Stationary Axisymmetric and Slowly Rotating Spacetimes in Hořava-Lifshitz Gravity

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Stationary, axisymmetric and slowly rotating vacuum spacetimes in the Hořava-Lifshitz (HL) gravity are studied, and shown that, for any given spherical static vacuum solution of the HL theory (of any model, including the ones with an additional U(1) symmetry), there always exists a corresponding slowly rotating, stationary and axisymmetric vacuum solution, which reduces to the former, when the rotation is switched off. The rotation is universal and only implicitly depends on the models of the HL theory and their coupling constants through the spherical seed solution. As a result, all asymptotically flat slowly rotating vacuum solutions are asymptotically identical to the slowly rotating Kerr solution. This is in contrast to the claim of Barausse and Sotiriou, Phys. Rev. Lett. **109**, 181101 (2012), in which slowly rotating black holes were reported (incorrectly) not to exist in the infrared limit of the non-projectable HL theory.

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Introduction:—Since Einstein proposed his general relativity (GR) in 1915, various experiments and observations have been carried out, and so far all of them are consistent with it [1]. Despite of these splendid achievements, it has been realized for a long time that GR is not (perturbatively) renormalizable [2], and thus may not be used to describe quantum effects of gravity in very short distances. On the other hand, because of the universal coupling of gravity to all forms of energy, it is expected that it should have a quantum mechanical description. Motivated by this anticipation, quantization of gravitational fields has been one of the main driving forces in physics in the past decades [3].

Recently, Hořava [4] proposed a theory of quantum gravity within the framework of quantum field theory, in which the fundamental variables are the metric. One of the essential ingredients of the theory is the inclusion of higher-dimensional spatial derivative operators, so that the ultraviolet (UV) behavior is dominated by them and that the theory is power-counting renormalizable. The exclusion of higher-dimensional time derivative operators, on the other hand, guarantees that the theory is unitary, a problem that has been faced in quantization of gravity for a long time [5]. However, this inevitably breaks Lorentz symmetry in the UV. Although such a breaking in the gravitational sector is much less restricted by experiments/observations than that in the matter sector [6] (See also [7]), it is still a challenging question how to prevent the propagation of the Lorentz violations into the Standard Model of particle physics [8]. In the infrared (IR) the lower dimensional operators take over, presumably providing a healthy low energy limit.

The Lorentz breaking in the UV are realized by invoking the anisotropic scaling between time and space, $t \to b^{-z}t,\ \vec{x} \to b^{-1}\vec{x}$. This is a reminiscent of Lifshitz scalars [9] in condensed matter physics, hence the theory is often referred to as the HL gravity. To be power-counting renormalizable, the critical exponent z has to be $z \geq 3$ [4, 10]. Hořava assumed that the symmetry is broken only down to the so-called foliation-preserving diffeomorphism,

$$t' = f(t), \quad {x'}^i = \zeta^i(t, x), \tag{1}$$

denoted often by Diff($M,\ \mathcal{F}$). With such a breaking, spin-0 gravitons in general appear, in addition to the spin-2 ones found in GR. This is potentially dangerous, and leads to several problems, such as instability, strong coupling and different speeds of massless particles [11]. To resolve these problems, various models have been proposed, including the healthy extension of the non-projectable HL theory [12], and a more dramatical modification, in which an extra local U(1) symmetry is introduced, so that the symmetry of the theory is enlarged to [13],

$$U(1) \ltimes \text{Diff}(M,\ \mathcal{F}). \tag{2}$$

Because of this extra symmetry, the spin-0 gravitons are eliminated [13, 14]. As a result, all the problems related to them, including the ones mentioned above, are resolved. This was initially done with the projectability condition [13, 14], in which the lapse function in the Arnowitt-Deser-Misner (ADM) decompositions [15] is a function of t only. Soon, it was extended to the case without it [16].

In this Letter, we shall investigate another important issue of the HL theory: Stationary axisymmetric and slowly rotating gravitational fields of black holes and stars. The existence of such fields are fundamental to the theory, since rotating objects are more common than non-rotating ones in our universe. In particular, observations show that rotating black holes very likely exist [17]. Certainly, the issue of black holes in the HL theory is very subtle, because of the Lorentz violations and modified dispersion relationship,

$$E^2 = c_p^2 p^2 \left(1 + \alpha_1 \left(\frac{p}{M_*}\right)^2 + \alpha_2 \left(\frac{p}{M_*}\right)^4\right), \quad (3)$$

where E and p are the energy and momentum of the particle considered, and $c_p,\ \alpha_i$ are coefficients, depending on the particular specie of the particle, while M_* denotes the suppression energy scale of the higher-dimensional operators. Then, one can see that both phase and group velocities of the particles are unbounded with the increase of energy. This suggests that black holes may not exist at all in the HL theory. However, in the IR the high-order terms of p are negligible, and the first term in Eq.(3) becomes dominant, so one may still define black holes, following what was done in GR. For more detail, we refer readers to [18] and references therein.

With the above in mind, in this Letter we shall show that for any given spherical static vacuum solution of the HL theory, there always exists a corresponding stationary axisymmetric and slowly rotating vacuum solution, which reduces to the former, when the rotation is switched off. This is true in all the models of the HL theory proposed so far, including the ones with the enlarged symmetry (2). In addition, the rotation is universal and only implicitly depends on the models of the HL theory and their coupling constants through the spherical seed solution [cf. Eq.(14)]. As a result, all asymptotically flat slowly rotating vacuum solutions are asymptotically identical to the slowly rotating Kerr solution found in GR, given by Eq.(17).

HL Theory without U(1) Symmetry:—A naturally starting point to formulate the HL theory is the ADM decompositions [15], $\left(N, N^i, g_{jk}\right)$, where $N,\ N^i$ and g_{jk} are, respectively, the lapse function, shift vector, and the 3-dimensional metric defined on the leaves $t =$ Constant. Then, the general action takes the form [12],

$$S_g = \frac{1}{16\pi G}\int dt d^3x N\sqrt{g}\left(\mathcal{L}_K - \mathcal{L}_V\right), \quad (4)$$

where $\mathcal{L}_K \equiv K_{ij}K^{ij} - \lambda K^2$, and $\mathcal{L}_V = \mathcal{L}_V\left(a_i, R_{jk},\ \nabla_l\right)$ denotes the potential made of all the operators constructed from a_i, R_{ij} and ∇_i. Its explicit form is irrelevant to the current problem, so we shall not present it here. R_{ij} denotes the Ricci tensor of g_{ij}, ∇_i the covariant derivative with respect to g_{ij}, and $a_i \equiv N_{,i}/N$. K_{ij} is the extrinsic curveture of the leaves $t =$ Constant, given by $K_{ij} = \left(-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i\right)/(2N)$. In addition, in the case with the projectability condition, we have $N = N(t)$, and then $a_i = 0$. In this Letter, we shall treat the case with the projectability condition as a particular case of the one without it, $N = N(t,x)$, whenever it is possible.

Then, variation of S_g with respect to N yields the Hamiltonian constraint,

$$(i)\ \int d^3x\sqrt{g}\mathcal{H}^{\perp} = 0, \qquad (ii)\ \mathcal{H}^{\perp} = 0, \quad (5)$$

respectively, for the projectable and non-projectble cases, where $\mathcal{H}^{\perp} \equiv \mathcal{L}_K + \mathcal{H},\ \mathcal{H} \equiv \delta\left(N\mathcal{L}_V\right)/\delta N = \mathcal{H}\left(a_i, R_{jk},\ \nabla_l\right)$. Variation of S_g with respect to N^i yields the momentum constraint,

$$\nabla^j \pi_{ij} = 0, \quad (6)$$

where $\pi_{ij} \equiv -K_{ij} + \lambda g_{ij}K$. Finally, the variation of S_g with respect to g_{ij} yields the dynamical equations,

$$\frac{1}{N\sqrt{g}}\frac{\partial}{\partial t}\left(\sqrt{g}\pi^{ij}\right) = \frac{1}{N}\left[\nabla_k\left(\pi^{ij}N^k\right) - 2\pi^{k(i}\nabla_k N^{j)}\right] - 2\left(K^{ik}K^j_k - \lambda K K^{ij}\right) + F^{ij} + \frac{1}{2}g^{ij}\mathcal{L}_K, \quad (7)$$

where $F^{ij} \equiv -\left[\delta\left(N\sqrt{g}\mathcal{L}_V\right)/\delta g_{ij}\right]/(N\sqrt{g}) = F^{ij}\left(a_k, R_{lm}, \nabla_n\right)$.

Slowly Rotating Spacetimes:—Let us consider the gravitational field of a body with slow and uniform rotation about an axis described by,

$$\begin{aligned} N &= N(r), \quad N^i = h(r)\delta^i_r + \omega(r,\theta)\delta^i_\phi, \\ g_{ij} &= \text{diag.}\left(f^{-1}(r), r^2, r^2\sin\theta^2\right), \end{aligned} \quad (8)$$

where slow rotation means $|\omega| \ll 1$. Thus, one can consider the gravitational field of a slowly rotating body as linear perturbations over the spherical background, $(N(r), f(r), h(r))$. Then, to the first-order of ω, we obtain

$$a_i = \bar{a}_i,\ R_{ij} = \bar{R}_{ij},\ \nabla_i = \bar{\nabla}_i,\ K_{ij} = \bar{K}_{ij} + \delta K_{ij}, \quad (9)$$

where quantities with bars denote the ones calculated from the spherical seed solutions (N, f, h), and

$$\begin{aligned} \bar{K}_{ij} &= \frac{1}{2Nf^2}\left(2fh' - hf'\right)\delta^r_i\delta^r_j + \frac{rh}{N}\Omega_{ij}, \\ \delta K_{ij} &= \frac{r^2\sin^2\theta}{2N}\left(\omega_{,i}\delta^\phi_j + \omega_{,j}\delta^\phi_i\right), \end{aligned} \quad (10)$$

with $\Omega_{ij} = \delta^\theta_i\delta^\theta_j + \sin^2\theta\delta^\phi_i\delta^\phi_j$, and $f' \equiv \partial f/\partial r$, etc. It is important to note that $a_i,\ R_{ij}$ and ∇_i all do not contain terms of first-order of ω. This is simply because that N and g_{ij} do not contain such terms. As a result, the potential $\mathcal{L}_V\left(a_i, R_{jk},\ \nabla_l\right)$ and terms resulted from it, such as $\mathcal{H}\left(a_i, R_{jk},\ \nabla_l\right)$ and $F^{ij}\left(a_i, R_{jk},\ \nabla_l\right)$, also do not contain such terms, that is $\mathcal{H} = \bar{\mathcal{H}} + \mathcal{O}(\omega^2),\ F^{ij} = \bar{F}^{ij} + \mathcal{O}(\omega^2)$. Moreover, since

$$\delta K = \bar{g}^{ij}\delta K_{ij} = 0, \quad \bar{K}^{ij}\delta K_{ij} = 0, \quad (11)$$

we find that $\mathcal{L}_K$ does not contain terms of first-order of ω either. Then, $\mathcal{L}_V = \bar{\mathcal{L}}_V\left(\bar{a}_i, \bar{R}_{jk},\ \bar{\nabla}_l\right) + \mathcal{O}(\omega^2)$, and $\mathcal{L}_K = \bar{\mathcal{L}}_K + \delta\mathcal{L}_K$, where

$$\begin{aligned} \bar{\mathcal{L}}_K &= \frac{1}{8r^2N^2f^2}\Big[(1-\lambda)r^2\left(hf' - 2fh'\right)^2 \\ &\quad + 8(1-2\lambda)f^2h^2 + 8\lambda rfh\left(hf' - 2fh'\right)\Big], \\ \delta\mathcal{L}_K &= \frac{\sin^2\theta}{2N^2}\left(r^2 f\omega^2_{,r} + \omega^2_{,\theta}\right). \end{aligned} \quad (12)$$

To zeroth order of ω, Eqs.(5), (6) and (7) yield the HL field equations for $(N(r), f(r), h(r))$.

When $h = 0$, from Eq.(10) - (12) we can see that $\bar{K}_{ij} = \bar{\mathcal{L}}_K = 0$. Thus, to firs-order of ω, Eqs.(5) and (7) are satisfied identically, simply because there are no non-vanishing terms of the first order of ω in these equations. On the other hand, we have $\pi_{ij} = -\delta K_{ij}$ where δK_{ij} is given by Eq.(10). Inserting it into Eq.(6), we find that it has only one independent equation, given by,

$$\frac{\sqrt{N^2 f}}{r^2}\left(r^4\sqrt{\frac{f}{N^2}}\,\omega'\right)' + \frac{1}{\sin^3\theta}\left(\sin^3\theta\omega_{,\theta}\right)_{,\theta} = 0. \quad (13)$$

It is easy to show that non-singular solutions for any θ exist only when $\omega = \omega(r)$. Then, we find that

$$\omega(r) = -3J\int\sqrt{\frac{N^2}{f}}\;\frac{dr}{r^4} + \omega_0, \quad (14)$$

where J and ω_0 are constants. Without loss of the generality, we can set $\omega_0 = 0$ by the coordinate transformation $\phi \to \phi + \omega_0 t$.

When $h \neq 0$, we find that to first-order of ω, the Hamiltonian constraint Eq.(5) is satisfied identically with the same arguments as those given above for the case $h = 0$. On the other hand, to first-order of ω, the momentum constraint (6) yields the same equation (13). For the dynamical equations (7), to the first-order of ω, only the first two terms on the right-hand side of Eq.(7) now are not zero. Taking Eq.(6) into account, we find that these two terms have two non-vanishing components, (r,θ) and (r,ϕ), and can be cast in the forms,

$$\left(r^4\sqrt{\frac{f}{N^2}}\,\omega'\right)' = 0, \quad (15)$$

$$\left(\frac{r^2 h\omega}{N\sqrt{f}}\right)'_{,\theta} = 0. \quad (16)$$

The combination of Eqs.(13) and (15) immediately yields, $\omega(r,\theta) = \omega_2(r)\int\sin^{-3}\theta d\theta + \omega_1(r)$, where $\omega_2(r)$ and $\omega_1(r)$ are two integration functions of r only. Clearly, ω becomes singular at $\theta = 0, \pi$, unless $\omega_2(r) = 0$. Then, substituting it into Eq.(15), we find that ω is also given by Eq.(14). Asymptotical flatness condition requires $f \simeq N^2 \sim 1$ and $h \sim 0$. Then, we find that

$$\omega \simeq \frac{J}{r^3}, \quad (r \gg 1). \quad (17)$$

Therefore, we conclude that, *for any given static spherical vacuum seed solution, (N, f, h), of the HL theory with or without the projectability condition, there always exists a slowly rotating vacuum solution (N, f, h, ω), where ω is given by Eq.(14). When the rotation is switched off, it reduces to the spherical seed solution.*

Note that our above conclusions hold for the case with any value of λ, including the one with $\lambda = 1$. In particular, the slowly rotating Kerr solution belongs to it. In fact, setting $\mathcal{L}_V = -R$ in the general action (4), where R denotes the Ricci scalar made of the 3D metric g_{ij}, we obtain two solutions, given, respectively, by $\left(N^2, f, h\right) = \left(1, 1, \sqrt{r_g/r}\right)$ and $\left(N^2, f, h\right) = \left(1 - r_g/r, 1 - r_g/r, 0\right)$, where r_g is the Schwarzschild radius. In both cases, we have $f = N^2$. Then, from Eq.(14), we find that ω takes the form of Eq.(17) for any r. Both of these two seed solutions are the Schwarzschild solution found in GR, but written in different coordinate systems and are related by [19] $t' = t + \int\sqrt{\frac{1-N^2}{fN^2}}\,dr$. However, this kind of coordinate transformations are forbidden by the foliation-preserving diffeomorphisms (1), so in the HL theory these two seed solutions and their corresponding slowly rotating ones actually represent different spacetimes [18].

Slowly Rotating Spacetimes with U(1) Symmetry:— The above conclusion can be easily generalized to the cases with the U(1) symmetry [13, 14, 16]. In fact, with the presence of the gauge field A and the Newtonian prepotential φ, the action can be still cast in the form (4), but now with the potential $\mathcal{L}_V$ being replaced by [14, 16],

$$\mathcal{L}_V - \mathcal{L}_A - \mathcal{L}_{\lambda,\varphi},$$

where $\mathcal{L}_A \equiv A(R - 2\Lambda_g)/N$, and Λ_g is a coupling constant. $\mathcal{L}_{\lambda,\varphi} = \mathcal{L}_{\lambda,\varphi}\left(a_i, g_{ij}, R_{ij}, K_{ij}, \varphi, \nabla_i\right)$, where its exact dependence on these variables is not important to our current discussions, as in the gauge

$$\varphi = 0, \quad (18)$$

to be chosen below, it vanishes identically [14, 16].

Then, the Hamiltonian constraint still takes the form of Eq.(5) but now with $\mathcal{H} \equiv \delta\left[N(\mathcal{L}_V - \mathcal{L}_A - \mathcal{L}_{\lambda,\varphi})\right]/\delta N = \delta\left[N(\mathcal{L}_V - \mathcal{L}_A)\right]/\delta N = \mathcal{H}\big(A, a_i, R_{jk}, \nabla_l\big)$ within the above gauge. Thus, to the first-order of ω, we have $\mathcal{H} = \bar{\mathcal{H}}\big(\bar{A}, \bar{a}_i, \bar{R}_{jk}, \bar{\nabla}_l\big)$, where $\bar{A} = A(r)$ is the spherical seed solution of the gauge field. Then, taking Eqs.(11) and (12) into account, we find that the Hamiltonian constraint is satisfied identically to first-order ω, if it is satisfied to its zeroth order, even with the U(1) symmetry.

Because of the presence of A and φ, the theory has two more constraints in comparison to the one without the U(1) symmetry, obtained from the variation of the action with respect to A and φ, given, respectively, by [14, 16]

$$R - 2\Lambda_g = 0, \quad (19)$$

$$\mathcal{G}^{ij}K_{ij} - \frac{1}{N}\left[\nabla_i\left(Na_jK^{ij}\right) - \nabla_i\left(Na^iK\right)\right] + \frac{1-\lambda}{N}\left[\nabla^2\left(NK\right) - \nabla_i\left(Na^iK\right)\right] = 0, \quad (20)$$

where $\mathcal{G}^{ij} \equiv R^{ij} + (\Lambda_g - R/2)g^{ij}$. Note that the above constraints hold for both the projectable and non-projectable cases. Then, considering Eqs.(9)-(12) and the fact that $\bar{\nabla}_i\left(\bar{N}\bar{a}_j\delta K^{ij}\right) = 0$, $\bar{g}_{ij}$ and $\bar{R}_{ij}$ are diagonal, one can see that the above equations are satisfied identically to the first-order of ω, if they are satisfied to its zeroth-order.

With the gauge (18), the momentum constraint takes the same form as that of Eq.(6). As a result, to the first-order of ω, it yields Eq.(13).

The dynamical equations, on the other hand, can be also cast in the form of Eq.(7), if one replaces $\mathcal{L}_K$ by $\mathcal{L}_K + \mathcal{L}_A$, and now defines F^{ij} as $F^{ij} \equiv -\left[\delta\left(N\sqrt{g}\mathcal{L}_g\right)/\delta g_{ij}\right]/(N\sqrt{g})$, where $\mathcal{L}_g \equiv \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_{\lambda,\varphi}$. Considering Eq.(18), we find $F^{ij} = F^{ij}\left(a_k, R_{lm}, \nabla_n\right)$. Since $\mathcal{L}_A\left(A, R\right) = \bar{\mathcal{L}}_A\left(\bar{A}, \bar{R}\right) + \mathcal{O}(\omega^2)$, and the rotation ω contributes to the dynamical equations only through the shift vector N^i and extrinsic curvature tensor K_{ij}, one can easily show that to the first-order of ω only the first two terms in the right-hand of Eq.(7) have non-vanishing contributions, exactly the same as those without the U(1) symmetry, and give rise to two equations, given, respectively, by Eqs.(15) and (16). Thus, in the present case we still have the same three field equations for $\omega(r,\theta)$ even with the enlarged symmetry (2). Therefore, *if* (N, f, h, A) *is a vacuum solution of the HL theory with the local U(1) symmetry, either with the projectability condition or without it, then,* (N, f, h, A, ω) *represents a corresponding slowly rotating vacuum solution of the same theory, where* ω *is given by Eq.(14).* Since physics does not depend on the gauge choice, the above conclusion is also true in any gauge.

Conclusions:—In this Letter, we have shown that, for any given spherical static vacuum solution of the HL theory, no matter it has the local U(1) symmetry or not, and whether it is with or without the projectability condition, there always exists a corresponding solution, representing a slowly rotating vacuum space-time, which will reduce to the former when the rotation is switched off.

It is remarkable to note that the rotation given by Eq.(14) depends on the models of the HL theory and their coupling constants only implicitly through the spherical seed solutions. All the other effects are high orders of ω. In this sense, the rotation is universal, and for all asymptotically flat seed solutions, it takes precisely the form (17) of the slowly rotating Kerr solution of GR.

When specifying to the particular cases considered in [20], we obtains the same results. On the other hand, our above conclusions are equally applicable to the IR limit of the non-projectable HL theory [12], since, as mentioned above, Eqs.(13), (15) and (16) do not contain explicitly the coupling constants of the HL theory, and setting the ones that corresponds to the high-order derivative terms to zero will not affect the forms of these equations, although they do affect the spherical seed solutions (N, f, h). Recently spherical static vacuum spacetimes were studied in the IR limit of the non-projectable HL theory by using the equivalence between it and the hypersurface-orthogonal Einstein-aether theory [12, 21], and a class of numerical solutions that represents black holes was found [22]. On the other hand, Blas and Sibiryakov also studied the same problem [23], and found that these black holes possess universal horizons. In all of these studies, the Eddington-Finkelstein metric, $ds^2 = F(r)dv^2 - 2B(r)dvdr - r^2d\Omega^2$, was used [22, 23], in which the four-velocity of the aether can be always parameterized as, $u^\alpha\partial_\alpha = \mathcal{A}(r)\partial_v - [1 - F(r)\mathcal{A}^2(r)]/[2\mathcal{A}(r)B(r)]\partial_r$, where $\mathcal{A}$ is an arbitrary function, to be determined by the field equations. In the spherical case, since the aether is always hypersurface-orthogonal [21], one can introduce the time-like variable t, so that u_μ takes the form $u_\mu = t_{,\mu}/\left|g^{\alpha\beta}t_{,\alpha}t_{,\beta}\right|^{1/2}$, from which we find that $dv = dt/t_{,v} + 2\mathcal{A}^2B/(1+\mathcal{A}^2F)dr$. The integrability condition requires $t_{,vr} = 0$. Without loss of generality, we choose $t_{,v} = 1$, so that $v = t + \int^r\left(2A^2B/(1+A^2F)\right)dr$. Inserting it into the Eddington-Finkelstein metric, we find that the corresponding ADM quantities are given by Eq.(8) with $\omega = 0$, where

$$N^2 = B^2 f = \frac{(1+\mathcal{A}^2F)^2}{4\mathcal{A}^2}, \quad h = \frac{1-\mathcal{A}^4F^2}{4\mathcal{A}^2B}.$$

Taking these black hole solutions as the seeds, from the results presented above one can see that slowly rotation black holes indeed exist in the IR limit of the non-projectable HL theory [12], in contrast to the claim presented in [24]. For detail, we refer readers to [25].

It should be noted that the equivalence between the hypersurface-orthogonal Einstein-aether theory and the IR limit of the non-projectable HL theory holds only in the level of action. In particular, the Einstein-aether theory still has the general diffeomorphisms as that of GR, while the HL theory has only Diff(M, $\mathcal{F}$). It is exactly because of the former that we are allowed to make coordinate transformations of the kind $t = t(v, r)$, which are forbidden by Eq.(1).

We also note that rotations (spins) of black holes are important not only because the observational fact that most of objects in our universe are rotating, as mentioned above, but also because they might provide important information on the evolution histories of black holes and the formation of their jets [17], among other things. Indeed, recently it was found evidence that the relativistic jet of a black hole might be powered by its spin energy [26]. Because of the universal form of the spin (17), it might be difficult to distinguish the HL theory from others only by measuring the spins of black holes (as far as slowly rotating black holes are concerned). However, a rotating source always drags space-time with it, and causes linearly polarized electromagnetic radiation to undergo polarization rotation - the gravitational Faraday effects [27]. Lately, it was shown that this leads to a new relativistic effect that imprints orbital angular momentum on such light [28]. Since the resulted spectra depend not only on the spin of the black hole but also on the detailed structure (null geodesics) of the space-time, they might provide important information to distinguish different models of gravity, including the HL theory.

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