# Ultrarelativistic Black Hole Formation

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We study the head-on collision of fluid particles well within the kinetic energy dominated regime ($\gamma = 8$ to 12) by numerically solving the Einstein-hydrodynamic equations. We find that the threshold for black hole formation is lower (by a factor of a few) than simple hoop conjecture estimates, and, moreover, near this threshold two distinct apparent horizons first form post-collision and then merge. We argue that this can be understood in terms of a gravitational focusing effect. The gravitational radiation reaches luminosities of $0.014\ c^5/G$, carrying $16 \pm 2\%$ of the total energy.

*Introduction.*—An important topic in high energy physics that remains poorly understand is the dynamics and outcome of super-Planck scale particle collisions. According to general relativity, kinetic energy, like all forms of energy, gravitates. This implies that at sufficiently high center of mass energies $E$ the gravitational force will eventually dominate any interaction. Suppose one can localize the particles' wavefunctions at the moment of interaction to be within a sphere of radius $R$; then, according to Thorne's hoop conjecture [1] (see also [2–4]), if $R$ is less than the corresponding Schwarzschild radius $R_s = 2GE/c^2$, the gravitational interaction will be so strong that a black hole (BH) will form. For particles satisfying the de Broglie relationship the threshold for BH formation occurs at Planck energies. There has been much interest in the past decade over the possible relevance of this to proton collisions at the Large Hadron Collider [5, 6] and cosmic ray collisions with the earth's atmosphere [7], spurred by theories of quantum gravity with small or warped extra dimensions [8–10] that present the possibility of a true Planck scale within reach of these processes. To date no evidence for BH formation has been found [11, 12], though since the theories do not make firm predictions for what the true Planck scale is, the high energy scattering problem is worthy (beyond intrinsic theoretical interest) of further study.

Here we explore the purely classical gravitational properties of head-on ultrarelativistic collisions (in 4-dimensional asymptotically flat spacetime). This ostensibly gives the leading order description of the process for energies sufficiently above the Planck scale, as all non-gravitational interactions will be hidden behind the event horizon, implying that the particular model for the particles is irrelevant. However, part of the motivation for this study is to test this notion, and begin to investigate how it breaks down approaching the threshold of BH formation (though again only at the classical level).

There have been several studies of ultrarelativistic collisions using BHs as model particles. Penrose [13] first considered the head-on collision of two Aichelburg-Sexl metrics [14], each representing the boost $\gamma \to \infty$ limit of the Schwarzschild metric (letting the mass $M$ go to zero such that the energy $E = \gamma M$ is fixed, and note throughout we use geometric units $G = c = 1$). Though the spacetime to the causal future is unknown, a trapped surface is present at the moment of collision, giving an upper bound of 29% for the radiated energy. Perturbative methods [15, 16] allowed a direct calculation, estimating 16.4% energy emitted. In [17], head-on collisions up to $\gamma \approx 3$ were studied using numerical solution of the field equations; extrapolating the results to $\gamma \to \infty$ gave a value of $14 \pm 3\%$. We briefly mention that studies of BH collisions for general impact parameters using the trapped surface method for infinite boosts [18], and numerical simulations of finite boosts [19, 20] show that considerably more energy can be radiated then.

However, as detailed in [21], the application of the infinite boost results to the collision of massive particles at ultrarelativistic but subluminal speeds is not entirely clear. In this limit the spacetime loses asymptotic flatness while the non-Minkowski part of the spacetime becomes a 2-dimensional shockwave. Moreover, BH collisions at any speed will necessarily produce a larger BH for sufficiently small impact parameter, and are not suitable for studying the threshold or dynamics of BH formation, nor whether BH formation is the generic outcome regardless of the nature or compactness of the colliding particles. Trapped surface calculations as in [18] can be used to infer the dependence of BH formation on impact parameter (which we do not consider here), however they do not provide information on the spacetime dynamics post-collision. In [21] a first attempt to address some of these questions was made, where the ultrarelativistic collision of boson stars (solitons of a minimally coupled complex scalar field) was studied numerically. It was found for boson stars with compactness $2M/r \approx 1/20$ that a BH forms for boosts greater than $\gamma \approx 2.9$, roughly one-third the value $\gamma_{\rm h} = 10$ predicted by applying the hoop conjecture at the time of collision. Whether the threshold is generically such a factor smaller than the hoop conjecture estimate was unclear, first because only a single matter model was considered, but also because though for $\gamma = 2.9$ there is almost twice as much kinetic as rest mass energy in the spacetime, this may not be high enough for the matter dynamics to be irrelevant. Furthermore, due to difficulties disentangling gauge from gravitational wave (GW) dynamics, no estimates of the radiated energy were made.

In this letter we also study black hole formation in head-on particle collisions. However, we use perfect fluid "stars" as the model particles. To begin with, this allows us to further test the generality of the above arguments in a case where gravity would be opposed by the tendency of the fluid to become highly pressurized on collision and disperse. Second, the nature of fluid stars, not having small-scale internal oscillations as boson stars, as well as a new method for constructing initial data [22], permits us to explore significantly higher boost collisions where the ratio of kinetic to rest mass energy is of order 10:1. An independent work with the same matter model used here was recently presented in [23], though as with [21] it focuses on regimes where this ratio is at most ≈ 2:1.

We find that BHs are formed above a critical boost $\gamma_c$ that is a factor of a few less than the hoop conjecture estimate. A new phenomenon we present here is, for boosts slightly above $\gamma_c$, we observe *two* separate apparent horizons (AHs) form shortly *after* the collision, which some time later are encompassed by a single horizon that rings down to a Schwarzschild BH. We argue that this can be qualitatively understood as due to the strong focusing of the fluid elements of one star by the boosted spacetime of the other, and vice versa, using a geodesic model similar to that in [24] for BH formation in the scattering problem. We also study the GWs emitted in this regime for the first time and find that for the $\gamma = 10$ BH forming case $16 \pm 2\%$ of the energy of the spacetime is radiated (the extrapolation described in [17] suggests this should be 94% of the $\gamma = \infty$ limit). For sub-threshold cases, the strong focusing leads to high fluid pressures that cause the stars to explode outward. In what follows, we outline the equations we are solving, the numerical methods for doing so, and the setup of the initial data. We then present the results of our simulations, compare them to geodesic focusing, and end with concluding remarks.

*Methodology.*—We numerically solve the Einstein field equations, in the generalized harmonic formulation, coupled to a perfect fluid using the code described in [25]. For simplicity we use the $\Gamma = 2$ equation of state. We use a variation of the damped harmonic gauge [21, 26] that corresponds to equation (A15) in [26] with $p = 1/4$.

We take advantage of the axisymmetry of a head-on collision to reduce the numerical grid to two dimensions and use seven levels of mesh-refinement where the finest level covers the equatorial and polar radii of the star by approximately 830 and (due to Lorentz contraction) $830/\gamma$ points, respectively. For the $\gamma = 10$ case, to estimate truncation error we also ran simulations with 1.5 and 2 times the resolution. Unless otherwise stated, results from this case are from the high resolution run.

Initial data is constructed using free data from two identical, boosted solutions of the Tolman-Oppenheimer-Volkoff equations with a polytropic condition, and then solving the constraint equations in the conformal thin-sandwich formulation as described in [22]. With this method, the "spurious" gravitational radiation is much smaller then the physical signal (see Fig. 3). We choose isolated star solutions with compaction $2M_*/R_* = 1/40$, where $M_*$ and $R_*$ are the gravitational mass and radius, respectively, of the star in its rest frame. They are boosted towards each other with Lorentz factor $\gamma$, at an initial separation of $d = 534M_*$. We consider cases with $\gamma = 8, 8.5, 9, 9.5, 10$ and 12, though most of our detailed results are from $\gamma = 8$ and 10.

*Results.*—We find that BH formation *does* occur in the ultrarelativistic collision of fluid particles with the aforementioned compaction for boost factors $\gamma \geq 8.5 \pm 0.5$ (the uncertainty is from the sampling resolution of our survey in $\gamma$). This is $\sim 2.4$ times smaller than the hoop conjecture threshold of $\gamma_{\rm h} \approx 20$. In Fig. 1 we show snapshots of the rest mass density for a subcritical case with $\gamma = 8$ and for a supercritical case with $\gamma = 10$. In the former, after the collision, the matter focuses down into two high density regions which then explode outward. In the latter, instead of exploding, two identical AHs appear surrounding these regions. (It should be noted that the existence of the initially disjoint AHs does not preclude the possibility of a single encompassing event horizon.) The AHs then fall towards each other with a third, encompassing AH appearing afterwards.

In Fig. 2 we show the irreducible mass of the AHs, proper distance between the smaller AHs, and the ratio of the proper equatorial and polar circumferences $C_{\rm eq}/C_{\rm p}$ for $\gamma = 10$. The two smaller AHs are born rather prolate with $C_{\rm eq}/C_{\rm p} \sim 0.6$. Together they have mass $> 0.4M$ where $M \approx 2\gamma M_*$ is the total spacetime mass, i.e. they contain a significant amount of what was originally kinetic energy. When the third encompassing AH appears it initially has less irreducible mass (though greater area) than the sum of the smaller AHs. It is also extremely distorted with $C_{\rm eq}/C_{\rm p} \sim 0.2$ and an equatorial circumference that is less than the smaller AHs, suggesting more of a dumbbell shape. This contrasts with what is found in ultrarelativistic black hole collisions where $C_{\rm eq}/C_{\rm p} \sim 1.5$ initially [17], consistent with a disk shaped AH.

In Fig. 3 we show the GW power associated with different spherical harmonics for $\gamma = 10$, and the early part of the GW power for $\gamma = 8$. (Because of the symmetries here only the even $l$, $m = 0$ harmonics are nonzero.) For $\gamma = 10$, $16 \pm 2\%$ of the initial spacetime energy is radiated as GWs, with a peak luminosity of $0.0137 \pm 1\%$ (the error bars include estimates of the truncation error and finite radius extraction effects). The mass of the final BH is $\approx 0.72M$, suggesting the remaining 12% of the energy is carried off by the $\approx 32\%$ of the initial rest mass that remains outside the final BH by the end of the simulation. Measuring the contributions to the total energy from higher $l$ modes relative to the $l = 2$ component we get that $E_4/E_2 = 0.19 \pm 0.01$, $E_6/E_2 = 0.073 \pm 0.001$, and $E_8/E_2 = 0.040 \pm 0.002$. The substantial amount of energy in higher modes is consistent with results from

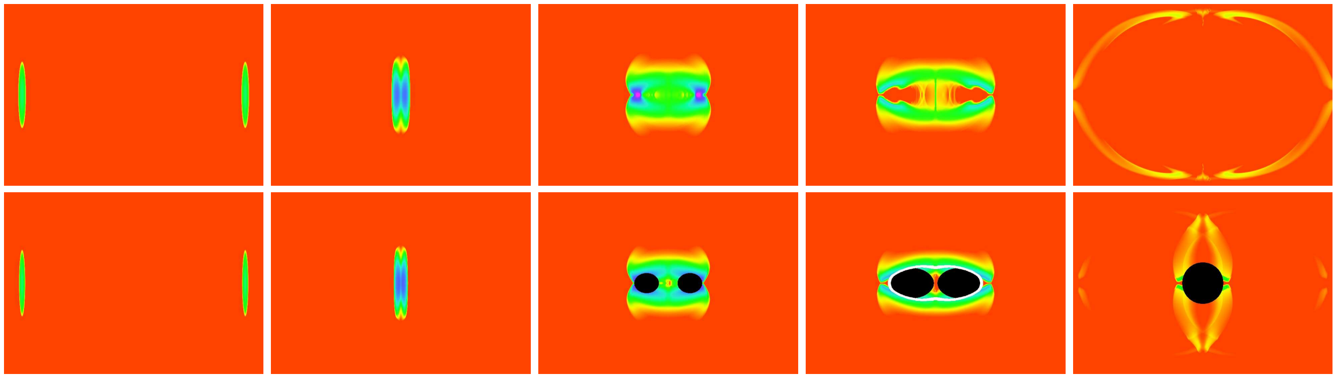

FIG. 1. Snapshots of rest mass density on a logarithmic scale from $10^{-2}$ to $10^{2}$ times the initial maximum density, for simulations with $\gamma = 8$ (top) and $\gamma = 10$ (bottom) at times (left to right) $t = 0$, the initial time; $t = 300M_*$, shortly *after* collision; $t = 375M_*$, after the appearance of the smaller AHs in the $\gamma = 10$ case; $t = 424M_*$, after the appearance of the third, encompassing AH (white outline) in the $\gamma = 10$ case; and $t = 700M_*$. The black regions are best-fit ellipses to the AHs.

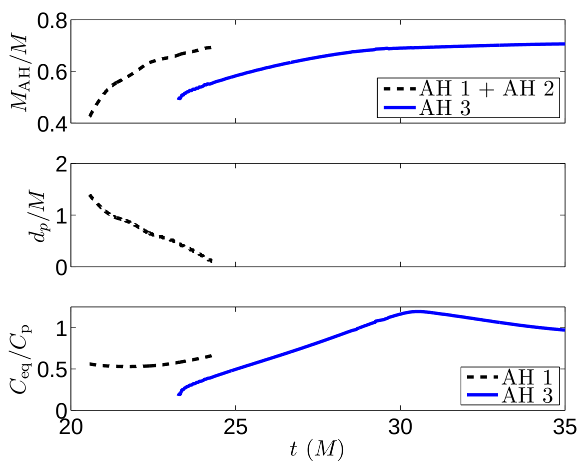


FIG. 2. Apparent horizon properties for the $\gamma = 10$ case. The AHs labeled 1 and 2 are the two identical (when mirrored about the collision plane) ones that appear first and are later encompassed by AH 3. We show (top to bottom): the irreducible masses, the proper distance $d_p$ between (and exterior to) AHs 1 and 2 measured along the collision axis, and the ratio of the equatorial to polar circumferences.

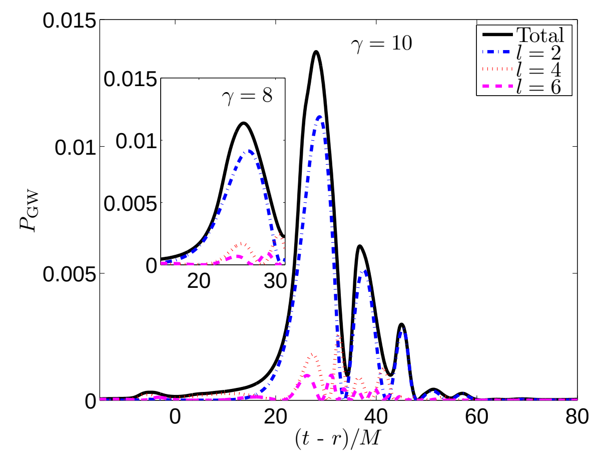


FIG. 3. Total and spin-weight 2 spherical harmonic decomposition of power in GWs from the $\gamma = 10$ case (with $\gamma = 8$ in the inset). The small feature at $(t - r) < 0$ is an artifact of the initial data and is not included in the estimate of energy.

ultrarelativistic BH collisions. Also, the zero-frequency limit combined with an $l$-dependent frequency cutoff $\omega_c = l/(3\sqrt{3}M)$ set by BH quasinormal frequencies [27] predicts corresponding values of 0.22, 0.09, and 0.05. For $\gamma = 8$ we can only extract the GW signal before the fluid outflow crosses the extraction sphere. Before this time, the GW signal looks qualitatively similar to the $\gamma = 10$ case and contains 10% of the energy of the spacetime.

Cases with $\gamma = 9.5$, 9.0, and 8.5 also first form two disjoint AHs with increasing initial separation the smaller the boost. However, we were unable to follow these cases through merger before numerical instabilities set in on the excision surface. The reason, we believe, is the smaller boosts form more distorted AH shapes, and our current approach of excising based on the best-fit ellipse to the AH shape is inadequate. We have also been unable to obtain robust results for significantly higher Lorentz factors due to high frequency numerical instabilities that develop at the surface of the boosted stars; however it seems that the third AH appears at nearly the same time in this gauge as the first two AHs at $\gamma \sim 12$ for the stars considered here, and for larger boosts we expect a single AH to form at collision.

*Geodesic focusing.*—To illustrate the manner in which a boosted star may act like a gravitational lens and, during collision, focus the matter of the other star, we consider a simplified scenario in a spacetime consisting of a single boosted star. We follow a set of geodesics coming

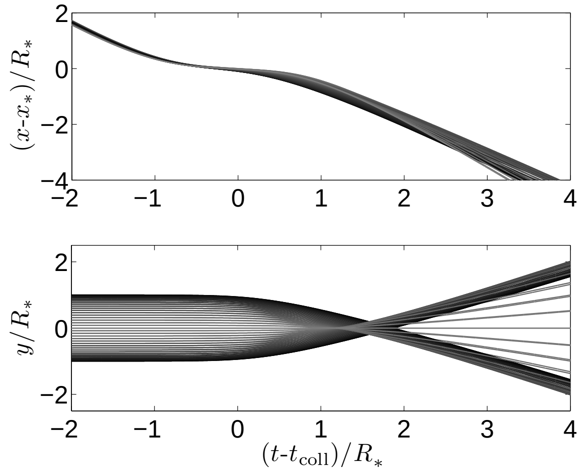


FIG. 4. Focusing of a set of geodesics in a boosted star spacetime with $\gamma = 10$ and $2M_*/R_* = 1/40$. Shown are the $x$-coordinate relative to the center of the boosted star which is at $x_* = vt$ (top) and the $y$-coordinate (perpendicular to the boost axis) of the geodesics as a function of coordinate time.

from the opposite direction with the same Lorentz factor, initially distributed to fill out the volume of what would have been the other boosted star (i.e., we replace the second star with tracer particles). These geodesics are shown in Fig. 4 for $\gamma = 10$ with the same compaction star described above. As these geodesics pass through the boosted star they become focused in the direction orthogonal to the boost axis while spreading out along the boost axis. The greatest focusing (i.e., when the separation between the geodesics in either direction is smallest) occurs at a distance of $\approx 1.5R_*$ away from the star and reduces the radius by a factor of $\approx 4$ (roughly consistent, with the caveat of coordinate differences, with the full problem—see Figs. 1 and 2). This contraction is sufficient to get to the BH formation threshold if we assume that this focusing also converts sufficient translational energy to radial inflow that it is valid to apply the hoop conjecture to the star's total energy in this frame. Evidence for this assumption comes from the temporary slowdown of the translational velocity seen in Fig. 4 (though somewhat before maximum focusing), and from the full simulations where in the $\gamma = 10$ case the two AHs move toward each other, and in the $\gamma = 8$ case post-compression the fluid flow is largely radial. In the ultrarelativistic limit, this geodesic focusing factor is mainly a function of the ratio $\gamma M_*/R_*$, and similar results are obtained for larger boosts with correspondingly less compact stars. This simplistic treatment of course ignores the effects of pressure and non-linear gravitational interactions.

*Conclusions.*— In this study we considered the head-on collision of self-gravitating fluid stars in the regime where the ratio of kinetic to rest mass energy in the spacetime is $\sim$10:1. We find above a critical boost $\gamma_c = 8.5 \pm 0.5$ that BHs do form. The dynamics of the solution, and a simple geodesic model similar to [24], suggest that near threshold the strong focusing nature of the spacetime sourced by one boosted star on the other, and vice versa, causes the energy to be concentrated post-collision around two focal points on axis. In the subcritical case, the material explodes outward from these points, consistent with [21, 23]; however just supercritical we find two distinct AHs that initially form around the focal points. This focusing also offers an intuitive explanation for why the threshold in cases studied to date is systematically less than hoop conjecture estimates (here $\gamma_c/\gamma_h \sim 0.4$, with the boson star collisions $\gamma_c/\gamma_h \sim 0.3$ for $\gamma_c \sim 2.9$ [21], and similar factors were found in [28–30] for the scattering problem using a perturbative model).

For the $\gamma = 10$ supercritical case, we find $16 \pm 2\%$ of the total energy is radiated gravitationally, consistent with results extrapolated from $\gamma \approx 3$ BH collisions [17], and perturbative calculations of the infinite boost limit [15, 16]. Moreover, the leading order spherical-harmonic multipole structure of the waves is consistent with point-particle approximations and the BH case [27], both super and subcritical, in the latter prior to obscuration of the waveform by matter outflow.

This suggests three different regimes in the head-on collision of ultra-relativistic, non-singular model particles in general relativity, for sources that have sufficiently *low* compactness such that $\gamma_c \gg 1$. For $\gamma \ll \gamma_c$ gravity plays little role, and the dynamics is governed by that of the matter; for $\gamma \gg \gamma_c$ we expect universal behavior, i.e. any particle model will give the same *quantitative* spacetime dynamics; however in the intermediate regime $\gamma \sim \gamma_c$ both gravitational and matter dynamics will be important. Ignoring quantum effects and studying the nature of super-Planck scale particles collisions using general relativity is arguably robust only when $\gamma \gg \gamma_c$, though perhaps some insights can still be drawn from classical general relativity in the intermediate regime.

The intermediate regime includes the threshold of BH formation and corresponding, matter-*dependent* critical phenomena [31]. We conjecture approaching $\gamma_c$ may generically result in two critical solutions unfolding post-collision about the geodesic focal points of the two colliding particles (we speculate the reason why two distinct AHs were not seen in [21, 23] is the compactness is not sufficiently low to have $\gamma_c \gg 1$.) For $\Gamma = 2$ fluid stars it would be interesting to see whether the critical solution is the type I unstable star-like solution found for lower $\gamma_c$'s [23], or the type II self-similar solution arising in the kinetic energy dominated regime [32]. It would also be interesting to explore collisions with nonzero impact parameters. This would allow a better comparison to BH collisions, which do not have a threshold for BH formation, but do have two distinct end states as a function of impact parameter: a large BH or two unbound BHs.

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