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CMB Power Asymmetry from Non-Gaussian Modulation

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Non-Gaussianity in the inflationary perturbations can couple observable scales to modes of much longer wavelength (superhorizon even), leaving as signature a large-angle modulation of the observed CMB power spectrum. This provides an alternative origin for a power asymmetry which is otherwise often ascribed to a breaking of statistical isotropy. The non-Gaussian modulation effect can be significant even for typical $\sim 10^{-5}$ perturbations, while respecting current constraints on non-Gaussianity, if the squeezed limit of the bispectrum is sufficiently infrared-divergent. Just such a strongly infrared-divergent bispectrum has been claimed for inflation models with a non-Bunch-Davies initial state, for instance. Upper limits on the observed CMB power asymmetry place stringent constraints on the duration of inflation in such models.

Large-scale features in the cosmic microwave background (CMB) offer interesting avenues for testing phenomena that occurred at very early times in the Universe's history. While most inflationary models predict approximately scale-invariant, Gaussian fluctuations, some amount of non-Gaussianity is invariably generated [1]. In this paper, we show that even for an almost scale-independent power spectrum of curvature perturbations (i.e. $\sim 10^{-5}$ in amplitude on all scales), primordial non-Gaussianity can lead to interesting, significant effects on the CMB, in particular a large angular scale modulation of the small scale power spectrum. This is achieved without violating stringent observational bounds on non-Gaussianity in the sub-horizon perturbations.

There are some observational indications for a dipolar modulation of the CMB power spectrum [2–4]. Such an anisotropic CMB sky can be described by [5]

$$\hat{\Theta}(\hat{\mathbf{n}}) = [1 + f(\hat{\mathbf{n}})]\Theta(\hat{\mathbf{n}}), \tag{1}$$

where $\hat{\Theta}(\hat{\mathbf{n}})$ is the observed, anisotropic temperature fluctuation $\delta T/T$, while $\Theta(\hat{\mathbf{n}})$ is a statistically isotropic temperature field, and $f(\hat{\mathbf{n}})$ is the modulating function. Note that while $\Theta(\hat{\mathbf{n}})$ is statistically isotropic and is thus (statistically) invariant under a rotation of the coordinate system, $f(\hat{\mathbf{n}})$ depends on fixed directions on the sky.

The lowest order modulation is a dipole, as any monopole of $f(\hat{\mathbf{n}})$ is absorbed in the angle-averaged CMB power spectrum. The most recent analysis of [2] obtain a statistically significant dipolar asymmetry, while the WMAP team do not confirm this finding [6]. Hanson et al. [7] find that beam asymmetries provide an explanation for the non-zero quadrupolar asymmetry. Several scenarios have been proposed in the literature to explain possible power asymmetries: [8, 9] considered remnants from the pre-inflationary phase, [10–12] proposed a single large-scale curvature perturbation, while [13] studied a spacelike vector field. These scenarios either involve a change in the inflation field $\Delta\varphi \sim A$ across the present horizon, many orders of magnitude larger than expected from the amplitude of fluctuations, or a breaking of the symmetries of the background.

Alternatively, one can interpret a large-scale modulation of the CMB temperature fluctuations as due to a non-Gaussian coupling between long and short wavemodes [14, 15]. The power spectrum of the Bardeen potential ϕ on short scales is modulated by the presence of long modes if the fluctuations are non-Gaussian. We can Taylor expand the power spectrum of short modes (k) in the presence of long modes ($k_\ell \ll k$) [1]:

$$P_\phi^{\rm mod}(\mathbf{k}) = P_\phi(k)\left[1 + \int \frac{d^3k_\ell}{(2\pi)^3} G(\mathbf{k},\mathbf{k}_\ell)\phi(\mathbf{k}_\ell)\right]$$
$$G(\mathbf{k},\mathbf{k}_\ell) \equiv \frac{B_\phi(|\mathbf{k}+\mathbf{k}_\ell/2|, |-\mathbf{k}+\mathbf{k}_\ell/2|, |-\mathbf{k}_\ell|)}{P_\phi(k_\ell)P_\phi(k)} \tag{2}$$

where $P_\phi^{\rm mod}(\mathbf{k})$ is the modulated power spectrum, and B_ϕ is the bispectrum [33]. $G(\mathbf{k},\mathbf{k}_\ell)$ can be understood as a scale- and orientation-dependent generalization of the dimensionless nonlinearity parameter $f_{\rm NL}$.

The scenario we are considering is not statistically anisotropic in any fundamental sense; rather, the observed power spectrum $P_\phi^{\rm mod}(\mathbf{k})$ depends on the direction of $\mathbf{k}$ because the long modes in our particular realization of the Universe statistically pick out certain directions $\mathbf{k}_\ell$, and non-Gaussianity couples these long modes to the observable ones. Also, this effect doesn't require having a large amplitude long wave-mode $\phi(\mathbf{k}_\ell)$; a large kernel G in the squeezed limit is sufficient.

Observational bounds on primordial non-Gaussianity are rather tight [16], which might lead one to expect the proposed effect must be small. The key point is that current observational constraints come from modes where both $\mathbf{k}_\ell$ and $\mathbf{k}$ are within our horizon. This is however not necessary for Eq. (2) to apply, allowing even superhorizon modes $\mathbf{k}_\ell$ which we cannot directly measure to have an impact on observable modes $\mathbf{k}$ in the form of an anisotropic modulation. Two conditions should be met for this effect to be interesting: 1. the kernel G should be anisotropic, i.e. a non-trivial function of $\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}_\ell$; 2. G has to grow in the squeezed limit, i.e. scale like k/k_ℓ to some positive power. Existing constraints effectively bound G

only for moderate ratios of k/k_ℓ, while the super-horizon modulation effect is sensitive to larger ratios. We will interpret claims of power asymmetries in the literature as upper limits, and use them to constrain models with such a strong coupling between short and long modes.

Power asymmetry: The fluctuations of a statistically isotropic Gaussian field $\Theta(\hat{\mathbf{n}})$ are specified through the spherical harmonic coefficients, $\langle \Theta_{lm}\Theta^*_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}C_l$. Adopting the notation of [17], the Θ_{lm} are related to the Bardeen potential perturbations $\phi(\mathbf{k})$ via

$$\Theta_{lm} = 4\pi \int \frac{d^3k}{(2\pi)^3}(-i)^l \phi(\mathbf{k})\Delta_l(k) Y^*_{lm}(\hat{\mathbf{k}}), \qquad (3)$$

where $\Delta_l(k)$ is the photon temperature transfer function. The power spectrum of the Gaussian temperature fluctuations is then given by,

$$C(l) \,=\frac{2}{\pi}\int k^2 dk\, P_\phi(k)|\Delta_l(k)|^2. \qquad (4)$$

On the other hand, the spherical harmonic coefficients of the modulated field $\hat{\Theta}$ [Eq. (1)] are

$$\hat{\Theta}_{lm} - \Theta_{lm} = \sum_{LM,l',m'} \Theta_{l'm'} f_{LM} \int d^2\Omega\, Y^*_{lm} Y_{LM} Y_{l'm'},$$

where we have expressed $f(\hat{\mathbf{n}})$ in terms of its multipole moments (with respect to a fixed coordinate system). The integral over three spherical harmonics can be written in terms of Wigner 3-j symbols, leading at linear order in f_{LM} to

$$\begin{aligned}\langle \hat{\Theta}_{lm}\hat{\Theta}^*_{l'm'}\rangle &= \delta_{ll'}\delta_{mm'}C_l + \sum_{LM} f_{LM}\mathcal{G}^{ll'L}_{-mm'M}\left[C_{l'} + C_l\right]\\ \mathcal{G}^{ll'L}_{-mm'M} &= (-1)^m\sqrt{\frac{(2l+1)(2l'+1)(2L+1)}{4\pi}}\\ &\quad\times\begin{pmatrix} l & l' & L\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l & l' & L\\ -m & m' & M\end{pmatrix}.\end{aligned} \qquad (5)$$

The 3-j symbols entail that $l+l'+L$ even, $m'-m+M=0$, and that $|l-l'| \leq L \leq l+l'$. The latter condition is particularly relevant since we are interested in the case where $L=1$. Eq. (5) gives the covariance matrix of $\hat{\Theta}$ in multipole space in terms of the (fixed) multipole moments f_{LM} and the statistics of Θ. As expected, the covariance is not diagonal, but it is very close to diagonal for $l, l' \gg L$, i.e. it is non-zero only if $|l-l'| \leq L$.

Non-Gaussianity: We assume that there is some general non-Gaussianity described to leading order by a bispectrum B_ϕ. We are interested in the limit $k_\ell \ll H_0 \lesssim k$, where H_0 is the Hubble scale today. Following Eq. (2), we expect that the presence of long-wavelength modes together with the mode-coupling induced by non-Gaussianity lead to a breaking of statistical isotropy through the preferred direction $\mathbf{k}_\ell$. Consequently, we now calculate the covariance of the temperature field given Eq. (2). Multiplying Eq. (3), with $\Theta_{l'm'}$, and integrating over one of the momenta leads to

$$\begin{aligned}\langle \Theta_{lm}\Theta^*_{l'm'}\rangle &\simeq \delta_{ll'}\delta_{mm'}C_l\\ &+ (4\pi)^2\int\frac{d^3k_\ell}{(2\pi)^3}\phi(\mathbf{k}_\ell)\int\frac{d^3k}{(2\pi)^3}\left[\Delta_l(k)\Delta^*_{l'}(k)\right]\\ &\times Y^*_{lm}(\hat{k})Y_{l'm'}(\hat{k})G(\mathbf{k},\mathbf{k}_\ell)P_\phi(k),\end{aligned} \qquad (6)$$

where we set $|\mathbf{k}-\mathbf{k}_\ell| \simeq k$ in the squeezed-limit approximation (corrections scale as k_ℓ/k and higher). We obtain

$$\begin{aligned}\langle \Theta_{lm}\Theta^*_{l'm'}\rangle &= \delta_{ll'}\delta_{mm'}C_l\\ &+ \int\frac{k_\ell^2 dk_\ell}{(2\pi)^3}\sum_{LM}\mathcal{G}^{ll'L}_{-mm'M}\mathcal{C}_{ll'}(k_\ell)\phi_{LM}(k_\ell),\end{aligned} \qquad (7)$$

where we have defined

$$\begin{aligned}\mathcal{C}_{ll'}(k_\ell) &= \frac{1}{\pi}\int k^2 dk\left[\Delta_l(k)\Delta^*_{l'}(k) + \Delta^*_l(k)\Delta_{l'}(k)\right]\\ &\qquad\times P_\phi(k)G_L(k,k_\ell)\\ G(\mathbf{k},\mathbf{k}_\ell) &= \sum_{LM} G_L(k,k_\ell)Y^*_{LM}(\hat{k}_\ell)Y_{LM}(\hat{k}), \qquad (8)\\ \phi_{LM}(k_\ell) &= \int d\Omega_{k_\ell}\,\phi(\mathbf{k}_\ell)Y^*_{LM}(\hat{k}_\ell), \qquad (9)\end{aligned}$$

using the fact that the kernel G only depends on the angle between $\mathbf{k}$ and $\mathbf{k}_\ell$. Comparing with Eq. (4), we see that $\mathcal{C}_{ll}(k_\ell)$ is equal to the temperature power spectrum obtained when replacing $P_\phi(k) \to G_L(k,k_\ell)P_\phi(k)$, i.e. with a different initial power spectrum of curvature fluctuations. Thus, apart from the fact that the non-Gaussian covariance involves $\mathcal{C}_{ll'}$, instead of $\mathcal{C}_{ll} + \mathcal{C}_{l'l'}$, it is identical in structure to the covariance obtained for the anisotropic field Eq. (5) [34]. The fractional difference between $\mathcal{C}_{ll'}$ and $\mathcal{C}_{ll}, \mathcal{C}_{l'l'}$ is of order $L/l \ll 1$. We will thus approximate $\mathcal{C}_{ll'}$ in Eq. (7) with $(\mathcal{C}_{ll} + \mathcal{C}_{l'l'})/2$.

We conclude that if $G_L(k,k_\ell)$ is significant in the limit $k_\ell/k \to 0$ for some $L > 0$, the temperature fluctuations of the CMB *appear as if they experience a (large-angle) modulation of multipole order L.* In particular, this necessitates an anisotropic coupling of long- and short-wavelength modes. We now calculate the amplitude of this modulation. For scale-free bispectrum shapes, the kernel moments in the squeezed limit ($k_\ell \ll k$) can be written as

$$G_L(k,k_\ell) = g_L\left(\frac{k_\ell}{k}\right)^{\alpha_L}, \qquad (10)$$

where g_L is a constant and α_L gives the scaling in the squeezed limit. We also define the temperature power spectrum with a tilted spectral index $n_s \to n_s + \alpha$,

$$C_l(\alpha) = \frac{2}{\pi}\int k^2 dk\left(\frac{k}{k_0}\right)^\alpha P_\phi(k)|\Delta_l(k)|^2, \qquad (11)$$

where $k_0 = 0.05\,\mathrm{Mpc}^{-1}$ is the pivot scale used to normalize $P_\phi(k)$. By comparing Eq. (7) with Eq. (5), we can then read off the relation between the long-wavelength perturbations and the anisotropy coefficients f_{LM}, for a given l considered:

$$f_{LM} = \frac{1}{2}\int \frac{k_\ell^2 dk_\ell}{(2\pi)^3}\phi_{LM}(k_\ell) g_L \left(\frac{k_\ell}{k_0}\right)^{\alpha_L} \frac{C_l(-\alpha_L)}{C_l(0)}. \quad (12)$$

The multipole coefficients which give the amplitude and direction of the modulation are thus related to the *given realization* of the large-wavelength modes $\phi(\mathbf{k}_\ell)$. The last factor in Eq. (12) encodes the fact that in general this modulation is l-dependent; i.e. one is effectively adding a tilted CMB power spectrum $C_l(-\alpha_L)$ with angular modulation to the angle-averaged CMB power spectrum. While we cannot predict the direction of the power modulation, we can calculate the expectation value of the *amplitude*, defined as $A \equiv (\sum_{M=-L}^{L} |f_{LM}|^2)^{1/2}$. Since the f_{LM} are proportional to $\phi(\mathbf{k}_\ell)$, they are Gaussian-distributed complex numbers with mean zero. The amplitude A thus follows a χ distribution for $2L+1$ degrees of freedom, with an expectation value of

$$\langle A \rangle = \frac{g_L}{\sqrt{2\pi}} \frac{(L!)^2 4^L}{(2L)!} \left[\int_{k_{\ell,\min}}^{k_{\ell,\max}} \frac{k_\ell^2 dk_\ell}{(2\pi)^3} P_\phi(k_\ell) \left(\frac{k_\ell}{k_0}\right)^{2\alpha_L}\right]^{1/2} \times \frac{C_l(-\alpha_L)}{C_l(0)} \quad (13)$$

where we have used

$$\langle \phi_{LM}(k)\phi^*_{L'M'}(k')\rangle = \delta_{LL'}\delta_{MM'}(2\pi)^3 \frac{\delta_D(k-k')}{k^2} P_\phi(k).$$

If α_L is sufficiently negative, $\langle A\rangle$ diverges as we let the lower integration bound go to zero. Such a prediction for A can be ruled out to high significance by the data if the observational limit $A_{\rm lim} \ll \langle A\rangle$. In general, if $P_\phi(k_\ell) \propto k_\ell^{n_s-4}$, then $\alpha_L < (1-n_s)/2$ for some $L > 0$ in Eq. (10) is necessary for a significant large-scale asymmetry of the CMB. Fig. 1 shows quantitative results for the expected asymmetry $\langle A\rangle$ with $L = 1$, as function of CMB multipole l. We adopt the ansatz Eq. (10) with $g_1 = 1$, and integrate from $k_{\ell,\min}$ to $k_{\ell,\max} = 1/\eta_{\rm lss}$, where $\eta_{\rm lss}$ is the comoving distance to the last scattering surface (the latter choice is unimportant numerically). We choose three different sets of $(\alpha_1, k_{\ell,\min})$ and use CAMB [18] for the computation of $C_l(\alpha)$. Clearly, a significant amplitude of power asymmetry can be achieved with a range of parameters. The closer α is to zero, the smaller $k_{\ell,\min}$ needs to be to generate a given amount of asymmetry (at fixed g_1). On the other hand, a more negative α_1 leads to a stronger scale-dependence: the amplitude of the modulation approximately scales as $l^{-\alpha_1}$.

Inflationary bispectra which consist of symmetrized polynomials in the three momenta k_1, k_2, k_3 do not lead to a power asymmetry since the coupling of modes is isotropic ($G_L = 0$ for $L > 0$). However, these simple bispectra are often only obtained as separable approximations to the more complicated exact bispectra, which may themselves in fact lead to $G_L \neq 0$. Hence, it is crucial to consider the full, exact bispectrum when determining whether a given inflationary model leads to a power asymmetry. It is clear however that such a power asymmetry requires a violation of the standard consistency relation [1], at least on the scales of interest, as it contains no anisotropic coupling between long and short modes. A recent example is solid inflation [19], which predicts a quadrupolar coupling between long and short modes. But since in this model G does not grow in the squeezed limit, the resulting quadrupolar modulation of the power spectrum is small.

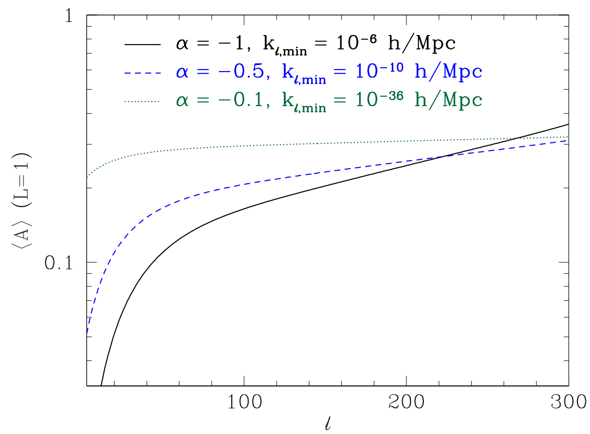

FIG. 1: Expected amplitude $\langle A\rangle$ of a dipole modulation ($L = 1$) as function of CMB multipole l, for three sets of values for $(\alpha_1, k_{\ell,\min})$ as indicated in the figure. We have used Eq. (10) with $g_1 = 1$. Predictions for different L can be obtained by multiplying with $g_L(L!)^2 4^L/2(2L)!$.

An example of a model that does produce a large-scale power modulation is the ekpyrotic ("case II") scenario of [20], which generates non-Gaussianities that in the squeezed limit lead to $\alpha_1 = -1-\epsilon$ and $\alpha_2 = -\epsilon$, where $\epsilon > 0$ is a red tilt. Thus, in this model one has divergent dipole and quadrupole modulations. Another case which has attracted recent interest is modifications to the initial state (non-Bunch-Davies) in single-field slow-roll inflation. These can lead to non-Gaussianity with $\alpha_1 = -1$ [21–27]. The squeezed bispectrum in the simple non-Bunch-Davies state considered in [26] reads

$$B_\phi(|\mathbf{k}+\mathbf{k}_\ell/2|, |-\mathbf{k}+\mathbf{k}_\ell/2|, |-\mathbf{k}_\ell|) = \mathcal{B}\, P_\phi(k_\ell)P_\phi(k)\frac{k}{k_\ell} \times \mathrm{Re}\left[\tilde{f}_1 \frac{1-e^{i(1+\mu)k_\ell/k_*}}{1+\mu} + \tilde{f}_2 \frac{1-e^{i(1-\mu)k_\ell/k_*}}{1-\mu}\right], \quad (14)$$

where $\mathrm{Re}(\tilde{f}_1+\tilde{f}_2)/2 \approx N_k$, and N_k is the occupation number of the momentum state k [35] , $\mu = -\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}_\ell$, $k_* \sim 1/|\eta_{\rm in}|$ is related to the conformal time at which the initial state is specified, and $k_\ell > k_*$ in order for this result to apply. $\mathcal{B}$ is a dimensionless constant equal to 4ϵ in the case studied in [26, 27], although it could take larger values in more general models. The kernel G_L scales as k/k_ℓ in this model, with $g_L \propto N_k$ for L even and $\alpha_L = -1$. For L odd, the modulation scales as $\tilde{f}_1 - \tilde{f}_2$ which is suppressed by k_ℓ/k. We use observational upper limits on the (primordial) quadrupole modulation amplitude $A \lesssim 0.1$ [7] to place constraints on $k_{\ell,\rm min} = k_*$. Numerical evaluation of Eqs. (13)–(14) leads to a 95% C.L. lower limit of [36]

$$k_* \gtrsim 2 \times 10^{-5}\, h\ \mathrm{Mpc}^{-1}\, N_k\, \mathcal{B}\,, \qquad (15)$$

implying no more than ~ 3 e-folds of inflation beyond our current horizon, for $N_k\mathcal{B} \sim 1$. This complements the bound on a non-Bunch-Davies initial state from backreaction arguments, which is sensitive to N_k but not $\mathcal{B}$.

Conclusions: Large-scale modulations of the CMB temperature fluctuations offer an interesting testing ground for the physics of the very early Universe. We have shown that certain types of primordial non-Gaussianity generically predict large power asymmetries in the CMB. The requisite non-Gaussianity can be thought of as an anisotropic, scale-dependent $f_{\rm NL}$ which grows in the squeezed limit.

Upper limits on such a modulation can put stringent constraints on this class of models, which includes scenarios with a non-Bunch-Davies initial state. One can roughly estimate the modulation amplitude from the dimensionless bispectrum amplitude $G(\mathbf{k}, \mathbf{k}_\ell)$ for the longest *observable* mode $k_\ell \sim H_0^{-1}$ through

$$\langle A \rangle \sim G\left(k, k_\ell = H_0^{-1}\right)\ 4 \times 10^{-5} \left(\frac{H_0}{k_{\ell,\rm min}}\right)^{-\alpha_L}\,, \qquad (16)$$

where $k_{\ell,\rm min}$ refers to the longest superhorizon mode responsible for the modulation, and α_L controls how fast G grows in the squeezed limit (Eq. 10). Conversely, observational hints of a power asymmetry provide motivation to further investigate such models. A convincing detection of a CMB power asymmetry, if interpreted in terms of this scenario, would open an observational window to scales much larger than the present horizon ($k_\ell \ll 1/\eta_{\rm lss}$), which are otherwise completely inaccessible to direct observation. This fact distinguishes this effect from a modulation of the temperature power spectrum by a horizon-scale mode.

We have shown that the power asymmetries are generally scale-dependent and increase towards smaller scales. Thus, unless one invokes a change in the shape or amplitude of non-Gaussianity on smaller scales, a non-detection of a similar power asymmetry in the large-scale structure [28, 29] puts further stringent constraints on models that produce such asymmetries. Furthermore, models with bispectra that peak more strongly in the squeezed limit than the local model will in fact generate a scale-dependent bias in large scale structure tracers [30–32] $\Delta b \propto k^{-n}$ with $n > 2$ [26, 27]. Observations of the large-scale structure will thus be of great importance in strengthening constraints on the possible non-Gaussian origins of a power asymmetry.

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[33] One can also derive the same by writing the non-Gaussian field as a convolution of Gaussian fields. See e.g. [31, 32].
[34] In Eq. (5), we have assumed a Gaussian covariance for the projected quantity Θ, while in Eq. (7) we are projecting

a non-Gaussian field, leading to this minor difference.
[35] This identification ignores interference terms. See [26].
[36] While [7] constrain the asymmetry up to $l = 1000$, we conservatively choose $l = 500$ here. We evaluate $P[A < 0.1] > 0.05$ using the χ distribution.