Event-by-Event Anisotropic Flow in Heavy-ion Collisions from Combined Yang-Mills and Viscous Fluid Dynamics

Charles Gale, Sangyong Jeon, Björn Schenke, Prithwish Tribedy, and Raju Venugopalan

Phys. Rev. Lett. **110**, 012302 — Published 2 January 2013

DOI: 10.1103/PhysRevLett.110.012302

Event-by-event anisotropic flow in heavy-ion collisions from combined Yang-Mills and viscous fluid dynamics

Charles Gale,[1] Sangyong Jeon,[1] Björn Schenke,[2] Prithwish Tribedy,[3] and Raju Venugopalan[2]

[1] *Department of Physics, McGill University, 3600 University Street, Montreal, Quebec, H3A 2T8, Canada*
[2] *Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*
[3] *Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700064, India*

Anisotropic flow coefficients v_1-v_5 in heavy ion collisions are computed by combining a classical Yang-Mills description of the early time glasma flow with the subsequent relativistic viscous hydrodynamic evolution of matter through the quark-gluon plasma and hadron gas phases. The glasma dynamics, as realized in the IP-Glasma model, takes into account event-by-event geometric fluctuations in nucleon positions and intrinsic sub-nucleon scale color charge fluctuations; the pre-equilibrium flow of matter is then matched to the MUSIC algorithm describing viscous hydrodynamic flow and particle production at freeze-out. The IP-Glasma+MUSIC model describes well both transverse momentum dependent and integrated v_n data measured at the Large Hadron Collider (LHC) and the Relativistic Heavy Ion Collider (RHIC). The model also reproduces the event-by-event distributions of v_2, v_3 and v_4 measured by the ATLAS collaboration. The implications of our results for better understanding of the dynamics of the glasma as well as for the extraction of transport properties of the quark-gluon plasma are outlined.

Heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) uniquely allow for systematic exploration of the high temperature many-body dynamics of a non-Abelian quantum field theory. Particularly intriguing is the prospect of disentangling the non-equilibrium strongly correlated dynamics of the early time glasma regime from those of late stage nearly equilibrated quark-gluon plasma and hadron gas phases by measurements of anisotropic flow harmonics v_n at both RHIC [1, 2] and LHC [3–5].

An excellent candidate for providing initial conditions for systematic flow studies is the IP-Glasma model described in detail in Refs. [6, 7]. It combines the IP-Sat (Impact Parameter Saturation Model) model [8, 9] of high energy nucleon (and nuclear) wavefunctions with the classical Yang-Mills (CYM) dynamics of the glasma fields produced in a heavy-ion collision [10–13]. We note that the IP-Sat model provides a good description of small x HERA deeply inelastic scattering (DIS) data off protons and fixed target nuclear DIS data [14]. Prior implementation of the IP-Sat model in proton-proton and nucleus-nucleus collisions at the LHC using a $k_\perp$-factorized expression approximating CYM dynamics was shown to give good agreement with bulk features of data [15]. The upcoming p+Pb run at the LHC should provide further constraints on the dynamics of the IP-Glasma model, in particular the energy dependence of the saturation scale Q_s.

In this letter, we couple the IP-Glasma model of the classical early time evolution of boost-invariant configurations of gluon fields to a relativistic hydrodynamic description of the system, using the energy density and flow velocity in the transverse plane at the switching time $\tau_{\rm switch} \sim 1/Q_s$ as input [16]. The hydrodynamic evolution in each event is described by MUSIC [17–20], a 3+1 dimensional relativistic viscous hydrodynamic simulation [21] that uses the Kurganov-Tadmor algorithm [22]. While this matching of glasma dynamics to viscous hydrodynamics is a significant improvement relative to previously employed initial conditions for heavy ion collisions, early stage dynamics is not fully included. Most notably, the hydrodynamic viscous tensor $\Pi^{\mu\nu}$ is too large to be described self-consistently by a gradient expansion. Instabilities triggered by quantum fluctuations, and subsequent strong scattering of over-occupied fields, may lead to rapid quenching of $\Pi^{\mu\nu}$ to reasonable values justifying the use of viscous hydrodynamics already at early times. In this letter, we will assume such an efficient mechanism to be at work and set the initial value of $\Pi^{\mu\nu}$ to zero. We note that one could also choose the Navier-Stokes value for the initial $\Pi^{\mu\nu}$, however, it will be very anisotropic, similar to the value given by the calculated $T^{\mu\nu}_{\rm CYM}$. We leave a detailed study of the dependence on changes in the initial $\Pi^{\mu\nu}$ for future work [23].

Recent progress in computing early-time quantum fluctuations will help eliminate this systematic uncertainty [24–28].

When we switch from the CYM description to hydrodynamics we construct the fluid's initial energy momentum tensor $T^{\mu\nu}_{\rm fluid} = (\epsilon + \mathcal{P})u^\mu u^\nu - \mathcal{P}g^{\mu\nu} + \Pi^{\mu\nu}$ from the energy density in the fluid's rest frame ε, the flow velocity u^μ, and, using an equation of state, the local pressure $\mathcal{P}$ at each transverse position. ε and u^μ are obtained by solving $u_\mu T^{\mu\nu}_{\rm CYM} = \varepsilon u^\nu$, using the fact that u^μ is a time-like eigenvector of $T^{\mu\nu}_{\rm CYM}$ and satisfies $u^2 = 1$.

Other important details of our analysis are as follows. Unless otherwise noted, $\tau_{\rm switch} = 0.2\,{\rm fm}/c$. We employ the *s95p-PCE* equation of state, obtained from fits to lattice QCD results and a hadron resonance gas model [29], with partial chemical equilibrium (PCE) setting in below a temperature $T_{\rm PCE} = 150\,{\rm MeV}$. Kinetic freeze-out

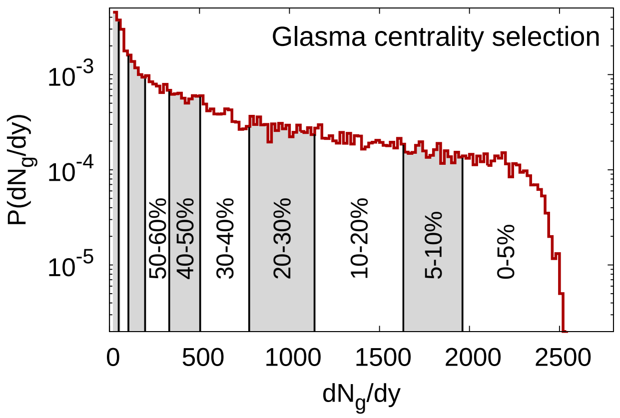

FIG. 1. (Color online) Gluon multiplicity distribution in the IP-Glasma model.

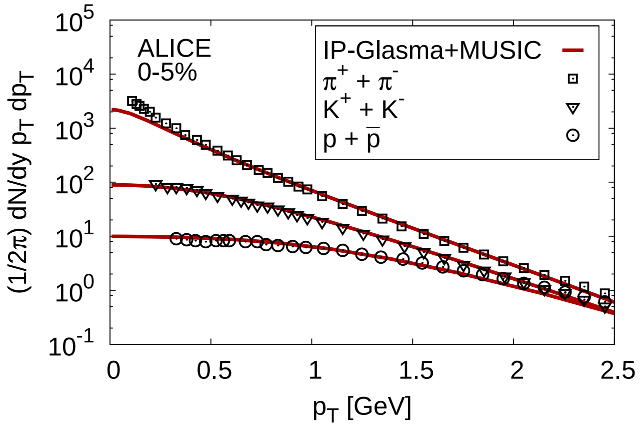

FIG. 2. (Color online) Identified particle transverse momentum spectra including all resonances up to 2 GeV compared to experimental data from the ALICE collaboration [33].

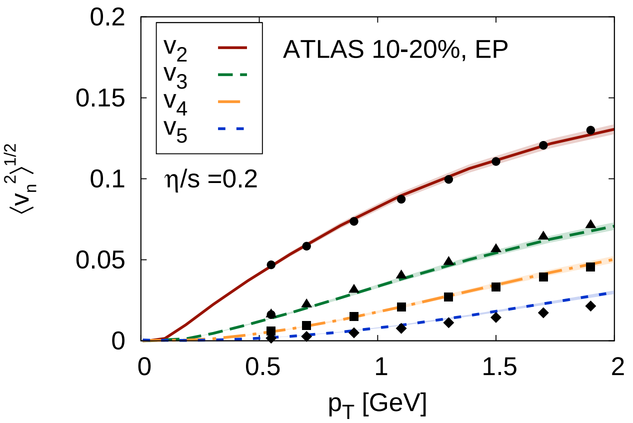

FIG. 3. (Color online) Root-mean-square anisotropic flow coefficients $\langle v_n^2\rangle^{1/2}$ as a function of transverse momentum, compared to experimental data by the ATLAS collaboration using the event plane (EP) method [4] (points). 200 events. Bands indicate statistical errors. Experimental error bars are smaller than the size of the points.

occurs at $T_{\rm FO} = 120\,{\rm MeV}$. At this temperature, we implement the Cooper-Frye prescription [30] for computing particle spectra. Unless otherwise noted, shown results include decays from resonances of masses up to 1.3 GeV.

A novel feature of our study is the determination of centrality classes using the multiplicity distribution of gluons much alike the procedure followed by the heavy ion experiments [31]. The gluon multiplicity distribution is shown in Fig. 1. Centrality classes are determined from the fraction of the integral over this distribution, beginning with integrating from the right. As a consequence of implementing this centrality selection, we properly account for impact parameter and multiplicity fluctuations.

Because entropy is produced during the viscous hydrodynamic evolution, we need to adjust the normalization of the initial energy density commensurately to describe the final particle spectra [32]. The obtained p_T-spectra of pions, kaons, and protons are shown for 0-5% central collisions at $\sqrt{s} = 2.76\,{\rm TeV/nucleon}$, using the shear viscosity to entropy density ratio $\eta/s = 0.2$, in Fig. 2, and compared to data from ALICE [33]. The results are for averages over only 20 events in this case, but statistical errors are smaller than the line width for the spectra. Overall, the agreement with experimental data is good. However, soft pions at $p_T < 300\,{\rm MeV}$ are underestimated.

We determine v_1 to v_5 in every event by first determining the *exact* event plane [34][35]

$$\psi_n = \frac{1}{n}\arctan\frac{\langle\sin(n\phi)\rangle}{\langle\cos(n\phi)\rangle}\,, \tag{1}$$

and then computing

$$\begin{aligned} v_n(p_T) &= \langle\cos(n(\phi-\psi_n))\rangle \\ &\equiv \frac{\int d\phi f(p_\perp,\phi)\cos(n(\phi-\psi_n))}{\int d\phi f(p_\perp,\phi)}\,, \end{aligned} \tag{2}$$

where $f(p_\perp,\phi)$ are the thermal distribution functions obtained in the Cooper-Frye approach (with additional contributions from resonance decays).

We first present the root-mean-square (rms) $v_n(p_T)$ for $10-20\%$ central collisions and compare to experimental data from the ATLAS collaboration [4] in Fig. 3. Agreement for v_2-v_5 is excellent. Note that the v_n from the experimental event-plane method used by ATLAS agree well with the rms values [36]. We also find excellent agreement over the whole studied centrality range when comparing the p_T-integrated rms v_2, v_3 and v_4 to the available $v_n\{2\}$ (obtained from two-particle correlations, corresponding to the rms values) from the ALICE collaboration [3], as shown in Fig. 4.

We studied the effect of initial transverse flow included in our framework by also computing $v_n(p_T)$ with u^μ set to zero at time $\tau_{\rm switch}$. The effect on hadron anisotropic

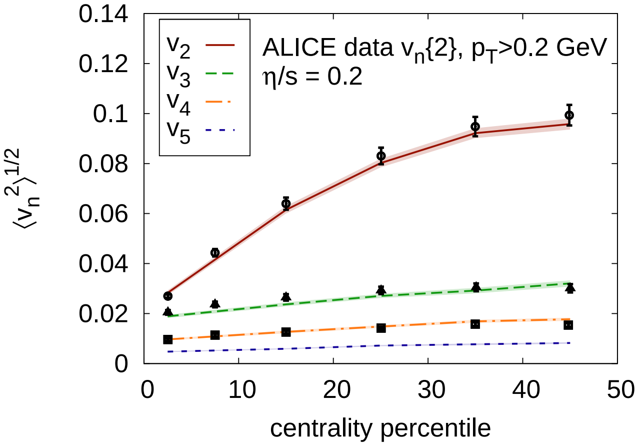

FIG. 4. (Color online) Root-mean-square anisotropic flow coefficients $\langle v_n^2\rangle^{1/2}$, computed as a function of centrality, compared to experimental data of $v_n\{2\}$, $n \in \{2, 3, 4\}$, by the ALICE collaboration [3] (points). Results are for 200 events per centrality with bands indicating statistical errors.

flow turns out to be extremely weak - results agree within statistical errors. Because photons are produced early on in the collision, we expect a greater effect on photon anisotropic flow; this will be examined in a subsequent work. We emphasize that pre-equilibrium dynamics that is not fully accounted for may still influence the amount of initial transverse flow.

The effect of changing the switching time from $\tau_{\rm switch} = 0.2\,{\rm fm}/c$ to $\tau_{\rm switch} = 0.4\,{\rm fm}/c$ is shown in Fig. 5. Results agree within statistical errors, but tend to be slightly lower for the later switching time. The nonlinear interactions of classical fields become weaker as the system expands and therefore Yang-Mills dynamics is less effective than hydrodynamics in building up flow at late times. Yet it is reassuring that there is a window in time where both descriptions produce equivalent results.

Because a constant η/s is at best a rough effective measure of the evolving shear viscosity to entropy density ratio, we present results for a parametrized temperature dependent η/s, following [37]. We use the same parametrization (HH-HQ) as in [37, 38] with a minimum of $(\eta/s)(T) = 0.08$ at $T = 180\,{\rm MeV}$, approximately at the cross-over from QGP to hadron gas in the used equation of state. The result, compared to $\eta/s = 0.2$ is shown for $20-30\%$ central collisions in Fig. 6. The results are indistinguishable when studying just one collision energy. The insensitivity of our results to two very different functional forms may suggest that the development of flow is strongly affected at intermediate times when η/s is very small. Also, since second order viscous hydrodynamics breaks down when $\Pi^{\mu\nu}$ is comparable to the ideal terms, our framework may be inadequate for too large values of η/s.

We compare results for top RHIC energies, obtained using a constant $\eta/s = 0.12$, which is about 40 % smaller than the value at LHC, to experimental data from STAR [39] and PHENIX [1] in Fig. 7. The data is well described given the systematic uncertainties in both the experimental and theoretical results [40]. A larger effective η/s at LHC than at RHIC was also found in [41]. The temperature dependent $(\eta/s)(T)$ used to describe LHC data works well for low-p_T RHIC data, but underestimates $v_2(p_T)$ and $v_3(p_T)$ for $p_T > 1\,{\rm GeV}$. The parametrizations of $(\eta/s)(T)$ in the literature are not definitive and significant improvements are necessary. Our studies suggest great potential for extracting the temperature dependent properties of QCD transport coefficients by performing complementary experiments extracting flow harmonics at both RHIC and LHC.

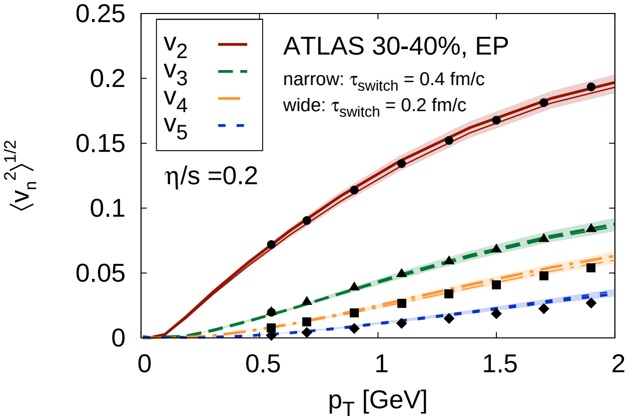

FIG. 5. (Color online) Comparison of $v_n(p_T)$ using two different switching times $\tau_{\rm switch} = 0.2\,{\rm fm}/c$ (wide), and $0.4\,{\rm fm}/c$ (narrow). Experimental data by the ATLAS collaboration using the event-plane (EP) method [4] (points). Bands indicate statistical errors.

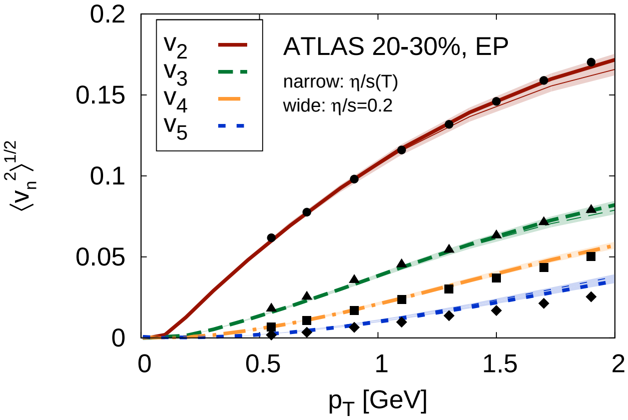

FIG. 6. (Color online) Comparison of $v_n(p_T)$ using constant $\eta/s = 0.2$ and a temperature dependent $(\eta/s)(T)$ as parametrized in [37]. Experimental data by the ATLAS collaboration using the event-plane (EP) method [4] (points). Bands indicate statistical errors.

In Fig. 8 we present results for $v_1(p_T)$ compared to experimental data from ALICE [42], extracted in [44], and

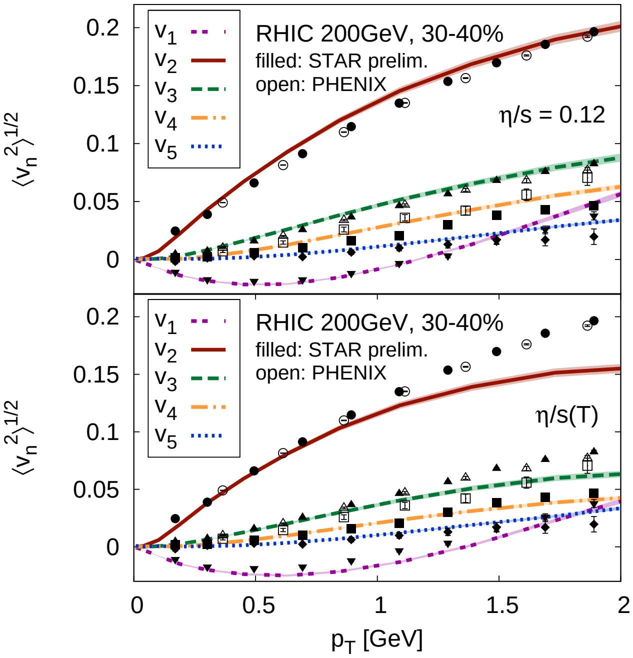

FIG. 7. (Color online) Comparison of $v_n(p_T)$ at RHIC using constant $\eta/s = 0.12$ and a temperature dependent $(\eta/s)(T)$ as parametrized in [37]. Experimental data by the PHENIX [1] (open symbols) and STAR [39] (preliminary, filled symbols) collaborations. Bands indicate statistical errors.

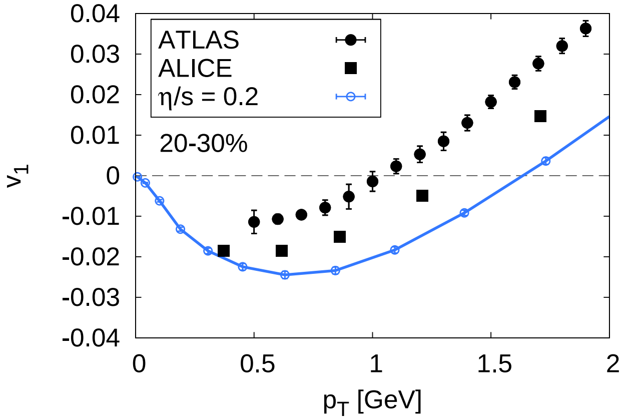

FIG. 8. (Color online) $v_1(p_T)$ compared to experimental data from the ALICE [42] and ATLAS [43] collaborations.

from ATLAS [43]. $v_1(p_T)$ cannot be positive definite because momentum conservation requires $\langle v_1(p_T)p_T\rangle = 0$. There is a disagreement between the experimental results (discussed in [43]) and between theory and experiment at LHC. On the other hand, $v_1(p_T)$ at RHIC is very well reproduced (see Fig. 7). One possible explanation for the data crossing $v_1(p_T) = 0$ at a lower p_T than the calculation at LHC could be the lower pion p_T-spectrum at very low p_T in the calculation – see Fig. 2. However, this is not necessarily the only explanation. In fact, for RHIC energies, calculated pion spectra also underestimate the data for $p_T < 300\,\text{MeV}$ but $v_1(p_T)$ is well reproduced.

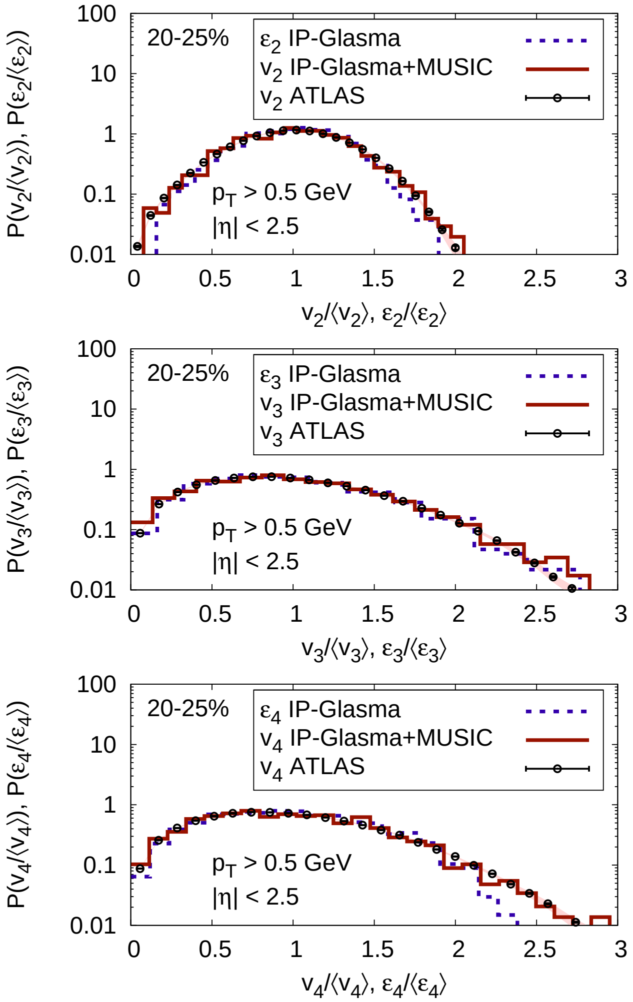

FIG. 9. (Color online) Scaled distributions of v_2, v_3, and v_4 (from top to bottom) compared to experimental data from the ATLAS collaboration [36, 45]. 1300 events. Bands are systematic experimental errors.

We present event-by-event distributions of v_2, v_3, and v_4 compared to results from the ATLAS collaboration [36, 45] in Fig. 9. We chose 20-25% central events because eccentricity distributions from neither MC-Glauber nor MC-KLN models agree with the experimental data in this bin [36]. To compare data with the distribution of initial eccentricities [46] from the IP-Glasma model and the final v_n distributions after hydrodynamic evolution, we scaled the distributions by their respective mean value. We find that the initial eccentricity distributions are a good approximation to the distribution of experimental v_n. Only for v_4 (and less so for v_2) the large v_n end of the experimental distribution is better described by the hydrodynamic v_n distribution than the ε_n distribution. This can be explained by non-linear mode coupling becoming important for large values of v_2 and v_4 [47].

In summary, we have shown that the IP-

Glasma+MUSIC model gives very good agreement to multiplicity and flow distributions at RHIC and LHC. By including properly sub-nucleon scale color charge fluctuations and their resulting early time CYM dynamics, this model significantly extends previous studies in the literature [20, 41, 48–52]. Omitted in all studies including ours is the stated dynamics of instabilities and strong scattering in over-occupied classical fields that can drive the system to isotropy and generate substantial flow well prior to thermalization. Ongoing work in this direction is promising and can be incorporated seamlessly in our framework. In addition, there are uncertainties in the equation of state, and in chemical and thermal freeze-out assumptions and parameters. We have not attempted a fine tuning of parameters – the sensitivity of our results to various parameters will be addressed in a follow up work. Despite these caveats, the successful description of a wide range of data in our model provides a framework to nail down key aspects of the complex dynamics of heavy ion collisions.

Acknowledgments BPS and RV are supported under DOE Contract No.DE-AC02-98CH10886 and acknowledge additional support from a BNL Lab Directed Research and Development grant. CG and SJ are supported by the Natural Sciences and Engineering Research Council of Canada. We gratefully acknowledge computer time on the Guillimin cluster at the CLUMEQ HPC centre, a part of Compute Canada HPC facilities. BPS gratefully acknowledges a Goldhaber Distinguished Fellowship from Brookhaven Science Associates. BPS thanks G. Denicol and J. Jia for helpful discussions.

---

[1] A. Adare *et al.* (PHENIX Collaboration), Phys.Rev.Lett. **107**, 252301 (2011).

[2] P. Sorensen (STAR Collaboration), J.Phys.G **G38**, 124029 (2011).

[3] K. Aamodt *et al.* (ALICE Collaboration), Phys.Rev.Lett. **107**, 032301 (2011).

[4] G. Aad *et al.* (ATLAS Collaboration), Phys.Rev. **C86**, 014907 (2012).

[5] CMS Collaboration CMS-PAS-HIN-11-005, 925214, (2011).

[6] B. Schenke, P. Tribedy, and R. Venugopalan, Phys. Rev. Lett. **108**, 252301 (2012).

[7] B. Schenke, P. Tribedy, and R. Venugopalan, Phys. Rev. **C86**, 034908 (2012).

[8] J. Bartels, K. J. Golec-Biernat, and H. Kowalski, Phys. Rev. **D66**, 014001 (2002).

[9] H. Kowalski and D. Teaney, Phys. Rev. **D68**, 114005 (2003).

[10] A. Kovner, L. D. McLerran, and H. Weigert, Phys. Rev. **D52**, 6231 (1995).

[11] Y. V. Kovchegov and D. H. Rischke, Phys. Rev. **C56**, 1084 (1997).

[12] A. Krasnitz and R. Venugopalan, Nucl. Phys. **B557**, 237 (1999); Phys. Rev. Lett. **84**, 4309 (2000); **86**, 1717 (2001).

[13] T. Lappi, Phys. Rev. **C67**, 054903 (2003).

[14] H. Kowalski, L. Motyka, and G. Watt, Phys. Rev. **D74**, 074016 (2006); H. Kowalski, T. Lappi, and R. Venugopalan, Phys. Rev. Lett. **100**, 022303 (2008).

[15] P. Tribedy and R. Venugopalan, Nucl. Phys. **A850**, 136 (2011); Phys.Lett. **B710**, 125 (2012).

[16] $Q_s(\mathbf{x}_\perp)$ is determined in the IP-Glasma model and $\tau_{\rm switch}$ should be chosen such that the order of magnitude relation $\tau_{\rm switch} \sim 1/Q_s$ holds for some Q_s averaged over the transverse plane.

[17] B. Schenke, S. Jeon, and C. Gale, Phys. Rev. **C82**, 014903 (2010).

[18] B. Schenke, S. Jeon, and C. Gale, Phys. Rev. Lett. **106**, 042301 (2011).

[19] B. Schenke, S. Jeon, and C. Gale, Phys. Lett. **B702**, 59 (2011).

[20] B. Schenke, S. Jeon, and C. Gale, Phys. Rev. **C85**, 024901 (2011).

[21] W. Israel and J. M. Stewart, Ann. Phys. **118**, 341 (1979).

[22] A. Kurganov and E. Tadmor, Journal of Computational Physics **160**, 214 (2000).

[23] Results for $v_2(p_T)$ using zero or Navier-Stokes $\Pi^{\mu\nu}$ were found to be indistinguishable when adjusting the initial entropy density to reproduce particle spectra in [53].

[24] K. Dusling, F. Gelis, and R. Venugopalan, Nucl.Phys. **A872**, 161 (2011).

[25] K. Dusling, T. Epelbaum, F. Gelis, and R. Venugopalan, Nucl.Phys. **A850**, 69 (2011).

[26] T. Epelbaum and F. Gelis, Nucl.Phys. **A872**, 210 (2011).

[27] K. Dusling, T. Epelbaum, F. Gelis, and R. Venugopalan, (2012), arXiv:1206.3336 [hep-ph].

[28] J. Berges and S. Schlichting, (2012), arXiv:1209.0817 [hep-ph].

[29] P. Huovinen and P. Petreczky, Nucl. Phys. **A837**, 26 (2010).

[30] F. Cooper and G. Frye, Phys. Rev. **D10**, 186 (1974).

[31] Strictly, this requires computing 10^4 complete events to determine charged particle distributions. Because this is computationally very demanding, we instead determine centrality from the distribution of produced gluons. We showed previously this gives good agreement with the uncorrected distribution in the STAR experiment [7].

[32] A more detailed study of the effect of viscosity on particle production relative to that in the glasma stage will be reported separately.

[33] B. Abelev *et al.* (ALICE Collaboration), (2012), arXiv:1208.1974 [hep-ex].

[34] We emphasize we are not using the experimental event-plane method. We compute a smooth average over particle distributions and there is no need for a resolution correction factor.

[35] We have checked that results are affected by less than 1% when using a p_T-dependent event plane definition, appropriate when comparing to two-particle correlation measurements.

[36] J. Jia (ATLAS Collaboration), (2012), arXiv:1209.4232 [nucl-ex].

[37] H. Niemi, G. S. Denicol, P. Huovinen, E. Molnar, and D. H. Rischke, Phys.Rev.Lett. **106**, 212302 (2011).

[38] H. Niemi, G. Denicol, P. Huovinen, E. Molnar, and D. Rischke, Phys.Rev. **C86**, 014909 (2012).

[39] Y. Pandit [for the STAR collaboration], Quark Matter 2012, (2012).

[40] For example, it is not clear that the experimental data

correpond exactly to the rms value in this case.

[41] H. Song, S. A. Bass, and U. Heinz, Phys.Rev. **C83**, 054912 (2011).

[42] K. Aamodt *et al.* (ALICE Collaboration), Phys.Lett. **B708**, 249 (2012).

[43] J. Jia (ATLAS Collaboration), (2012), arXiv:1208.1874 [nucl-ex].

[44] E. Retinskaya, M. Luzum, and J.-Y. Ollitrault, Phys.Rev.Lett. **108**, 252302 (2012).

[45] ATLAS Collaboration ATLAS-CONF-2012-114, https://cdsweb.cern.ch/record/1472935 (2012).

[46] We define $\varepsilon_n = \sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}/\langle r^n\rangle$, where $\langle\cdot\rangle$ is the energy density weighted average.

[47] D. Teaney and L. Yan, (2012), arXiv:1206.1905 [nucl-th].

[48] M. Luzum and P. Romatschke, Phys. Rev. **C78**, 034915 (2008).

[49] J. Takahashi *et al.*, Phys. Rev. Lett. **103**, 242301 (2009).

[50] H. Holopainen, H. Niemi, and K. J. Eskola, Phys.Rev. **C83**, 034901 (2011).

[51] Z. Qiu and U. W. Heinz, Phys.Rev. **C84**, 024911 (2011).

[52] P. Bozek, Phys.Rev. **C85**, 034901 (2012).

[53] C. Shen, U. Heinz, P. Huovinen, and H. Song, Phys.Rev. **C84**, 044903 (2011).