Origin and Consequences of ^{12}C+^{12}C Fusion Resonances at Deep Sub-barrier Energies

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Phys. Rev. Lett. **110**, 072701 — Published 11 February 2013

DOI: 10.1103/PhysRevLett.110.072701

Origin and consequences of ^{12}C+^{12}C fusion resonances at deep sub-barrier energies

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(Dated: October 23, 2012)

Previous explanations for the resonance behavior of ^{12}C+^{12}C fusion at low energies were based on a non-resonant compound-nucleus background and an additional contribution from a series of resonances. This separation into 'resonance' and 'background' contributions of the cross section is artificial. We propose to explain this phenomenon through the impact on the cross section of the relatively large spacings and the narrow widths of ^{24}Mg compound levels in the corresponding excitation-energy region.

PACS numbers: 25.70.Jj, 26.30.-k, 24.10.Eq, 24.30.Gd

According to our current understanding of the late stages of the evolution of giant stars, large concentrations of ^{12}C lead to a rapid carbon burning phase in which two ^{12}C nuclei fuse to produce ^{24}Mg [1, 2]. At present, the reaction rate for this process can be estimated only by extrapolating the measured cross sections down into the region of the Gamow window corresponding to center-of-mass collision energies of ∼2 MeV and temperatures of ∼ 0.85 GK. However, this extrapolation is complicated by the fact that substantial fluctuations (resonances) in the ^{12}C+^{12}C fusion excitation function are present below E$_{cm}$ ∼7 MeV and these appear to become more pronounced at the lower energies, *e.g.,* close to the Gamow window.

The present work is motivated by the astrophysical importance and the fact that these resonances, and ^{12}C+^{12}C fusion itself, have received considerable experimental attention. Indeed, many different methods have been used to obtain the fusion cross sections including the detection of charged particles (α's and protons) [3–5] and γ rays [6, 7] emitted from the fused ^{24}Mg compound system [8]. Recently, a method of detecting coincidences between charged particles and γ rays [9] has shown promise for extending measurements down into the astrophysically interesting region by providing a strong suppression of the backgrounds that plague other measurements. However, until such measurements can be performed, extrapolations to this energy region are necessary in order to estimate the rate of ^{12}C+^{12}C fusion for astrophysical scenarios.

For this reason, it is important to understand the origin of the resonances and why they are especially pronounced in the ^{12}C+^{12}C system. In the analysis of their measurements, Aguilera *et al.* [10] proposed a description of the excitation function of ^{12}C + ^{12}C in terms of a non-resonant background, σ_{bkg} calculated with the Krappe-Nix-Sierk potential [11] using the Incoming Wave Boundary Condition (IWBC), and an additional contribution from a series of resonances of Breit-Wigner form, σ_{BW}, with the relation

$$\sigma_{fus} = \sigma_{bkg} + \sigma_{BW}. \qquad (1)$$

In this description, it is implicitly assumed that the appearance of resonances provides an additional mechanism *over and above* that associated with the normal fusion cross section, the latter being represented by the IWBC calculation which assumes that the square of the amplitude of an incoming partial wave appearing inside the fusion barrier (at the incoming wave boundary) leads automatically to fusion. A more natural explanation for the resonance peaks in this system has been given in terms of molecular-resonance doorway states that absorb the incoming flux and allow for a competition of the decay strength between complete fusion and re-scattering into outgoing channels (*e.g.* Ref. [12, 13]).

In this paper, we will argue that the separation into 'resonance' and 'background' contributions to the cross section is artificial and unproven. We will show that, compared to the neighboring systems (^{12}C+^{13}C and ^{13}C+^{13}C), the ^{12}C+^{12}C system exhibits a cross section deficit at energies below E_{cm} < 7 MeV and that this deficit is reasonably explained by a paucity of ^{24}Mg compound states into which the system can fuse below this energy threshold. The observed resonance structure is, therefore, expected to be associated with the distribution of ^{12}C+^{12}C parentage strength among compound nuclear levels or, at lower excitation energies, with the individual levels themselves.

Previous papers have made similar observations. Almqvist *et al.* [14] suggested that the resonance structure observed in ^{12}C+^{12}C may be due to the low excitation energy available in the ^{24}Mg system, but Erb *et al.* (from the same collaboration, [15]) concluded that, apart from the narrow resonances, the average energy dependence of the ^{12}C+^{12}C system can be described adequately by the IWBC model calculations of Christensen *et al.* [16]. However, in this case, no attempt was made to simultaneously reproduce the excitation functions for all three C+C systems in a self-consistent manner.

Other fusion systems in the same mass range exhibit smooth fusion excitation functions and astrophysical S factors, as illustrated in Fig 1. Here, the astrophysical S factor is given by $S(E) = \sigma E \exp(2\pi\eta)$, and η is the Sommerfeld parameter. Focusing on the C+C systems, it becomes clear that the resonance structure appears only in the ^{12}C+^{12}C case; the systems ^{12}C+^{13}C and ^{13}C+^{13}C

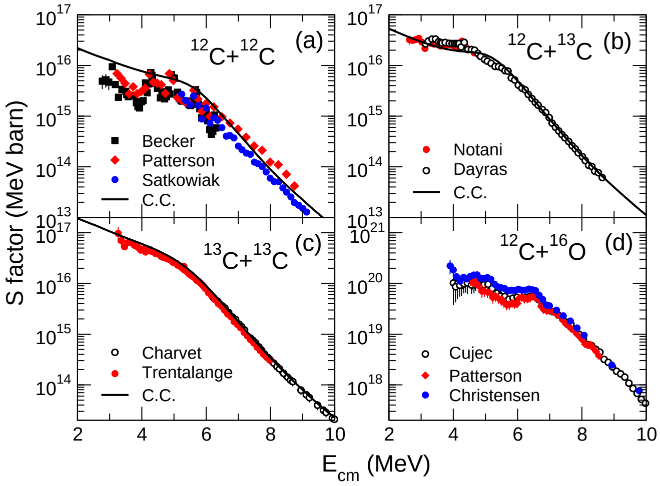

FIG. 1: Comparisons of the S factors for four fusion systems. The data for systems in (a) - (d) are taken from references [3–7], [17, 18], [19, 20] and [16, 21, 22], respectively.

both show smooth S factors as a function of the center-of-mass energy, E_{cm}. In a recent paper [23], Esbensen *et al.,* analyzed the three C+C fusion systems in terms of Coupled-Channels (CC) calculations with M3Y potentials augmented with a repulsive core meant to include the effects of nuclear incompressibility [24] required to explain the hindrance of heavy-ion fusion at extremely low energies [25, 26]. Using a consistent set of channel couplings and ion-ion potentials, these authors obtain quite accurate descriptions of the data for ^{12}C+^{13}C and ^{13}C+^{13}C, but the ^{12}C+^{12}C system exhibits a clear cross section deficit, especially in the resonance region of $E_{cm} < 7$ MeV (Fig. 1). Here, the measured S-factor data fall substantially below the calculations and only the resonance peaks reach the predicted values. The system ^{12}C+^{16}O is also included in Fig. 1, since this system and ^{16}O+^{16}O exhibit similar, but less pronounced, resonance structures at low energies.

In this paper, we suggest that the observed suppression of the average cross section, or S factor, at energies below $E_{cm} < 7$ MeV in ^{12}C+^{12}C finds its origin in the relatively large spacing D and narrow widths Γ of the levels in the ^{24}Mg compound system populated in the fusion process. In this view, the fusion process is simply blocked or hindered at energies between compound states of the appropriate spin and parity. A similar situation occurs for slow neutron capture on heavy nuclei, where the compound level structure is clearly visible in the cross sections of (n,fission) processes [27]. For essentially all heavy-ion fusion reactions, the height of the Coulomb barrier, V_B, and the fusion Q value are such that it is impossible to reach the low excitation energies where this effect would manifest itself. The effect is, therefore, never taken into account in heavy-ion fusion models, which assume that complete fusion occurs with 100% probability once the potential barrier has been traversed. Thus, it is typical and appropriate to employ the IWBC to estimate the probability that fusion occurs for a specific partial wave.

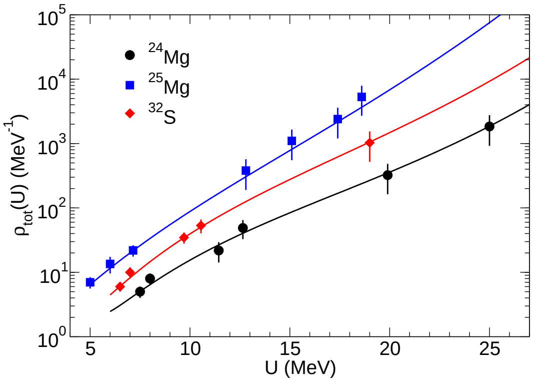

FIG. 2: Total level densities for ^{24}Mg, ^{25}Mg, and ^{32}S as a function of the excitation energy U; the curves are calculations described in the text.

For the ^{12}C+^{12}C system, several factors conspire to make the paucity of compound levels an effect to consider in the fusion cross section. First, the fusion Q value for this system is smaller than that for the two other C+C systems considered here, leading to relatively low excitation energies in the fused ^{24}Mg system relative to the Coulomb barrier. Secondly, because ^{24}Mg is an even-even system, the level spacing at a given excitation is increased because of the pairing gap, and thirdly, since the entrance channel is comprised of identical spin-zero nuclei, only states with positive parity and even spin in ^{24}Mg can be populated. These three factors result in a substantial reduction by more than an order of magnitude of the effective compound-nucleus level density for this system relative to, *e.g.*, ^{12}C+^{13}C.

In order to quantify the effect of the discrete nature of compound levels in ^{24}Mg, we estimate in the following the average spacing, $\langle D\rangle$, and total width $\langle\Gamma\rangle$, as a function of spin, J, and excitation energy, U. The level density, $\rho = 1/D$, of a large range of systems has been studied by Iljinov *et al.* [28]. The total level densities ρ_{tot} for the three systems relevant for the present study are found in Fig.2. These data are taken from a compilation by Beckerman [29]. The density of levels of specific spins and parity is given by the relation

$$\rho(U, J^{\pi}) = \frac{(2J+1)e^{-(J+1/2)^2/2\sigma^2}}{4\sigma^3\sqrt{2\pi}}\rho(U), \qquad (2)$$

where $\sigma^2 = 0.088aTA^{2/3}$ is the spin cutoff parameter, $T = \sqrt{U/a}$ the nuclear temperature, and $\rho(U)$ is defined as

$$\rho(U) = \frac{\sqrt{\pi}}{12}\frac{\exp 2\sqrt{aU}}{a^{1/4}U^{5/4}}. \qquad (3)$$

The level density parameter, a, was taken from fits to the values given in Table II of Ref.[28]. Clearly, as demonstrated in Fig. 2, these average curves describe the

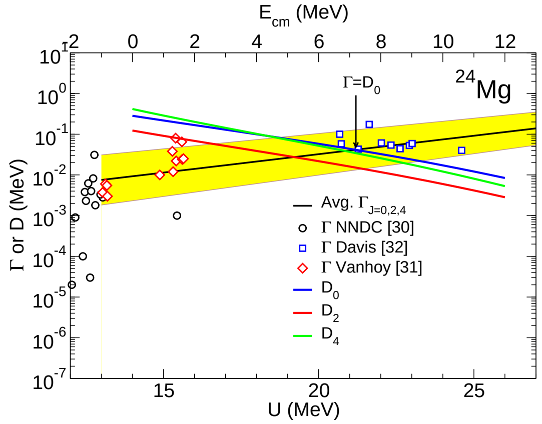

FIG. 3: The experimental widths of J=0^+, 2^+, and 4^+ levels in ^{24}Mg are shown as open symbols. The solid black line and the shaded region represent the estimated average trend and range above U >13 MeV. The average level spacings, D_J, for levels of spin, $J^\pi = 0^+, 2^+$ and 4^+ are given as blue, red, and green curves, respectively. The corresponding center-of-mass energy, E_{cm}, for ^{12}C+^{12}C fusion is given on the top. The energy, E_g at which $\Gamma = D_0$ is indicated by an arrow.

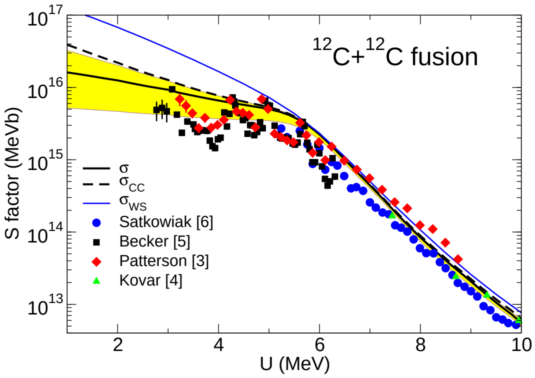

FIG. 4: Calculations of the S factor corresponding to ^{12}C+^{12}C fusion cross sections for the Woods-Saxon potential (blue curve), CC(M3Y+rep) IWBC (dashed black curve) and Eq. 5 (solid black curve) including the range of Γ values indicated in Fig. 3 (shaded region) are compared with experimental values from Refs. [3–6].

experimental level densities well, including in the energy region of interest in this work.

The other characteristic of the compound levels that comes into play is their width Γ_J. There are no detailed comparisons in the literature between direct experimental measurements and calculations of Γ_J for lighter systems. For such systems, decays from the compound nuclei to residual nuclei may not be well described in terms of statistical properties. In the following discussion, we will therefore use experimental values of level widths for ^{24}Mg from the NNDC compilation, Vanhoy and Davis [30–32], as shown in Fig. 3 for J = 0, 2 and 4. The spread of experimental Γ_J values, *e.g.* for $J = 2$ is more than one order of magnitude around U = 13 MeV, just below the ^{12}C+^{12}C threshold energy. Resonances with higher spin are not included since they are not significantly populated at energies below ∼7-8 MeV. From these values, we construct an approximate, J-averaged Γ band as shown by the shaded region in Fig. 3, in comparison with the corresponding average level spacings obtained from Eq.(2) for $J = 0, 2$ and 4.

The energy, E_g, at which the average level width, $\overline{\Gamma}_{J=0,2,4}$, equals the average spacing of J = 0 levels, D_0, is indicated by the arrow in Fig. 3. Around and below this energy, it is expected that the fusion cross section is reduced relative to the one obtained from the CC(M3Y+rep), IWBC calculation. In recent work, Notani [18] and Esbensen [23] discussed the issue of discrete compound resonances and their effect on compound-nucleus formation cross sections in ^{12}C+^{12}C. Vandenbosch [33] also studied the origin of oscillations in the ^{12}C+^{12}C fusion cross sections (higher momentum components) at energies higher than those investigated in this work and stated that "if there are not overlapping compound states of the relevant angular momentum ($\Gamma_J/D_J > 1$), fusion will not compete successfully with direct reaction processes". Haas and Abe [34] have discussed this issue as well in relation to the onset of higher angular momentum transfer with energy. In an earlier publication, Moldauer [35] analyzed the issue of partially overlapping resonances within the context of R-matrix theory and obtained an average reduction factor of

$$P_J = 1 - \exp(-2\pi\overline{\Gamma}_J/D_J). \qquad (4)$$

Based on this analysis, we therefore suggest that the average fusion cross section be given as

$$\sigma = \sum_J \sigma^J_{CC} P_J, \qquad (5)$$

where the summation includes the possible compound-nuclear spins, J= 0, 2, 4 ..., σ^J_{CC} is the CC(M3Y+rep), IWBC cross section for partial wave, $L = J$ [23], and P_J accounts for the correction for the level-density effect. The results are illustrated in Fig. 4 and they show that the non-overlapping behavior of the compound levels in ^{24}Mg needs to be considered for ^{12}C + ^{12}C fusion at E_{cm} < 7 MeV. It is well known that weaker resonance structures persist at higher energies in this system as well as in ^{12}C+^{16}O and ^{16}O+^{16}O, see Fig. 1. These cross section fluctuations are likely associated with Ericson fluctuations, or at even higher energies, the effects of molecular resonances come into play.

In Table I, some of the relevant parameters are given for neighboring collision systems. Here, the center-of-mass energy, E_g, at which $\Gamma_0 = D_0$, are listed along with the corresponding excitation energy, U_g. The fusion Q-value, the Coulomb barrier and the expected value of Γ/D for compond-nucleus states populated at the Coulomb barrier are given as well. Table I shows that the effect of non-overlapping compound levels is expected only for the ^{12}C+^{12}C system. The cross-over point, E_g, is

TABLE I: The fusion Q value, the center-of-mass energy E_g at which $\Gamma = D_0$, the corresponding excitation energy, U_g, the Coulomb barrier V_c and the Γ/D at the Coulomb barrier are listed for five light heavy-ion systems. Here, E_g, U_g and $(\Gamma/D)_c$ are all given for the lowest J for the compound system.

System	Q(MeV)	E_g(MeV)	U_g(MeV)	V_c(MeV)	$(\Gamma/D)_c$
^{12}C + ^{12}C	13.934	7.3	21.2	6.66	0.7
^{12}C + ^{13}C	16.318	−1.1	15.2	6.56	120
^{13}C + ^{13}C	22.465	−8.5	14.0	6.48	2210
^{12}C + ^{16}O	16.765	1.8	8.5	8.45	12
^{16}O + ^{16}O	16.542	0.8	17.3	10.76	94

negative for the other two C+C systems such that fusion always leads to strongly overlapping compound levels in the Mg compound nucleus. For ^{12}C+^{16}O and ^{16}O+^{16}O, E_g is 6.6 and 10 MeV below the Coulomb barrier, respectively, and it is therefore unlikely that the weak cross section fluctuations seen in these systems [16] are associated with this effect.

In conclusion, we suggest that the appearance of strong resonances and the suppression of sub-barrier fusion cross sections in ^{12}C + ^{12}C is associated with the non-overlapping nature of the compound states in ^{24}Mg at the relevant excitation energies. Below a center-of-mass energy of $\sim$ 7 MeV we estimate that the widths of the compound states are smaller than their average spacing; *i.e.,* $\Gamma/D < 1$. Thus, the probability to find an available compound state is less than unity. The ^{12}C + ^{12}C system is rather unique in this respect because of its small fusion Q value and the symmetry property in the entrance channel. The fact that these effects are not observed in neighboring systems, such as ^{12}C + ^{13}C, ^{13}C + ^{13}C, is shown to be consistent with this explanation. The effect of the non-overlapping compound states is expected to play an important role when extrapolating the measured fusion cross sections to the region corresponding to temperatures of $\sim$ 0.85 GK occurring in explosive carbon burning during the late stages of giant stars. Compared to extrapolations based on coupled-channels calculations using the incoming wave boundary conditions, the effect of non-overlapping compound resonances will lead to the requirement of higher densities and higher temperatures in order to achieve ignition.

This work is supported by the US Department of Energy, Office of Nuclear Physics, under Contract No. DE-AC02-06CH11357 and DE-FG02-87ER-40316, and the NSF under Grant No. PHY-0758100 and PHY-0822648. The authors thank X.D. Tang, M. Hussein, and A. Karpov for valuable discussions.

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