



This is the accepted manuscript made available via CHORUS, the article has been published as:

Crustal Entrainment and Pulsar Glitches

N. Chamel

Phys. Rev. Lett. **110**, 011101 — Published 2 January 2013

DOI: [10.1103/PhysRevLett.110.011101](https://doi.org/10.1103/PhysRevLett.110.011101)

Crustal Entrainment and Pulsar Glitches

N. Chamel

*Institut d'Astronomie et d'Astrophysique, CP-226,
Université Libre de Bruxelles, 1050 Brussels, Belgium*

Large pulsar frequency glitches are generally interpreted as sudden transfers of angular momentum between the neutron superfluid permeating the inner crust and the rest of the star. Despite the absence of viscous drag, the neutron superfluid is strongly coupled to the crust due to non-dissipative entrainment effects. These effects are shown to severely limit the maximum amount of angular momentum that can possibly be transferred during glitches. In particular, it is found that the glitches observed in the Vela pulsar require an additional reservoir of angular momentum.

Introduction. Since their fortuitous discovery by Jocelyn Bell and Anthony Hewish in 1967, more than 2000 pulsars have been found [1]. Their identification as neutron stars [2], the compact residues of type II supernova explosions predicted by Baade and Zwicky in 1933 [3], was definitively established the next year after the discoveries of pulsars in the Crab and Vela supernova remnants. Pulsars are among the most accurate clocks in the universe with periods ranging from 1.4 milliseconds up to several seconds. The delays associated with the spin-down of the star are at most of a few of milliseconds per year.

Nevertheless, irregularities have been detected in long-term pulsar timing observations [4]. In particular, some pulsars exhibit sudden increases in their rotational frequency Ω . These “glitches”, whose amplitude varies from $\Delta\Omega/\Omega \sim 10^{-9}$ up to $\sim 10^{-5}$ are generally followed by a relaxation over days to years and are sometimes accompanied by a sudden change of the spin-down rate from $|\Delta\dot{\Omega}/\dot{\Omega}| \sim 10^{-6}$ up to $\sim 10^{-2}$ (see e.g. Section 12.4 in [5]).

Soon after the first observations of glitches in the Vela and Crab pulsars, several scenarios were advanced [6]. In particular, glitches were thought to be the manifestations of starquakes, but this could not explain the frequent occurrence of Vela pulsar glitches [7]. A corequake model of Vela pulsar glitches was proposed [12], but the existence of a solid core later appeared to be highly speculative (see e.g. [2]). The long relaxation times following glitches provided strong evidence for the presence of superfluids in neutron star interiors and hinted at its possible role in the glitch mechanism itself [8, 9]. Neutron-star superfluidity had been predicted and studied even before the discovery of pulsars [10, 11]. Anderson and Itoh developed the fruitful idea that Vela like glitches are related to the dynamics of the neutron superfluid permeating the inner crust of neutron stars [13].

Vortex-mediated glitches. The neutron superfluid is weakly coupled to the crust by mutual friction forces and thus follows its spin-down via a radial motion of quantized vortices away from the rotation axis unless vortices are pinned to the crust. In this case, the superfluid can rotate more rapidly than the crust. The lag between the

superfluid and the crust induces a Magnus force acting on the vortices thereby producing a crustal stress. When the lag exceeds a critical threshold, the vortices are suddenly unpinned. As a result, the superfluid spins down and, by the conservation of the total angular momentum, the crust spins up leading to a glitch. This scenario found some support from laboratory experiments in superfluid helium [14, 15]. The good fit to the glitch data triggered further developments to explain the postglitch relaxation by the motion of vortices [16–18].

In the meantime, it was argued that the core (supposed to contain superfluid neutrons and type I superconducting protons) is unlikely to play any role in glitch events [19] (see also [20]). Due to non-dissipative entrainment effects similar to those arising in superfluid ^3He - ^4He mixtures, neutron superfluid vortices carry a fractional magnetic quantum flux. Electron scattering off the magnetic field of the vortices leads to a (dissipative) mutual friction force acting on the superfluid. As a result, the core superfluid is strongly coupled to the crust and to the charged particles, thus following the long-term spin-down of the star caused by electromagnetic radiation.

The confidence in the vortex-mediated glitch interpretation led to a new constraint on the structure of neutron stars hence also on the equation of state of dense matter [21]. The latest models like the “snowplow” model [22] can reproduce various observations of pulsar glitches. However, many fundamental aspects of these models remain poorly understood. For instance, the strength of vortex pinning, which is one of the crucial microscopic inputs, has been a controversial issue over the past years (see e.g. Section 8.3.5 of [5]). The mechanism that triggers the unpinning of vortices like superfluid instabilities [23] is also a matter of debate.

More importantly, these models ignore the non-dissipative entrainment effects in neutron-star crusts which have been shown to be very strong [24–29]. In this Letter, the impact of crustal entrainment on pulsar glitches is studied. Using the latest pulsar glitch data [4], it is shown that the neutron superfluid in the crust does not carry enough angular momentum to explain the Vela pulsar glitches.

Entrainment in neutron-star crusts. It has been re-

alized only recently that entrainment arises not only in the core of a neutron star but also in the crust because unbound neutrons can be elastically scattered by the crustal lattice for specific wave vectors, as determined by Bragg's law [24–29]. A neutron that is Bragg reflected cannot propagate and is therefore entrained by the crust. Unlike viscous drag, entrainment is non-dissipative. Even if a neutron is not Bragg reflected, its motion will still be affected by the crustal lattice. Neutron diffraction experiments are routinely performed to study crystal structures. The specificity of neutron-star crusts is that neutrons form a highly degenerate quantum liquid. Due to the Pauli exclusion principle, neutrons have different wave vectors and are therefore diffracted differently. Entrainment can be characterized by the density n_n^c of conduction neutrons, i.e. neutrons that are effectively “free”, or equivalently by an effective neutron mass $m_n^* = m_n n_n^f / n_n^c$ where m_n is the bare neutron mass and n_n^f the density of unbound neutrons. Neutron conduction has been systematically studied in all regions of the inner crust using a state-of-the-art crust model based on the band theory of solids [29]. Entrainment has been found to be very strong, especially in the intermediate region of the inner crust at average baryon densities $\bar{n} \sim 0.02 - 0.03 \text{ fm}^{-3}$ where $n_n^c \ll n_n^f$ or equivalently $m_n^* \gg m_n$.

Pulsar glitch constraint. Large pulsar glitches are usually interpreted as sudden transfers of angular momentum between the neutron superfluid in the crust and the rest of the star [17]. This model predicts that the ratio of their respective moments of inertia must obey the constraint [21]

$$\frac{I_s}{I_c} \geq \mathcal{G} \equiv A_g \frac{\Omega}{|\dot{\Omega}|} \quad (1)$$

where A_g is the glitch activity parameter defined by the sum over glitches occurring during a time t

$$A_g = \frac{1}{t} \sum_i \frac{\Delta\Omega_i}{\Omega} \quad (2)$$

while $\dot{\Omega}$ is the average spin-down rate. Both A_g and $\dot{\Omega}$ can be measured from pulsar-timing observations. Since $I_s \ll I_c$, I_c can be replaced by the moment of inertia $I = I_s + I_c$ of the entire star. Approximating I_s by the moment of inertia I_{crust} of the crust, a constraint on the mass and radius of the Vela pulsar was derived in [21]. This approximation treats all unbound neutrons as conducting ($n_n^c = n_n^f$), an assumption which turns out to be unrealistic [24, 28, 29]. Due to entrainment, the angular momentum J_s of the superfluid depends not only on the angular velocity Ω_s of the superfluid, but also on the observed angular velocity Ω of the star and can be expressed as [30]

$$J_s = I_{\text{ss}}\Omega_s + (I_s - I_{\text{ss}})\Omega, \quad (3)$$

with

$$I_s = \int m_n n_n^f \varrho^2 d^3r, \quad I_{\text{ss}} = \int m_n^* n_n^f \varrho^2 d^3r, \quad (4)$$

where ϱ is the cylindrical radius. The constraint (1) thus becomes [30]

$$\frac{(I_s)^2}{II_{\text{ss}}} \geq \mathcal{G}. \quad (5)$$

This inequality is much more stringent than (1) because $I_{\text{ss}} \gg I_s$.

Results. The ratio appearing in the left hand side of Eq. (5) can be decomposed as

$$\frac{(I_s)^2}{II_{\text{ss}}} = \frac{I_{\text{crust}}}{I_{\text{ss}}} \left(\frac{I_s}{I_{\text{crust}}} \right)^2 \frac{I_{\text{crust}}}{I}. \quad (6)$$

In the thin crust approximation [31], I_{ss} is given by

$$I_{\text{ss}} \approx \frac{8\pi R^6}{3GM} \left(1 - \frac{2GI}{R^3c^2} \right) \int_{P_{\text{drip}}}^{P_{\text{core}}} \frac{n_n^f(P)m_n^*(P)}{\bar{\rho}(P)} dP, \quad (7)$$

where M and R are the neutron-star mass and radius, $\bar{\rho}$ is the average mass density, P_{core} is the pressure at the crust-core transition and $P_{\text{drip}} (\ll P_{\text{core}})$ is the neutron-drip pressure. Corresponding expressions for I_s and I_{crust} are obtained by replacing $n_n^f m_n^*$ in Eq. (7) by $n_n^f m_n$ and $\bar{\rho}$ respectively. Note that $I_{\text{ss}}/I_{\text{crust}}$ and I_s/I_{crust} depend only on the crust properties, and can be written as

$$\frac{I_{\text{ss}}}{I_{\text{crust}}} \approx \frac{1}{P_{\text{core}}} \int_{P_{\text{drip}}}^{P_{\text{core}}} \frac{n_n^f(P)^2}{\bar{n}(P)n_n^c(P)} dP, \quad (8)$$

$$\frac{I_s}{I_{\text{crust}}} \approx \frac{1}{P_{\text{core}}} \int_{P_{\text{drip}}}^{P_{\text{core}}} \frac{n_n^f(P)}{\bar{n}(P)} dP \quad (9)$$

where $\bar{n} = \bar{\rho}/m_n$ is the average baryon density. Integrating Eqs. (8) and (9) with the trapezoidal rule using the results of [29] summarized in Table I, we find $I_{\text{ss}} \simeq 4.6I_{\text{crust}}$ and $I_s \simeq 0.89I_{\text{crust}}$ leading to $(I_s)^2/II_{\text{ss}} \simeq 0.17I_{\text{crust}}$. The ratio I_{crust}/I depends on the global structure of neutron stars. We have made use of Eq. (47) of [32]. This formula was obtained by solving the equations of General Relativity using a set of realistic dense-matter equations of state (EoS). Results for $(I_s)^2/(II_{\text{ss}})$ are shown in Fig 1. Note that microscopic calculations based on chiral effective field theory [33] (and more generally any realistic EoS as illustrated in Fig. 3) as well as observations of X-ray binaries [34] indicate that neutron stars with $M = M_{\odot}$ have a radius $R \lesssim 13 \text{ km}$.

Because it was the first observed pulsar to exhibit glitches, Vela has become the testing ground for glitch theories. Since 1969, 17 glitches have been detected [4]. As shown in Fig. 2, the cumulated glitch amplitudes given by $\sum_i \Delta\Omega_i/\Omega = tA_g$ (with an appropriate choice of time origin) increases almost linearly with the time t . A linear fit yields $A_g \simeq 2.25 \times 10^{-14} \text{ s}^{-1}$. With the angular

P (MeV fm $^{-3}$)	n_n^f/\bar{n} (%)	n_n^c/n_n^f (%)
0.0004575	15.0	82.6
0.0009886	61.1	27.3
0.006097	82.6	17.5
0.01507	86.0	15.5
0.03820	87.9	7.37
0.06824	89.1	7.33
0.1068	86.6	10.6
0.1561	89.1	30.0
0.2183	89.2	45.9
0.2930	89.4	64.6
0.3678	100	64.8

TABLE I. Entrainment parameters in the inner crust of cold non-accreting neutron stars as obtained in [29]. \bar{n} is the average baryon density, n_n^f is the density of unbound neutrons, and n_n^c is the density of conduction neutrons. The pressure P was calculated using the formulae in Appendix B of [37].

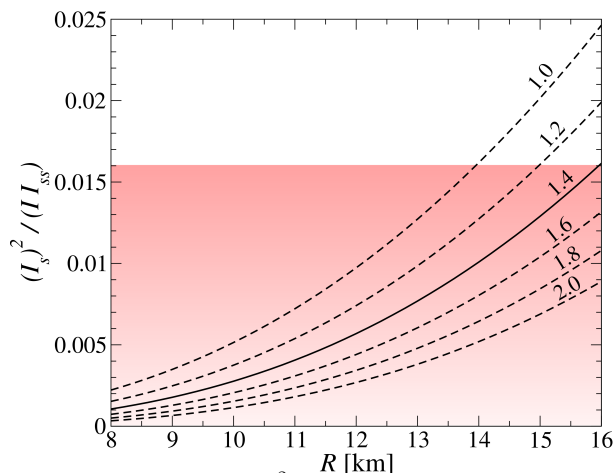


FIG. 1. (Color online) $(I_s)^2/(I I_{ss})$ for different neutron-star radii R and masses M from $1M_\odot$ (upper curve) to $2M_\odot$ (lower curve). The shaded area is excluded if Vela pulsar glitches originate in the inner crust.

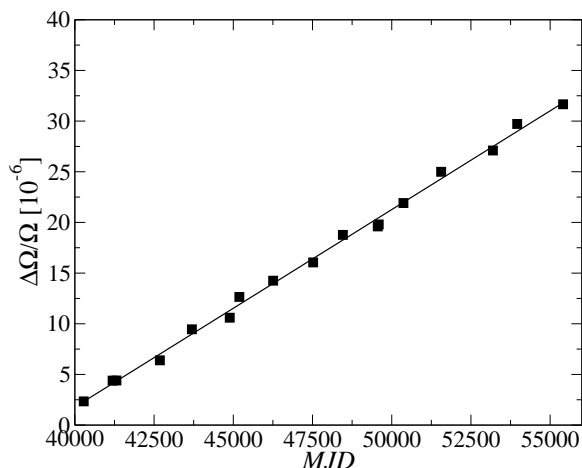


FIG. 2. Cumulated glitch amplitudes as a function of the modified Julian date for the Vela pulsar from [4] (square symbols) and linear fit (solid line).

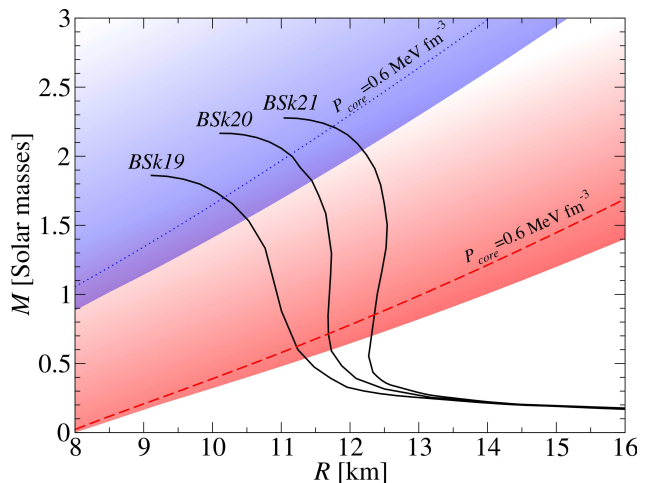


FIG. 3. (Color online) Neutron-star mass-radius diagram for three different unified equations of state. The shaded area is excluded by Vela pulsar glitch data, assuming that only the neutron superfluid in the crust is involved. The red (blue) area is the constraint obtained with (without) taking into account crustal entrainment. The sensitivity of these constraints with respect to the crust-core transition pressure P_{core} is indicated by the dashed line and the dotted line.

frequency $\Omega = 11.1946499395$ Hz and average spin-down rate $\dot{\Omega} = -1.5666 \times 10^{-11} \text{ s}^{-2}$ [1], we find $\mathcal{G} \simeq 1.6\%$. A similar estimate has been obtained from a statistical analysis of Vela like pulsars [35]. As illustrated in Fig. 1, combining the glitch data with Eq. (5) leads to a constraint on the global neutron-star structure. This constraint, which can be approximately written as $R \geq 8.51 + 5.23M$ or equivalently $M \leq 0.190R - 1.61$ with M in M_\odot and R in km, is plotted in Fig. 3, together with three representative unified EoS spanning different degrees of stiffness of dense neutron matter, from the softest (BSk19) to the stiffest (BSk21), as obtained from microscopic calculations [36]. The sensitivity of the glitch constraint with respect to the corresponding crust-core transition pressure is also shown. This analysis implies that Vela (and more generally pulsars with Vela like glitches) should be less massive than our Sun ($M < 0.6M_\odot$ for the softest EoS). Such a low mass neutron star is unlikely to be formed in a type II supernova explosion [38]. However, the association of the Vela pulsar with the eponymous supernova remnant is well established.

Discussion. We are thus led to conclude that the neutron superfluid in neutron-star crusts does not carry enough angular momentum to explain large pulsar glitches like those observed in Vela, unless crustal entrainment and crust-core coupling are much weaker than considered here. A similar conclusion has been reached in [39].

The estimates of m_n^* obtained in [29] agree closely with previous calculations [24] using a different model thus suggesting that strong crustal entrainment is generic. Moreover, m_n^* was found to be weakly dependent on

the crystal structure [24]. The existence of nuclear “pasta” phases near the crust bottom (see e.g. Section 3.3 of [5]), which we have ignored, might enhance the neutron conduction owing to the low dimensionality of such configurations. However, it has been argued that these pastas (if any) could only exist in a small region of the crust, at baryon densities above $\sim 0.06 \text{ fm}^{-3}$, if the lowest-frequency quasiperiodic oscillation observed in giant flares from soft gamma-ray repeaters (SGRs) is to be interpreted as the fundamental torsional crustal mode [40]. Setting $n_n^c = n_n^f$ for $\bar{n} \geq 0.06 \text{ fm}^{-3}$, the impact of pastas is found to be small since the ratio $I_{\text{ss}}/I_{\text{crust}}$ is reduced from 4.6 to 4.3 whereas $(I_s)^2/(I_{\text{crust}}I_{\text{ss}})$ is raised from 0.17 to 0.19. In reality, n_n^c is never equal to n_n^f in any region of the crust, even in the presence of pastas [24]. On the other hand, the spin-orbit coupling (which was neglected in [29]), would increase the number of entrained neutrons [24] and could be more important than pastas since it operates at all densities. We also anticipate that quantum and thermal fluctuations of ions about their equilibrium positions, crystal defects, impurities and more generally any kind of disorder would presumably reduce (but not cancel) entrainment effects. Further work is needed to confirm these expectations.

On the other hand, the strong coupling of the core to the crust that we have considered here relies on the assumption of type I superconductivity [19]. The observed rapid cooling of the neutron star in Cassiopeia A has recently provided strong evidence for core neutron superfluidity and proton superconductivity, but not on its type [41, 42]. If the superconductor is of type II, the coupling time could be much longer [43]. On the other hand, type II superconductivity has not only been argued to be incompatible with observations of long-period precession in pulsars [44] but has also been questioned on theoretical grounds [45]. In fact, the superconductor might neither be of type I nor of type II [46]. In addition, neutron-star cores might contain other particle species with various superfluid and superconducting phases.

Removing the discrepancy between glitch models and observations thus requires a closer examination of crustal entrainment and crust-core coupling. The regularity of glitches illustrated in Fig. 2 and the fact that $\mathcal{G} \lesssim 2\%$ suggest the existence of a reservoir of angular momentum in a limited region of the star, possibly in the outermost part of the core just below the crust (e.g. [47, 48]). This warrants further studies.

This work also shed light on the importance of crustal entrainment, which has been generally overlooked even though they may have implications for other astrophysical phenomena like quasiperiodic oscillations in SGRs [49].

The author thanks B. Link and A. Alpar for valuable discussions. This work was financially supported by FNRS (Belgium) and CompStar, a Research Networking

Programme of the European Science Foundation. The author thanks the Institute for Nuclear Theory at the University of Washington for its hospitality and the Department of Energy for partial support.

-
- [1] R. N. Manchester, G. B. Hobbs, A. Teoh and M. Hobbs, *Astron. J.* **129**, 1993 (2005); <http://www.atnf.csiro.au/research/pulsar/psrcat>
 - [2] P. Haensel, A. Y. Potekhin, D. G. Yakovlev, *Neutron Stars 1: Equation of state and structure*, Springer (2007).
 - [3] W. Baade and F. Zwicky, *Phys.Rev.***45**, 138 (1934).
 - [4] C. M. Espinoza, A. G. Lyne, B. W. Stappers, M. Kramer, *Mon.Not.Roy.Astron.Soc.* **414**, 1679 (2011) ; <http://www.jb.man.ac.uk/pulsar/glitches.html>
 - [5] N. Chamel and P. Haensel, “Physics of Neutron Star Crusts”, *Living Rev. Relativity* **11**, (2008), 10. <http://www.livingreviews.org/lrr-2008-10>
 - [6] M. Ruderman, *Annu. Rev. Astron. Astrophys.***10**, 427, (1972).
 - [7] M. Ruderman, *Nature* **223**, 597 (1969).
 - [8] G. Baym, C. J. Pethick, and D. Pines, *Nature* **224**, 673 (1969).
 - [9] R. E. Packard, *Phys. Rev. Lett.* **28**, 1080 (1972).
 - [10] A. B. Migdal, *Nucl. Phys.* **13**, 655 (1959).
 - [11] V. L. Ginzburg and D. A. Kirzhnits, *Zh. Eksp. Teor. Fiz.* **47**, 2006, (1964).
 - [12] D. Pines, J. Shaham, M. Ruderman, *Nature Phys. Sci.* **237**, 83 (1972).
 - [13] P. W. Anderson and N. Itoh, *Nature* **256**, 25 (1975).
 - [14] L. J. Campbell, *Phys. Rev. Lett.***43**, 1336 (1979).
 - [15] J. S. Tsakadze and S. J. Tsakadze, *J. Low Temp. Phys.***39**, 649 (1980).
 - [16] M. A. Alpar, P. W. Anderson, D. Pines, J. Shaham, *Astrophys. J.* **249**, L29 (1981).
 - [17] D. Pines and M. A. Alpar, *Nature* **316**, 27 (1985).
 - [18] P. B. Jones, *Mon. Not. R. Astron. Soc.***263**, 619 (1993).
 - [19] M. A. Alpar, S. A. Langer and J.A. Sauls, *Astrophys. J.* **282**, 533 (1984).
 - [20] N. Andersson, T. Sidery, G.L. Comer, *Mon. Not. R. Astron. Soc.***368**, 162 (2006).
 - [21] B. Link, R. I. Epstein and J. M. Lattimer, *Phys. Rev. Lett.* **83**, 3362 (1999).
 - [22] P. M. Pizzochero, *Astrophys. J. Lett.* **743**, L20 (2011).
 - [23] K. Glampedakis and N. Andersson, *Phys. Rev. Lett.* **102**, 141101 (2009).
 - [24] N. Chamel, *Nucl.Phys.* **A747**, 109 (2005).
 - [25] B. Carter, N. Chamel, P. Haensel, *Nucl. Phys.* **A748**, 675 (2005).
 - [26] B. Carter, N. Chamel, P. Haensel, *Nucl.Phys.* **A759**, 441 (2005).
 - [27] B. Carter, N. Chamel, P. Haensel, *Int.J.Mod.Phys.***D15**, 777 (2006).
 - [28] N. Chamel, *Nucl. Phys.* **A773**, 263 (2006).
 - [29] N. Chamel, *Phys. Rev.* **C85**, 035801 (2012).
 - [30] N. Chamel and B. Carter, *Mon.Not.Roy.Astron.Soc.* **368**, 796 (2006).
 - [31] C. P. Lorenz, D. G. Ravenhall and C. J. Pethick, *Phys. Rev. Lett.* **70**, 379 (1993).
 - [32] J. Lattimer and M. Prakash, *Phys.Rep.***333**,121 (2000).
 - [33] K. Hebeler, J. M. Lattimer, C. J. Pethick, A. Schwenk,

- Phys. Rev. Lett.**105**, 161102 (2010).
- [34] A. W. Steiner, J. M. Lattimer and E. F. Brown, *Astrophys. J.* **722**, 33 (2010).
- [35] A. G. Lyne, S. L. Shemar, F. G. Smith, *Mon. Not. R. Astron. Soc.***315**, 534 (2000).
- [36] N. Chamel, A. F. Fantina, J. M. Pearson, S. Goriely, *Phys. Rev.* **C84**, 062802 (2011).
- [37] J. M. Pearson, N. Chamel, S. Goriely, and C. Ducoin, *Phys. Rev.* **C85**, 065803 (2012).
- [38] K. Strobel and M. K. Weigel, *Astron. Astrophys.* **367**, 582 (2001).
- [39] N. Andersson, K. Glampedakis, W.C.G. Ho, C.M. Espinoza, eprint arXiv:1207.0633.
- [40] H. Sotani, K. Nakazato, K. Iida, K. Oyamatsu, *Phys. Rev. Lett.***108**, 201101 (2012).
- [41] D. Page, M. Prakash, J. M. Lattimer, A. W. Steiner, *Phys. Rev. Lett.***106**, 081101 (2011).
- [42] P. S. Shternin, D. G. Yakovlev, C. O. Heinke, W. C. G. Ho, D.J. Patnaude, *Mon.Not.Roy.Astron.Soc.* **412**, L108 (2011).
- [43] A. D. Sedrakian, D. M. Sedrakian, J. M. Cordes, Y. Terzian, *Astrophys. J.***447**, 324 (1995).
- [44] B. Link, *Phys. Rev. Lett.* **91**, 101101 (2003).
- [45] J. Charbonneau and A. Zhitnitsky, *Phys. Rev.* **C76**, 015801 (2007).
- [46] E. Babaev, *Phys. Rev. Lett.* **103**, 231101 (2009).
- [47] A. Sedrakian and J. M. Cordes, *Mon. Not. Roy. Astron. Soc.* **307**, 365 (1999).
- [48] C. Peralta, A. Melatos, M. Giacobello, A. Ooi, *Astrophys. J.* **651**, 1079 (2006).
- [49] A. Passamonti and N. Andersson, *Mon. Not. Roy. Astron. Soc.* **419**, 638 (2012).