Using the Relativistic Two-Stream Instability for the Generation of Soft-X-Ray Attosecond Radiation Pulses

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Using the relativistic two-stream instability for the generation of soft x-ray atto-second radiation pulses

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In this paper we discuss a novel method for generating ultra-short radiation pulses using a broad-band two-stream instability in an intense relativistic electron beam. This method relies on an electron beam having two distinct two energy bands. The use of this new high brightness electron beam scenario, in combination with ultra-short soft x-ray pulses from high harmonic generation in gas, allows the production of high power attosecond pulses for ultra-fast pump and probe experiments.

The successful operation of the Linac Coherent Light Source [1] and other free-electron laser (FEL) facilities around the world [2, 3] has established the FEL as by far the most brilliant current source of coherent x-rays. In a high-gain FEL [4], a high-brightness electron beam travels in an undulator magnet and amplifies to saturation a co-propagating resonant radiation pulse. The main features of FEL light sources are the very high power (up to several tens of gigawatts [1]), transverse coherence [5], narrow bandwidth and tunability over a continuous range of wavelengths (see e.g. [2]). The generation of coherent hard X-rays enables new methods, such as diffraction imaging, that may examine atomic and molecular systems at their characteristic length scale (Angstrom). Further, FEL pulse lengths are now obtained at the femtosecond level, thus resolving much of the dynamics of such systems. While this is an impressive achievent, there is demand for generating yet shorter X-ray pulses for pump-probe experiments. In this case the narrow bandwidth of an XFEL, a highly desirable feature for many applications, limits the capability of the FEL to achieve amplification in ultra-short pulses.

Thus, in this paper we discuss an alternative amplification scheme based on a relativistic two-stream instability driven by longitudinal space-charge forces. As we shall see, the two stream instability is a broad-band exponential amplification process and may represent an important alternative to the FEL amplifier in cases in which broad-band operation is needed. Indeed, it may allow the generation and amplification of few cycle pulses at X-ray wavelengths.

The two-stream instability is a well-known physical effect in the context of fusion plasmas, space plasmas and high-energy accelerators. The instability is driven by the longitudinal Coulomb field generated by a plasma with two distinct peaks in the longitudinal velocity distribution (see e.g. [6]). This type of velocity distribution can present itself in a wide variety of forms, such as, for example, a particle beam being injected in a fusion plasma [7] or an ion beam propagating in the presence of a background plasma that is employed for transverse focusing and stablization [8].

In the current case of interest, we study the the two-stream instability in a relativistic electron beam which has an energy distribution with two narrow peaks, representing a beam with two distinct energy strata or bands. This type of scenario was examined in a different context having much different physical goals by Bekefi and Jacobs [9], to explore use of the two-stream instability to enhance the gain and efficiency of low-energy, *mm*-wave FELs. In contrast, in this paper we discuss the exploitation of the the broad-band nature of the two-stream instability in a modern FEL in the VUV and soft x-ray regions, in order to allow the generation and amplification of attosecond pulses.

Figure 1 shows a schematic layout of the proposed amplification scheme. A pre-modulated (meaning weakly micro-bunched by inverse FEL-derived seeding or shot-noise) electron beam with two distinct energy bands propagates in a focusing channel. This bifurcated energy distribution may be created in many beam pulse compression processes (e.g. Ref. [10]), as is discussed further below. The two-stream instability then serves to amplify the initial density modulation. After saturation of the instability, the beam is sent to a broad-band radiator such as a short undulator or a transition radiation screen and the strong microbunching obtained induces the emission of coherent radiation. Contrary to the FEL, in which the undulator field couples the electron motion to the radiation field, thus driving a collective instability, in this case the undulator only serves the purpose of extracting energy from the microbunched beam in the form of electromagnetic radiation (super-radiant emission). The exponential growth of the microbunching, instead, is driven by the longitudinal space-charge forces outside of the undulator.

The two-stream instability for a relativistic electron beam with two energy levels can be well described by a simple one-dimensional model. We assume a coasting (non-accelerating) beam with an average energy of γmc^2. The beam is described by a distribution function in longitudinal phase-space $f(z,\eta,\tau)$ where z is the longitudinal coordinate along the electron beam with respect to a reference particle traveling at the normalized speed $\beta_z = \sqrt{1-\gamma^{-2}}$, $\eta = \delta\gamma/\gamma$ is the relative energy deviation with respect to the mean beam energy and $\tau = ct$

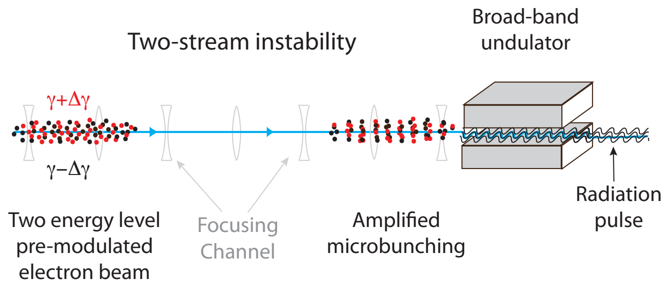

FIG. 1: Layout of the two-stream amplifier.

where t is the time variable and c is the speed of light. The collective longitudinal beam dynamics is described by the Vlasov equation in the relevant two-dimensional phase space. We expand the distribution function to first order in perturbation theory: $f = f_0 + f_1$ with $|f_1| \ll f_0$. Furthermore, we assume that the lowest order distribution function is factorable as $f_0 = n_0 f_v(\eta)$ where n_0 is the beam volume density and f_v is the beam's energy distribution function. The resulting linearized Vlasov equation is:

$$\frac{\partial f_1}{\partial \tau} + \frac{\eta}{\gamma^2}\frac{\partial f_1}{\partial z} + \frac{eE_z}{\gamma mc^2}\frac{\partial f_0}{\partial \eta} = 0 \tag{1}$$

where e is the electron charge and E_z is the longitudinal electric field, which can be computed by solving the one-dimensional Poisson equation,

$$\frac{\partial}{\partial z}E_z = \frac{e}{\epsilon_0}\int f_1 d\eta. \tag{2}$$

It is convenient to solve Eqs. (1) and (2) in the Laplace-Fourier domain. We give the following definitions: $\hat{f}_1 = \int f_1 e^{-ikz}dz$; ans $\tilde{\hat{f}}_1 = \int_0^\infty \hat{f}_1 e^{i\frac{\omega}{c}\tau}d\tau$. With these definitions, the Fourier-Laplace transforms of Eqs.(1,2) yield

$$-i\frac{\omega}{c}\tilde{\hat{f}}_1 - \hat{f}_1\Big|_{\tau=0} + \frac{ik\eta}{\gamma^2}\tilde{\hat{f}}_1 + \frac{e}{\gamma mc^2}n_0\tilde{\hat{E}}_z\frac{\partial f_v}{\partial \eta} = 0 \tag{3}$$

and

$$\tilde{\hat{E}}_z = -\frac{i}{k}\frac{e}{\epsilon_0}\int \tilde{\hat{f}}_1 d\eta. \tag{4}$$

After some algebraic manipulation (see e.g. Ref. [11]), it can be shown that the phase-space perturbation f_1 can be expressed as

$$\tilde{\hat{f}}_1 = \frac{1}{-i\frac{\omega}{c} + \frac{ik\eta}{\gamma^2}}\left(\hat{f}_1\Big|_{\tau=0} - \frac{1}{\epsilon_p}\frac{\omega_p^2}{c^2}\frac{\partial f_v}{\partial \eta}\frac{\gamma^2}{ik}\int \frac{\hat{f}_1\Big|_{\tau=0}}{-i\frac{\omega}{c} + ik\frac{\eta'}{\gamma^2}}d\eta'\right), \tag{5}$$

where ϵ_p is the beam's plasma dielectric function, given by

$$\epsilon_p = 1 + \frac{\omega_p^2}{c^2}\frac{\gamma^2}{ik}\int \frac{\frac{\partial f_v}{\partial \eta}}{-i\frac{\omega}{c} + \frac{ik\eta}{\gamma^2}}d\eta, \tag{6}$$

and $\omega_p^2 = \frac{e^2 n_0}{\epsilon_0 m\gamma^3}$ being the relativistic beam plasma frequency.

The Laplace transform in Eq.(5) can be inverted by using the residue theorem. In doing so we will only consider the poles associated to the zeroes of the dielectric function ϵ_p since those are the poles that describe the collective response of the electrons.

The two-stream energy structure can be modeled with the following energy distribution function $f_v = (2\sqrt{2\pi}\sigma_\eta)^{-1}\left(e^{\frac{(\eta-\Delta\eta)^2}{2\sigma_\eta^2}} + e^{\frac{(\eta+\Delta\eta)^2}{2\sigma_\eta^2}}\right)$ where $\Delta\eta \ll 1$. We introduce the following dimensionless variables: $K = \frac{kc\sigma_\eta}{\gamma^2\omega_p}$ is the energy spread parameter, $\Delta = \frac{kc\Delta\eta}{\gamma^2\omega_p}$ is the normalized energy energy separation and $\Omega = \frac{\omega}{\omega_p}$ is the normalized Laplace variable. The resulting plasma dielectric function is

$$\epsilon_p = 1 - \frac{1}{4K^2}\left(Z'\left(\frac{\Omega-\Delta}{\sqrt{2}K}\right) + Z'\left(\frac{\Omega+\Delta}{\sqrt{2}K}\right)\right), \tag{7}$$

where Z' is the complex derivative of the plasma dispersion function defined as: $Z(\zeta) = \frac{1}{\sqrt{\pi}}\int_{\tilde{c}} dx\frac{e^{-x^2}}{x-\zeta}$, where $\tilde{c}$ is the Landau contour which runs in the complex plane from $-\infty$ to $+\infty$ and below the singularity at $x = \zeta$.

To study the stability of the system we will focus on the cold beam limit, i.e. the limit for vanishing energy spread: $K \to 0$. In this limit the dispersion equation for the system reduces to

$$1 - \frac{1}{2(\Omega-\Delta)^2} - \frac{1}{2(\Omega+\Delta)^2} = 0. \tag{8}$$

Equation (8) has the following solutions:

$$\Omega_{\pm,\pm} = \pm\frac{1}{\sqrt{2}}\sqrt{2\Delta^2 + 1 \pm \sqrt{1+8\Delta^2}} \tag{9}$$

For $\Delta < 1$ the root $\Omega_{+,-} = i\Gamma = \frac{i}{\sqrt{2}}\sqrt{\sqrt{1+8\Delta^2} - 2\Delta^2 - 1}$ is purely imaginary with a positive imaginary part, leading to an exponential growth of the phase-space perturbation $\hat{f}_1$ as a function of time. In analogy with the theory of free-electron lasers, we define Γ as the gain parameter and the gain-length $l_g = \frac{c}{\omega_p\Gamma}$. Figure 2 shows the unstable root as a function of Δ for the for the cold beam limit and for a warm beam with different values of K. The gain parameter has an optimum value for $\Delta_{opt} = \sqrt{3}/2\sqrt{2}$ corresponding to $\Gamma_{opt} = 1/2\sqrt{2}$. In terms of physical units, the streaming instability provides a broadband amplification mechanisms for beam microbunching at

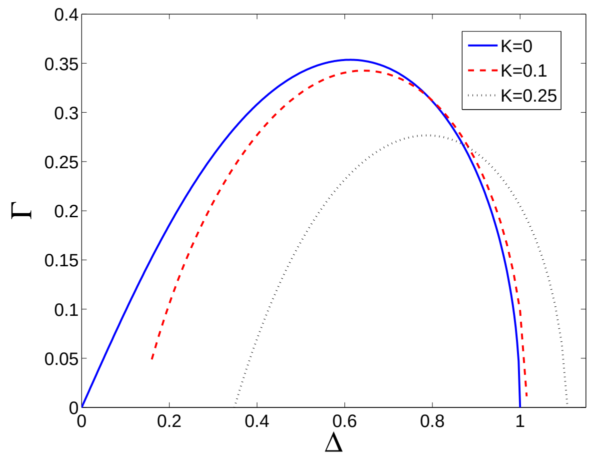

FIG. 2: Imaginary part of the normalized unstable root Γ as a function of the normalized energy separation Δ for three different values of the energy-spread parameter K.

wavelengths larger than $\lambda_{th} = \frac{\lambda_p \Delta\eta}{\gamma^2}$ with an optimum gain-length $l_{g,opt} = \sqrt{2}\lambda_p/\pi$ (where $\lambda_p = 2\pi c/\omega_p$ is the plasma period) at $\lambda_{opt} = \frac{2\sqrt{2}\lambda_p\Delta\eta}{\sqrt{3}\gamma^2}$.

To quantify the spectral properties of the amplification process, we define the bunching factor as $b(k) = \frac{1}{N}\int \hat{f}_1 d\eta$, where N is the number of particles in the beam. In the cold beam limit, explicit inversion of the Laplace transform in Eq.(5) gives, in dimensionless units,

$$b(\Delta) = \frac{b_0(\Delta)}{2}\frac{\Gamma^2 - \Delta^2}{\Gamma^2 - 3\Delta^2}e^{\Gamma T} \tag{10}$$

where $T = \frac{\omega_p \tau}{c}$ and b_0 is the bunching factor at $T = 0$. To study the behavior of the amplification spectrum around the optimum energy separation we note that, to second order, $\Gamma(\Delta) \simeq \frac{1}{2\sqrt{2}} - \frac{3\sqrt{2}}{4}(\Delta - \Delta_{opt})^2$, giving $|b|^2 \propto e^{\frac{T}{\sqrt{2}} - \frac{(\Delta-\Delta_{opt})^2}{2\sigma_\Delta^2}}$, with a relative root-mean-square (RMS) power amplification bandwidth of $\frac{\sigma_\Delta}{\Delta_{opt}} = \frac{\sigma_k}{k} = \frac{2^{5/4}}{3\sqrt{T}}$. The amplification bandwidth has a rather weak dependence on the normalized interaction time T, yielding a broad amplification bandwidth for most cases of practical interest. Note that, from basic Fourier analysis, the shortest rms pulse duration achievable with the amplifier is given by: $\sigma_{z,min} = \frac{1}{2\sigma_k} = \lambda_{opt}\frac{k}{4\pi\sigma_k} = \lambda_{opt}\frac{3\sqrt{T}}{4\pi 2^{5/4}}$. It follows that the two-stream instability can amplify few optical cycle pulses without significant lengthening, unlike the free-electron laser instability in which the radiation slippage limits the minimum pulse duration to a cooperation length [12]. For example, assuming $T = 14$ (corresponding to roughly five gain-lengths) we have $\frac{\sigma_k}{k} \simeq 20\%$, giving a minimum rms pulse length of $\sigma_{z,min} \simeq 0.4\lambda_{opt}$. This feature makes the two-stream amplifier an attractive method for the generation of attosecond pulses.

A central challenge in the operation of a two-stream amplifier lies in the generation of intense electron beams with a two-energy-band structure. This problem may be addressed in several ways and a detailed discussion of the generation of two-stream beams will be left for future publications. However, we mention here that this type of phase-space structure can be generated by illuminating a photo-cathode with a pulse train and imparting a z-energy correlation by accelerating the resulting multi-bunch beam off-crest. At this point the micro-pulses can be overlapped in time with using either velocity bunching or, again, magnetic compression. It is also worth mentioning that the peak current amplification induced by non-linear wave-breaking, demonstrated in Ref. [13], could greatly enhance this scheme. Alternatively, an ultra-short two-stream structure could be induced with an E-SASE compression scheme [14] by adding a beam mask in the center of the magnetic chicane.

The longitudinal space-charge microbunching instability has also recently been proposed as an amplifier for the generation of broad-band radiation pulses, in a scheme known as the Longitudinal Space-Charge Amplifier (LSCA) [15, 16]. In a LSCA, an electron beam travels in a focusing channel and the collective longitudinal space-charge fields, generated by shot-noise or by a pre-existing density modulation, induce an energy modulation. After the focusing channel, the energy modulation is transferred into density modulation with a magnetic chicane, generating a bunching factor that is much greater than its starting value. One obvious advantage of the two-stream instability with respect to the LSCA is the absence of a magnetic chicane in the amplification process. Another key advantage of the two-stream amplifier over the LSCA is the flexibility in the wavelength tuning. The central amplification wavelength of a LSCA is given by the condition $k_{opt}\sigma_x/\gamma \simeq 1$, where σ_x is the RMS transverse size of the electron beam. This condition is difficult to attain at very short wavelengths and requires drastic changes in the beam transport in order to be tuned over a wide range of wavelengths. On the other hand, the two-stream amplifier can be simply tuned by varying the energy-separation of the two beamlets.

As pointed out in [15], broad-band amplifiers based on relativistic electrons have a wide range of applications, due to their robustness to beam stability requirements and their unique spectral properties. In particular, the broad-band nature of the two-stream instability makes it an attractive method for the generation and amplification of tunable ultra-short pulses. The generation of intense attosecond VUV radiation pulses from high-harmonic generation in gas has recently been demonstrated [17]. Such radiation pulses cannot be amplified in a conventional seeded FEL due to the effect of slippage, which limits the time duration of an FEL pulse to a cooperation length (which is typically several radiation periods).

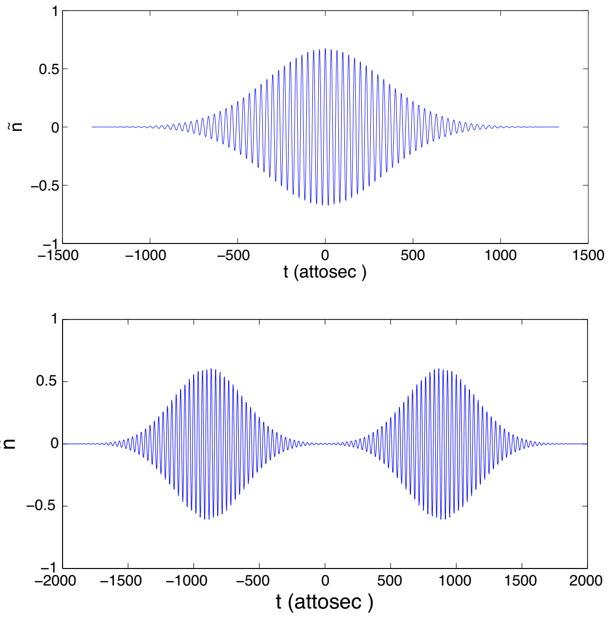

FIG. 3: Longitudinal density perturbation at saturation for the single pulse (upper plot) and double pulse (bottom plot) configurations.

Few-cycle pulses can be amplified with a broad-band amplifier, such as the two-stream amplifier discussed in this letter. We discuss an example corresponding to the generation of an ultra-short soft x-ray pulse starting from high harmonic generation (HHG) in gas. We assume a seed pulse at $\lambda_s = 30nm$ with a power of 1 MW and a RMS duration of 150 attoseconds. To quantify the amplification process, we define the density perturbation $\tilde{n} = \int d\eta f_1/n_0$. The seed interacts with a resonant electron beam of energy $E = 750$ MeV in a magnetic undulator. Assuming an undulator with two periods of length $\lambda_w = 5$ cm, an undulator parameter of $K_w = 1.3$ and a beam uncorrelated energy-spread of $\sigma_\eta = 0.75 \times 10^{-4}$, the resonant interaction, followed by a magnetic chicane with longitudinal dispersion $R_{56} = \partial z/\partial\eta = 25$ μm, we can assume a third harmonic density perturbation of the type $\tilde{n} \simeq \tilde{n}_{max} cos(kz) e^{-\frac{(kz)^2}{2\sigma_z^2}}$ at $\lambda = 2\pi/k = 10nm$, with $\tilde{n}_{max} \simeq 2 \times 10^{-3}$ and $\sigma_z \simeq 350asec \times c$. The induced energy modulation is smaller than the assumed uncorrelated energy-spread, which means that to generated strong microbunching, amplification through the two-stream instability is needed. Due to the two-energy structure, the two beamlets will shift with respect to each other as a result of dispersion in the bunching chicane, leading to de-phasing of the microbunches and to an overall lengthening of the microbunching structure. For an efficient short-pulse seeding the chicane dispersion has to fulfill the following conditions: $2R_{56}\Delta\eta = n\lambda$, which means that the relative shift of the two beamlets has to be a multiple of the microbunching wavelength, and $2R_{56}\Delta\eta < \sigma_z$ which ensures that the relative shift of the two streams does not wash out the ultrashort pulse structure. The final rms length of the microbunched structure is estimated by adding in quadrature the length of the radiation pulse ($\simeq 66nm$ for the field distribution), the undulator slippage ($2\lambda_r = 60nm$) and the relative shift of the two beamlets in the dispersive section ($2\Delta\eta R_{56} = 60nm$). Alternatively, one could choose a larger R_{56} to generate two separate pulses with a shorter length. With the parameters chosen, assuming $R_{56} = 220\mu m$, the relative shift of the beamlets is $2\Delta\eta R_{56}/c \simeq 1760asec$, which results in two isolated microbunching pulses of amplitude $\tilde{n}_{max} \simeq 1.5 \times 10^{-3}$ and duration $\sigma_z \simeq 300asec \times c$.

We assume a beam current of $I = 500A$, a transverse radius of $r_x = 11\mu m$ with an energy separation of $\Delta\eta = 0.0012$. With these parameters, the gain length is optimized at $\lambda_{opt} \simeq 10nm$ with $l_g = 5.2m$. Figure 3 shows the longitudinal density perturbation $\tilde{n} = \int d\eta f_1/n_0$ as a function of the position along the electron bunch, at saturation for the case of a single pulse (roughly after 6 gain lengths) and for the double pulse case (roughly after 6.5 gain lengths). Note that the attosecond structure is not washed out during the exponential gain due to the broadband nature of the instability. Note also that for a bunching factor larger than $\tilde{n} \simeq 50\%$, nonlinear effects become important, leading to saturation of the exponential gain. In this non-linear regime a numerical particle tracking code would be needed to describe this phenomenon.

After the amplification process, the microbunched electron beam can be sent into a broad-band radiator for the emission of coherent radiation. The radiator can be a broad-band undulator (i.e. an undulator with few periods) or a metal foil, which causes the emission of coherent transition radiation. The undulator radiation mechanism induces a pulse-lengthening process due to the slippage of the radiation over the electrons. The condition to preserve the short pulse structure in the undulator is that the slippage length, defined as the resonant wavelength times the number of unulator periods, be much shorter than the length of the density perturbation in the beam, i.e. $N_w\lambda_r < \sigma_z$. The peak power emitted by a pre-bunched electron beam in a helical undulator can be estimated with the following expression, derived in [18]:

$$P = W_b \frac{\pi^2 b_m^2}{2} \frac{I}{\gamma I_a} \frac{K_w^2}{1+K_w^2} N_w F(\hat{N}), \qquad (11)$$

where $\hat{N} = kr_x^2/4L_w$, with L_w being the undulator length, $F(\hat{N}) = \frac{2}{\pi}(arctan(\frac{1}{2\hat{N}}) + \hat{N}ln(\frac{4\hat{N}^2}{1+4\hat{N}^2}))$, $W_b = \gamma mc^2 I/e$ is the beam power, b_m is the peak bunching factor, $I_a = 17kA$ is the Alfven current, K_w is the undulator parameter and N_w is the number of undulator periods. For the examples chosen, a four-period undulator with a period of $\lambda_w = 2cm$ and $K_w = 1.15$ yields a peak power of $W \simeq 15MW$, with a pulse energy of $U \simeq 10nJ$, for the single pulse case, and $U \simeq 8.6nJ$ per pulse in the double pulse case, with negligible pulse lengthening. The final radiation pulse duration is $\sigma_z/c\sqrt{2} \simeq 250asec$ for the single pulse and $\sigma_z/c\sqrt{2} \simeq 214asec$ for each of the two pulses in the double pulse case.

In conclusion, in this Letter we have discussed the generation of attosecond, short wavelength electromagnetic pulses using the relativistic two-stream instability. The relativistic two-stream amplifier is a tunable broad-band amplifier, which holds great promise for the generation of ultrashort VUV and soft x-ray pulses. The two-stream amplifier relies on the formation of a beam with two distinct energy levels and is continuously tunable due to the flexibility in the choice of the energy separation $\Delta\eta$. We have discussed a one-dimensional model of the two-stream amplifier and identified simple scaling laws for the optimal wavelength, gain-length and for the coupling to an initial bunching factor. Finally, we have discussed an example corresponding to the generation of an attosecond pulse at $\lambda = 10nm$, thus illustrating the practical application of this novel amplification scheme in the context of creating an unprecedented 250 attosecond, coherent soft-X-ray pulse.

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