu\nu} + a_{\mu\nu}\,,$$

into its symmetric and antisymmetric parts, we learn

$$0 = \gamma^\perp_\mu + a^\perp_\mu\,. \qquad (6)$$

Together, Equations (5) and (6) give $2\mathcal{E}_{\mu\nu} = a^\perp_{\mu\nu} - 2\xi_\mu\gamma^\perp_\nu + \xi_\mu\xi_\nu\,\xi.\xi.\gamma$ so that Equation (3) becomes

$$0 = f^\perp_\mu{}^\rho a^\perp_{\nu\rho} + \xi_\mu\big(2f^\perp_\nu{}^\rho\gamma^\perp_\rho - f^\perp_\rho a^\perp_\nu{}^\rho - \xi.\xi.\gamma\,f^\perp_\nu\big) - \big(\mu\leftrightarrow\nu\big)\,. \qquad (7)$$

The terms perpendicular and parallel to ξ_μ must vanish separately so

$$f^\perp_\mu{}^\rho a^\perp_{\nu\rho} - f^\perp_\nu{}^\rho a^\perp_{\mu\rho} = 0 = 2f^\perp_\nu{}^\rho\gamma^\perp_\rho - f^\perp_\rho a^\perp_\nu{}^\rho - \xi.\xi.\gamma\,f^\perp_\nu\,. \quad (8)$$

The first set of these equations *generically* gives 3 independent linear conditions on as many unknowns ($a^\perp_{\mu\nu}$) so enforces $a^\perp_{\mu\nu} = 0$. The second set then gives 3 conditions on the 4 remaining non-vanishing unknowns, $\gamma^\perp_\mu$ and $\xi.\xi.\gamma$. Thus, *generically* 3 linear combinations of these vanish, leaving one non-zero linear combination. If this were to vanish, we would have established the absence of shock wavefronts Σ with timelike normal ξ_μ. (Of course, one still would have to verify the absence of special cases for the two italicized appearances of "generically" in the preceding argument, but those are irrelevant in the face of the generic acausality we are about to exhibit.)

The model is left requiring one more condition on $\mathcal{E}_\mu{}^m$ for its causal consistency. That condition can only derive from the remaining scalar constraint $\mathcal{C}$, whose discontinuity across Σ we compute next. To begin with, to better exhibit the problem we are about to find, let us make the assumption that the background is flat and that the contorsion vanishes so that the remaining constraint implies $f = 0$ whose discontinuity across Σ implies $f^{\mu\nu}\mathcal{E}_{\mu\nu} = 0$. This provides the remaining independent linear relation between $\xi.\xi.\gamma$ and $\gamma^\perp_\mu$ required to establish that $\mathcal{E}_\mu{}^m = 0$ and in turn the absence of superluminal shocks–*so long as the contorsion vanishes.*

However, the contorsion does not vanish as a consequence of the field equations (in fact, as discussed above this would imply too many conditions on the field theoretic degrees of freedom). Thus a proper computation of the discontinuity of $\mathcal{C}$ reads

$$\Big[\xi^\alpha\partial_\alpha\big(\mathcal{C} - \frac{1}{2m^2}\,(\mathcal{C}.\ell_\nu)^2\big)\Big]_\Sigma \;=\; \frac{m^2}{2}\,\xi^\alpha\big[\partial_\alpha\big(K_{\mu\nu\rho}K^{\nu\rho\mu}\big)\big]_\Sigma$$
$$= -\frac{m^2}{2}\,\xi_\nu K^{\mu\nu\rho}\mathcal{E}_{\rho\mu}$$
$$= \frac{m^2}{4}\,\xi_\nu K^{\mu\nu\rho}a^\perp_{\mu\rho}\,.$$

Thus, instead of a relation involving $\xi.\xi.\gamma$ and $\gamma^\perp_\mu$, we find the seemingly additional, but in fact redundant, requirement $\xi_\nu K^{\mu\nu\rho}a^\perp_{\mu\rho} = 0$ on $a^\perp_{\mu\nu}$. Therefore, since some linear combination of $\xi.\xi.\gamma$ and $\gamma^\perp_\mu$ does not vanish, timelike shock normals are allowed. This establishes the promised presence of acausal characteristics for any choice of background.

DISCUSSION

We have just shown that one otherwise ghost-free, acceptable finite range gravity model is excluded. How far does this no-go result extend to all three possible such combinations, quite apart from other previously mentioned obstacles to these models? Very recently, causality for models with mass terms quadratic in the f-bein has been ruled out [8] using methods similar to the present ones. This leaves only a third candidate mass, cubic in f. Any model of the form $G_{\mu\nu}(g) = T_{\mu\nu}(f,e)$ with algebraic T universally yields Equation (5) for the shock; the structure of the fifth constraint is at the root of the acausality [24]. Its covariant version for the third mass is as yet unknown, but if it takes the generic form $f^3 + f^2K^2$ where K is the contorsion, the argument of [8] already establishes its acausality. Even if it does not, there is potentially a new source of discontinuity, closer to that of the charged massive spin 3/2 and 2 systems [13–15, 17]. Namely, zeros in the characteristic matrix can allow superluminal characteristics, just as critical values of the background E/M field permit superluminal signal propagation in the charged $s = (3/2, 2)$ models. In fact, for those models, acausality can be traced to non-positivity of equal time commutators, a fatal physical flaw [18]. We conclude therefore that the acausality we have exhibited is an unavoidable pathology of f-g massive gravity barring some miracle of the cubic model or some (hitherto unknown) underlying "rescue" modification of the model [25] that also yields a smooth massless limit [26].

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[23] An earlier massive gravity causality study [9] found superluminal behavior in the auxiliary fields of the model's Stückelberg formulation. However, these superluminal modes amount to unphysical background metrics [10]. Nonetheless, this effect could well be related to known horizon and null energy difficulties of one of either one of the metrics of a bimetric theory [11]. Our classical results are valid for ALL (nonzero) values of m; while it has been argued that at the quantum level, graviton masses small enough to be observationally consistent force a cut-off that removes predictivity of the model [12].
[24] Given the scalar constraint's nefarious role, one might try to turn it into a harmless Bianchi identity by taking a de Sitter background and a partially massless (PM) limit where the scalar helicity does not propagate [19]. However, very recently it has been shown that no PM limit of ghost-free massive f-g theories exists [8].
[25] One example, in a different context, is the use of string theoretical non-minimal couplings for charged higher spins [20].
[26] The massless vDVZ discontinuity can actually be averted by introducing a cosmological constant and setting the mass to zero before limiting to flat space [21]. Also, it was suggested long ago that a similar mechanism applies to the interchange of massless and free limits in a putative non-linear massive theory [22].