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Phys. Rev. Lett. **110**, 035702 — Published 15 January 2013

DOI: [10.1103/PhysRevLett.110.035702](https://doi.org/10.1103/PhysRevLett.110.035702)

Quantum Phase Transition in a Clean Two-Dimensional Electron System

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(Dated: November 26, 2012)

A quantum phase transition that was recently observed in a high-mobility silicon MOSFET is analyzed in terms of a scaling theory. The most striking characteristic of the transition is a divergence of the thermopower, according to an inverse linear law, as a critical value of the electron density is approached. A scaling description of this transition yields predictions about the critical behavior of other observables, e.g., the specific heat. We also explore the possibility that this transition realizes a recently predicted transition from a Fermi liquid to a non-Fermi-liquid state.

PACS numbers: 72.15.Jf; 05.30.Rt; 73.40.Qv

The question of the existence or otherwise of a metal-insulator transition in the bulk of a two-dimensional ($2-d$) electron system has been an issue of great interest for some time. It had seemed settled after it became clear that the lower critical dimension for the Anderson metal-insulator transition of noninteracting electrons is $d_c^- = 2$ – that is, the system is insulating and no transition to a normal-metal phase can take place in $d = 2$ [1–3] – and generalizations of the pertinent theory concluded that this remains true for interacting electrons as well [4, 5]. The observation of what appeared to be a metal-insulator transition in a strongly interacting $2-d$ electron system, namely, a silicon metal-oxide-semiconductor field-effect transistor (MOSFET) [6] therefore came as a considerable surprise, as it indicated the possibility of an interaction-driven quantum phase transition that involves a normal metallic phase. However, despite a large amount of experimental and theoretical work, the physics underlying these observations, and even the existence of a true phase transition, continues to be debated [7, 8].

One problem is that the electrical conductivity, which is the most obvious observable to focus on in the context of a metal-insulator transition, is susceptible to complicated and non-universal scattering mechanisms that can either mimic or mask true critical behavior. It is therefore very interesting that a recent experiment on low-disorder MOSFETs has found what appears to be critical behavior of the thermopower, or Seebeck coefficient, S as a function of the electron density [9]. The thermopower is a ratio of two transport coefficients [10], and therefore presumably less susceptible to the non-critical scattering that can make transport coefficients hard to interpret. The experiment of Ref. 9 found that $1/S$, at temperatures between $T = 300$ mK and 800 mK, is a linear function of $1/T$ with a prefactor that vanishes approximately linearly as the electron density n approaches a critical value n_c from above:

$$S(T, n) \equiv eT s(n) \propto T (n - n_c)^{-\mu} \quad (1)$$

with e the electron charge, $\mu = 1.0 \pm 0.1$ and $n_c =$

$(0.78 \pm 0.01) \times 10^{11} \text{ cm}^{-2}$. The sample in question also shows what Ref. 9 interpreted as a resistive transition at a density very close to n_c . This feature is very similar to the putative metal-insulator transition in higher-disorder samples, and the density where it occurs is strongly disorder dependent, whereas the critical density n_c for the thermopower is independent of the disorder. This led the authors of Ref. 9 to suggest that their system, at the temperatures that were accessible in their experiment, can be reasonably interpreted as a clean electron fluid with a normal metallic phase. Under these assumptions, the critical behavior of S/T signifies a transition from a Fermi liquid to an unknown phase that is driven by electron-electron interactions, is not sensitive to weak disorder, and that is characteristic of, and would still be present in, a true clean system with no disorder. If this interpretation of the data given in Ref. 9 is correct, then this transition joins the ranks of, but is distinct from, other quantum phase transitions in the bulk of clean $2-d$ electron systems, e.g., the antiferromagnetic quantum phase transition [11], or transitions involving stripe phases [12]. In what follows, we assume that this is indeed the case.

In Ref. 9 the observations summarized by Eq. (1) and the above discussion were interpreted in terms of an effective mass that diverges at n_c . We will come back to this interpretation below. However, if they represent a true quantum phase transition, then it is natural to first perform a general scaling analysis [13, 14]. To do so is the purpose of the present Letter. We will initially give a very general analysis that relies on a minimal set of assumptions, and then explore the possibility that the observed transition is related to a recently proposed transition between a Fermi liquid and a non-Fermi-liquid in clean electron systems [15]. Within the framework of this suggestion, the order parameter for the transition is the density of states at the Fermi level, and the low-density phase is a “strange metal” or “interaction-induced semimetal”, characterized by a pseudogap in the density of states, which represents the phase that Ref. 9 postulated to precede a Wigner crystal.

Let us assume that the observations expressed by Eq. (1) reflect a quantum phase transition with some unknown order parameter density \mathcal{N} [16]. We assign scale dimensions $[L] = -1$, and $[t] = -z$ to factors of length and time, respectively. We use units such that $\hbar = k_B = 1$, so energy and temperature both carry scale dimensions $[E] = [T] = z$. We denote the (unknown) field conjugate to \mathcal{N} by h , with a scale dimension $[h] = y_h$. Finally, let the dimensionless distance from the critical point be r with a scale dimension $[r] = 1/\nu$, where ν is the correlation length exponent. For the experiment in question, $r = (n - n_c)/n_c$. ν , y_h , and z are in general three independent critical exponents, and all other exponents can be expressed in terms of these three [17]. Then the free energy density f , which dimensionally is an energy per volume, satisfies the homogeneity law

$$f(r, T, h) = b^{-(d+z)} f(r b^{1/\nu}, T b^z, h b^{y_h}), \quad (2)$$

where b is the length rescaling coefficient. Now consider the thermopower S , which is defined as the ratio of the induced voltage gradient to an applied temperature gradient in the absence of an electrical current [10]. eS is thus dimensionless by power counting [18]. At a phase transition where it displays critical behavior it is expected to obey a homogeneity law

$$eS(r, T) = eS(r b^{1/\nu}, T b^z). \quad (3a)$$

If we use the experimental observation that the thermopower is proportional to T , and define $s = S/eT$, we have

$$s(t) = b^z s(r b^{1/\nu}) \propto r^{-\nu z}. \quad (3b)$$

Comparing with the results of Ref. 9, we can draw a first nontrivial conclusion, namely, the correlation length exponent ν and the dynamical exponent z are related by [19]

$$\nu z = 1. \quad (4)$$

We next show how scaling leads to a prediction for the specific heat coefficient $\gamma \equiv C_V/T = -\partial^2 f/\partial T^2$. From Eq. (2) we find

$$\begin{aligned} \gamma(r, T) &= b^{z-d} \gamma(r b^{1/\nu}, T b^z) = r^{d\nu-\nu z} f_\gamma(T/r^{\nu z}) \\ &= r^{2\nu-1} f_\gamma(T/r). \end{aligned} \quad (5)$$

Here f_γ is a scaling function, and in the second line we have specialized to $d = 2$ and used Eq. (4). This predicts that a scaling plot of γ/r^x versus T/r , with x suitably adjusted, will make data for various values of T and r collapse onto one curve [20]. In particular, the specific heat coefficient extrapolated to $T = 0$ will vanish as $r^{2\nu-1}$ as the transition is approached from high densities. This will allow for an experimental determination of the correlation length exponent ν , as well as the scaling function f_γ . From Eq. (4) one then obtains z .

A determination of ν will also provide a check for whether the transition can possibly be dominated by disorder – which we assume it is not, see the discussion after Eq. (1). The Harris criterion [21, 22] states that for a critical fixed point to be stable with respect to quenched disorder the inequality $\nu \geq 2/d$ must hold. While no conclusion can be drawn if ν is found to satisfy the Harris criterion, a value $\nu < 1$ would rule out a disorder-dominated transition.

Let us now turn to the unknown order-parameter density $\mathcal{N} = -(\partial f/\partial h)/T$. From Eq. (2) we have

$$\begin{aligned} \mathcal{N}(r, T) &= b^{y_h-d} \mathcal{N}(r b^{1/\nu}, T b^z) = r^{\nu(d-y_h)} f_{\mathcal{N}}(T/r^{\nu z}) \\ &= r^{\nu(2-y_h)} f_{\mathcal{N}}(T/r), \end{aligned} \quad (6)$$

with $f_{\mathcal{N}}$ another scaling function. In the second line, we have again used Eq. (4) and specialized to $d = 2$. If the nature of the order parameter were known, then a scaling plot in addition to the one described by Eq. (5), combined with Eq. (4), would thus provide the third independent critical exponent at the quantum phase transition. All other critical exponents can be expressed in terms of ν , z , and y_h [5]. For instance, from Eq. (6) we see that the order-parameter exponent β , defined by $\mathcal{N}(r, T = 0) \propto r^\beta$, is given by $\beta = \nu(2 - y_h)$, the exponents γ and η that govern the dependence of the order-parameter susceptibility on r and the wave number, respectively, are given by $\gamma = \nu(2 - \eta) = \nu(2y_h - 2)$, etc.

We finally explore the possibility that the observed transition is the one from a Fermi liquid to a non-Fermi liquid proposed recently in Ref. 15. The order parameter for this transition is the density of states N at the Fermi level, which dimensionally is an inverse energy times an inverse volume. If it is critical, it is thus expected to have a scale dimension, in $d = 2$, $[N] = [N] = 2 - z$, which implies

$$y_h = z. \quad (7)$$

That is, at this transition there are only two independent critical exponents. The scaling behavior of the density of states then becomes the same as that of the specific heat coefficient, only with a different scaling function:

$$N(r, T) = r^{2\nu-1} f_N(T/r). \quad (8)$$

This behavior of the density of states, if it exists, is measurable by means of tunneling. For all $\nu > 1/2$ Eq. (8) predicts a vanishing density of states at the Fermi surface, or a pseudogap [23]. Note that this is consistent with the notion of a diverging effective mass m^* : Consider the standard quasi-particle picture of Landau Fermi-liquid theory [24]. Let m be the bare electron mass, m^* the renormalized, or effective, mass, and Z the quasiparticle weight. For simplicity, consider a wave-number-independent self energy, in which case $m^*/m = 1/Z$. Therefore, $Z \rightarrow 0$, which implies a vanishing density of

states, corresponds to a diverging m^* . (This is not the only mechanism that can lead to a diverging m^* [25, 26], but it is one possibility.) The electrical resistivity also shows critical behavior at this transition [15], but presumably this would be masked by non-critical scattering mechanisms as discussed above.

Finally, we come back to the point that there are multiple dynamical exponents, one of which is $z_c = 1$, which describes the density dynamics [19]. If this is the dominant dynamical exponent for the observables discussed above, then $\nu = y_h = z = 1$, and both the specific heat coefficient and the density of states at $T = 0$ will vanish linearly as a function of r [27].

We now summarize and discuss our results. Our starting point is the interpretation of the data presented in Ref. 9 as put forward in that reference, namely, that the thermopower in a $2-d$ electron system displays critical behavior at a quantum phase transition that is not sensitive to weak disorder and would be present in a truly clean system as well. We have employed simple scaling arguments to make predictions about other observables that allow to confirm or refute these notions. Our scaling analysis shows that the thermopower experiment yields the product of the correlation length exponent ν and the dynamical exponent z . It predicts that the specific heat coefficient will also display scaling behavior, which will allow for a separate determination of ν . If ν were found to violate the Harris criterion, this would definitively rule out a disorder-dominated nature of the transition. The unknown order-parameter density is predicted to obey scaling characterized by a third independent exponent y_h . These predictions are all very general, and hinge only on the assumption that the observed behavior of the thermopower does indeed reflect a true quantum phase transition. In addition, we have given a criterion to check whether this transition is a manifestation of the Fermi-liquid-to-non-Fermi-liquid transition discussed in Ref. 15, which is caused by strong correlations in the electron fluid. In that case, the density of states at the Fermi level is predicted to be critical, and its scaling behavior will be the same as that of the specific heat coefficient, except for a different functional form of the scaling function. In particular, at $T = 0$ both quantities will vanish as $(n - n_c)^{2\nu-1}$, with ν the correlation length exponent, as the transition is approached from high densities.

This work was supported by the National Science Foundation under Grant Nos. DMR-09-29966 and DMR-09-01907.

spin-orbit scattering, that displays a normal metal phase in $d = 2$ [28]. However, this is not believed to be relevant for the systems under consideration here, see Ref. 7. We also note that by “metal-insulator transition” we mean a transition involving a normal-metal phase, as opposed to the well-established superconductor-insulator transition in thin metal films [29], and that the Quantum Hall Effect is caused by delocalized surface states and does not constitute a bulk phase transition [30].

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implies that there is a time scale for the charge or density dynamics that is characterized by $z_c = 1$, whereas the critical dynamical exponent is in general $z \neq 1$. Which z enters in any given context cannot be determined from scaling considerations alone.

- [20] For scaling plots of this sort one has to keep in mind that the dynamical critical scaling can be expected to hold only at temperatures small compared to the Fermi temperature, which sets the microscopic energy scale. Since the Fermi temperature in the samples in question is only on the order of a few Kelvin [7], the temperatures of a few hundred mK achieved in Ref. 9 may not suffice to demonstrate the characteristic T/r scaling behavior.
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