Plasma-Based Accelerator with Magnetic Compression

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Electron dephasing is a major gain-inhibiting effect in plasma-based accelerators. A novel method is proposed to overcome dephasing, in which the modulation of a modest ($\sim \mathcal{O}(10$ kG$)$), axial, uniform magnetic field in the acceleration channel leads to densification of the plasma through magnetic compression, enabling direct, time-resolved control of the plasma wave properties. The methodology is broadly applicable and can be optimized to improve the leading acceleration approaches, including plasma beat-wave, plasma wakefield, and laser wakefield acceleration. The advantages of magnetic compression are compared to other proposed techniques to overcome dephasing.

PACS numbers:

Introduction.— Charged-particle acceleration in plasma employs short, intense laser pulses or high energy electron bunches to excite longitudinal plasma waves capable of accelerating relativistic particles to high energies over very short distances [1–6]. One major factor limiting energy gain in plasma-based accelerators is *phase slippage*, in which a particle eventually outruns the segment of the wave providing a positive accelerating force (see, e.g., Ref. [3]). Methods to improve gain require that particles remain in phase with the forward accelerating component of a plasma wave for an extended period of time. The *surfatron* employs a static, transverse applied magnetic field to control the axial phase of an accelerating particle in a beat-wave accelerator [7, 8], while a stationary, axial plasma density gradient can be used to synchronize the advance of a wakefield and an accelerating ultrarelativistic electron [9–13]. The use of active media to modulate the wake phase velocity has also been proposed [14].

In this Letter, we propose a new method to improve energy gain by modulating the phase velocity, $v_{\rm ph}$, of an accelerating plasma wave using an externally generated, time-varying, uniform axial magnetic field. Within a bounded parameter regime, uniform transverse magnetic compression of the plasma column leads to a tunable, time-varying density profile. With compression, $v_{\rm ph}$ can be increased beyond the subluminal driver pulse velocity in the cases of plasma beat-wave (PBWA), laser wakefield (LWFA), and plasma wakefield (PWFA) acceleration (up to and even beyond c). Static, axial magnetic fields have been shown to enhance electron injection, trapping, beam stability, optical guidance, and energy gain in LWFA [15–17] and PBWA [18]; however, this is the first time a time-varying field is proposed as a precise control mechanism for the plasma wave dynamics.

For PBWA, dephasing is mitigated with only a small fractional density increase, and no cross-beam electron motion is induced, unlike the surfatron [7]. For wakefield acceleration, the density increase required is much more gradual compared to the axial density gradient method [9–13], and the wakefield amplitude actually increases with propagation distance in some cases. Also, generating a time-varying, uniform density profile with magnetic compression should be technologically considerably easier than generating a stationary density gradient.

Plasma beat-wave acceleration with compression.— In PBWA, two co-propagating lasers combine to form a subluminal ponderomotive beat-wave. Here, the laser frequencies $\omega_{1,2} = \omega_d \pm \Delta\omega/2$ and wavenumbers $k_{1,2} = k_0 \pm \Delta k/2$, with $\Delta\omega = \omega_p$, $\Delta k \equiv k_p$, $\omega_d \gg \omega_p$, and ω_p is the plasma frequency. For simplicity, we consider the 1D limit, i.e., $r_{\rm s} \gg k_p^{-1}$, where $r_{\rm s}$ is the characteristic laser spot size. The beat-wave resonantly drives a long (many k_p^{-1}), high-amplitude plasma wave whose phase velocity is set by the driver group velocity, i.e., $v_{\rm ph} = \Delta\omega/\Delta k \simeq c(1 - \omega_p^2/2\omega_d^2)$ [6]. For PBWA, autoresonant phase-locking of driven plasma waves to frequency chirped laser beat-waves has been shown to drive plasma waves to high amplitudes [19, 20], but the dephasing problem is not addressed. Our proposed method modulates both ω_p and $v_{\rm ph}$.

Consider homogeneous, uniformly magnetized plasma, i.e., $\mathbf{B} = B(t)\hat{\mathbf{z}}$, where $B(t)$ changes with time. For example, this could be realized for plasma inside a solenoid. Magnetization implies that the plasma density $n \propto |\mathbf{B}|$. For slow variation of plasma parameters, a relativistic plasma wave, i.e., $v_{\rm ph} = \omega/k \lesssim c$, with wavevector $\mathbf{k} \parallel \mathbf{B}$, obeys the cold plasma eikonal equation, $\omega = \omega_p$ [21]. Since $\omega_p^2 \propto n \propto B$, we have $\omega_p = \omega_p(t)$, while k remains constant (neglecting nonlinear effects [3]), since the compression is perpendicular to the wavevector. Note, when $v_{\rm ph}/c \equiv \beta_{\rm ph} \approx 1$, only small changes in n are needed to produce large changes in $\gamma_{\rm ph} \equiv (1 - \beta_{\rm ph}^2)^{-1/2}$.

The axial dynamics of a relativistic electron interacting with a sinusoidal potential are given by [22]:

$$\frac{d\gamma}{dt} = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} \left(\frac{eE}{m_e c}\right) \cos\xi, \tag{1}$$

$$\frac{d\xi}{dt} = ck\left(1 - \frac{1}{\gamma^2}\right)^{1/2} - \dot{\Psi}, \tag{2}$$

where $\xi = kz - \Psi(t)$, $\gamma m_e c^2$ is the electron energy in the laboratory frame, e is the elementary charge, and $\Psi(t) = \int_0^t \omega(k; t')\, dt'$. Because minimal compression is anticipated, the electric field amplitude $E \approx$ const.

The compression profile required to overcome phase slippage in PBWA can be calculated from Eqs. (1) and (2). Suppose $\Psi(t)$ is configured such that a stable fixed point arises in the phase space associated with the rest frame of the accelerating plasma wave. Then, combining Eqs. (1) and (2) by eliminating the square root gives $\dot{\gamma} = \Phi_0(\dot{\xi} + \dot{\Psi})\cos\xi$, with $\Phi_0 \equiv eE/km_ec^2$. If $\xi = \xi_0$, corresponding to the fixed point, there exists an energy-like integral of motion: $d/dt(\gamma - \Phi_0\Psi\cos\xi_0) = 0$. Equation (2), with $\dot{\xi} = 0$, yields the necessary plasma compression profile. Noting that $\dot{\Psi} = \omega_p(t)$, and that the distance traversed by a phase-locked particle $D(t) = (\Psi - \Psi_0)/k$, the required normalized density profile, $\tilde{n} = \omega_p^2/\omega_{p0}^2$, is:

$$\tilde{n}[D(t)] = \frac{\gamma_0^2\left[(\gamma_0 + \Phi_0 kD\cos\xi_0)^2 - 1\right]}{(\gamma_0^2 - 1)(\gamma_0 + \Phi_0 kD\cos\xi_0)^2}, \tag{3}$$

where $\gamma_0 = \gamma_{\mathrm{ph},0}$ implies exact initial wave-particle resonance. Equation (3) is monotonic in D, asymptotically approaching $\tilde{n}_{\max} = \gamma_0^2/(\gamma_0^2 - 1)$ as $D \to \infty$, at which point $v_{\mathrm{ph}} \to c$. For instance, injection of a 2 MeV electron bunch ($\gamma_0 \approx 4$) requires a peak density shift of only $\tilde{n}_{\max} \approx 1.07$ to maintain proper phasing indefinitely (in principle) as the bunch accelerates. Equation (3) can be expressed as an explicit function of time by integrating $D(t) = \int_0^t v\,dt'$, with $v(t)$ the velocity of a relativistic particle accelerated by the constant force attributable to the fixed wave-particle relative phase, yielding

$$D(t) = \frac{1}{\alpha}\left[\sqrt{\left(\alpha ct + \sqrt{\gamma_0^2 - 1}\right)^2 + 1} - \gamma_0\right], \tag{4}$$

with $\alpha = eE\cos\xi_0/m_ec^2$. The calculation assumes zero transverse momentum, which equates to a compression profile optimized to trap relativistic particles with a narrow transverse energy spread.

Peak acceleration occurs when $\xi_0 = 0$, for which an electron starting at $\xi = 0$ obeys $\gamma - \Phi_0\Psi =$ const. In fact, for a wave of specified E and k, this is the *maximum* achievable acceleration, in which the electron remains in phase with the peak accelerating field. More generally, choosing $0 < \xi_0 < \pi/2$ in Eq. (3) enables electron trapping over a broader range of initial electron energies. Since the fixed point in the wave rest frame, ξ_0, lies ahead of the peak accelerating field, at $\xi = 0$, some particles that slip behind ξ_0 can catch up to ξ_0 once again.

The increased gain arises from maximizing the dephasing length scale, $L_d \simeq (2/\pi)(\omega_d^2/\omega_p^2)\lambda_p$, with $\lambda_p \approx 2\pi c/\omega_p$ [3]. The part of the plasma wave furthest upstream from the driver pulse will become turbulent through ion instabilities on the time scale $\omega_{pi} = (4\pi ne^2/M_i)^{1/2}$, with M_i the ion mass [6]. Then, the new, *turbulent* length scale over which positive acceleration is achieved becomes $L_t \simeq 2\pi c/\omega_{pi}$. The ratio of achievable gains with and without compression, $\tilde{G}$, is simply the ratio of the two length scales, $\tilde{G} = L_t/L_d \simeq (\pi/2)(\omega_p/\omega_{pi})\omega_p^2/\omega_d^2$. Thus, compression offers the greatest benefit to PBWA employing heavier plasma ions. For example, $\tilde{G} > 1$ for any $\omega_d < 20.6\omega_p$ using singly-ionized argon, or $\omega_d < 24.8\omega_p$ for singly-ionized krypton.

Slight compression does not detune substantially the driver from the optimal resonant plasma response, so plasma wave generation can continue beyond L_t. If the pump is not depleted after length L_t, then the gain scales with the pump depletion length, $L_{pd} \simeq (\omega_d^2/\omega_p^2)\lambda_p/a_0^2$, with $a_0 \equiv eA_0/m_ec^2$, and A_0 the characteristic initial vector potential magnitude of the laser drivers [3].

Wakefield acceleration with compression.— Mitigating phase slippage through magnetic compression in (linear) wakefield acceleration, including LWFA and PWFA, is a somewhat different process. Here, a time-varying density profile during wake excitation results in an axial gradient in the plasma wake parameters, which was not the case with PBWA. Electron dephasing is often the dominant effect limiting energy gain in wakefield acceleration when the driver amplitude is no more than weakly relativistic, i.e., $a < 1$ for LWFA [13], or $n_b/n < 1$ for PWFA, where n_b is the peak driver beam density [3].

In wakefield acceleration, a subluminal wakefield is excited by an ultrarelativistic driver, i.e., $\gamma_d = (1 - \beta_d^2)^{-1/2} \gg 1$, with $\beta_d = v_d/c$, and v_d is the driver pulse velocity. For PWFA, the longitudinal velocity of the electron beam driver is unaffected by perpendicular magnetic compression. Because only modest density changes will be needed, the laser pulse group velocity, v_{gr}, is mostly unaffected as well, since a change in plasma frequency, $\Delta\omega_p$, leads to a change in wave phase velocity $\Delta v_{\mathrm{ph}}/v_{\mathrm{ph}} \simeq \Delta\omega_p/\omega_p$, which is large compared to the change in laser group velocity, $\Delta v_{\mathrm{gr}}/v_{\mathrm{gr}} \simeq (\omega_p/\omega_d)^2\Delta\omega_p/\omega_p$, where ω_d is the laser frequency, and $\omega_p/\omega_d \ll 1$ in underdense plasma. Thus, both PWFA and LWFA exhibit $v_d \approx$ const.

We follow the technique of Ref. [9] to derive the compression profile (in the 1D limit) needed to maintain a luminal wakefield phase front initially at a distance $w\lambda_{p0}$ behind the lead pulse, with the plasma wavelength $\lambda_p = 2\pi v_d/\omega_p$, and w an arbitrary constant. This luminal front will remain approximately in phase with an accelerating ultrarelativistic bunch of electrons also traveling at velocity $v \simeq c$. First, we review the calculation of the optimal stationary, but inhomogeneous, axial density profile required to perform the same task. The rate of advance of the wake is given by $\Delta z_w/\Delta t = v_d - w\Delta\lambda_p/\Delta t = v_d - wv_d(\partial\lambda_p/\partial n)(dn/dz)$. An ultrarelativistic particle advances at $\Delta z/\Delta t \simeq c$. Setting the two rates of advance equal gives the equation for the optimized density profile:

$$\frac{d\omega_p}{dz} = \frac{\omega_p^2(z)}{2\pi wv_d}\left(\beta_d^{-1} - 1\right), \tag{5}$$

where $(1/n)dn/dz = (2/\omega_p)d\omega_p/dz$ was used. We define the overtaking time, $T \equiv ct_0/(c - v_d)$, and the overtaking length, $L \equiv cT$, where $t_0 = 2\pi w/\omega_{p0}$ is the particle injection time. After time T, an ultrarelativistic electron overtakes the slower lead pulse. In dimensionless variables, $\tilde{z} \equiv z/L$ and $\tilde{\omega}_p \equiv \omega_p/\omega_{p0}$, Eq. (5) becomes

$$\frac{d\tilde{\omega}_p}{d\tilde{z}} = \beta_d^{-2}\tilde{\omega}_p^2, \tag{6}$$

which has the solution, $\tilde{\omega}_p = (1 - \beta_d^{-2}\tilde{z})^{-1}$. This is the well-known stationary axial density profile to sustain the luminous phase front [9], and, since $\beta_d^{-2} > 1$, the density always becomes singular just prior to $\tilde{z} = 1$.

For a uniform plasma densifying through magnetic compression, $\Delta z_w/\Delta t = v_{\rm ph} - w\Delta\lambda_p/\Delta t$. At each point, the wakefield wavevector satisfies $k[z_d(t)] = \omega_p(t)/v_d$ [3], where $z_d(t) = v_d t$ is the axial position of the driver amplitude maximum at time t. For a point in the electron trajectory within the wake, $z(t)$, $k(z)$ has been set by the driver at a previous time, $t' = (t - t_0)/\beta_d$, whereas ω_p has *increased* through densification since t'. Accordingly, $v_{\rm ph}[z(t)] = v_d\,\omega_p(t)/\omega_p(t')$. Also, $\Delta\lambda_p/\Delta t \to (\partial\lambda_p/\partial n)(dn/dt)$. Setting equal the rates of advance of the wake and the accelerating electron gives

$$\frac{d\omega_p}{dt} = \frac{\omega_p^2(t)}{2\pi w}\left[\beta_d^{-1} - \frac{\omega_p(t)}{\omega_p[(t - t_0)/\beta_d]}\right], \tag{7}$$

in which $\dot{\omega}_p$ now depends on ω_p at a previous time. In dimensionless variables, $\tilde{t} \equiv t/T$ and $\tilde{\omega}_p$, Eq. (7) becomes

$$\frac{d\tilde{\omega}_p}{d\tilde{t}} = \frac{\tilde{\omega}_p^2}{\chi}\left[\beta_d^{-1} - \frac{\tilde{\omega}_p(\tilde{t})}{\tilde{\omega}_p[(\tilde{t} - \chi)(1 + \chi\beta_d^{-1})]}\right], \tag{8}$$

where $\chi \equiv 1 - \beta_d \ll 1$. Since $\chi \ll 1$, Eq. (8) can be approximated by expanding the past-time form of $\tilde{\omega}_p$ about $\chi = 0$: $\tilde{\omega}_p[(\tilde{t} - \chi)(1 + \chi\beta_d^{-1})] \approx \tilde{\omega}_p(\tilde{t}) - \chi(1 - \tilde{t}\beta_d^{-1})(d\tilde{\omega}_p/d\tilde{t})$. Plugging this into Eq. (8), expanding the denominator, and rearranging yields

$$\frac{d\tilde{\omega}_p}{d\tilde{t}} \approx \frac{\beta_d^{-2}\tilde{\omega}_p^2}{1 + \tilde{\omega}_p(1 - \tilde{t}\beta_d^{-1})}. \tag{9}$$

For an ultrarelativistic driver, $\beta_d \to 1$, and Eq. (9) turns out to be negligibly dependent on the driver velocity.

Figure 1 shows the solutions for $\tilde{\omega}_p^2 = \tilde{n}$ corresponding to both methods, given by Eqs. (6) and (9), in the limit, $\beta_d \to 1$. It is clear that magnetic compression requires substantially less densification than a stationary, axial density gradient to maintain a luminal wake front. Moreover, the optimized compression profile does not exhibit a density singularity as the accelerating electron approaches the driver pulse, unlike the optimized stationary density gradient profile.

In PWFA, magnetic compression does not produce the loss of wakefield amplitude with propagation distance exhibited by a stationary density gradient [9]. From

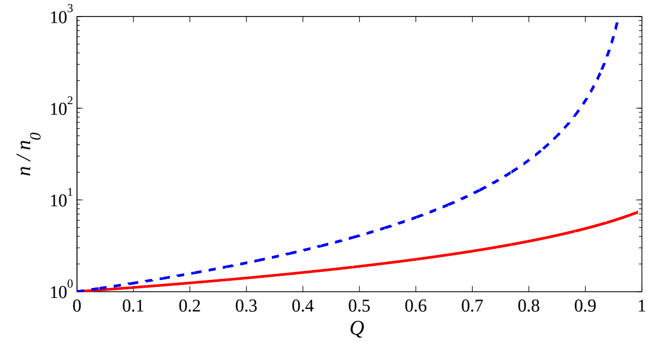

FIG. 1: (color online) Optimized density profiles for: (a) the axial density gradient method (dashed line), with $Q = z/L$ signifying an axially inhomogeneous density profile; and (b) the perpendicular magnetic compression method (solid line), with $Q = t/T$ signifying a time-varying, but axially uniform, density profile. Note that $L = cT$.

Eq. (16) of Ref. [9], the peak electric field $E_{\rm max} \propto (n_b/n)n^{1/2}$. So, at fixed n_b, a wakefield excited in a stationary axial density gradient obeys $E_{\rm max} \propto n^{-1/2}$, while magnetic compression causes the background and beam to densify together, i.e., $n_b/n = $ const, leading to $E_{\rm max} \propto n^{1/2}$ as $n(t)$ increases in time. Remarkably, although $E_{\rm max}$ increases with propagation distance in compressing plasma, the driver depletion length, L_{dp}, is essentially unaffected. From Ref. [3], $L_{dp} \approx \gamma_d m_e c^2/eE_-$, where $E_- = E_{\rm max}/R_t$ is the retarding electric field excited within the driver, and R_t is the transformer ratio. For a long, triangular-shaped beam with a linear density rise over the length $L_b = N_b\lambda_p$, with $N_b > 1$, followed by a sharp termination, the transformer ratio is given by $R_t \simeq \pi N_b \propto \omega_p$ [23]. Hence, the ratio $E_{\rm max}/R_t = $ const, and thus, $L_{dp} = $ const. Also, the average electric field an ultrarelativistic particle feels over the total overtaking length for the compression scheme, calculated numerically from Fig. 1, is given by $\langle E\rangle_{\rm comp} \approx 1.5E_{\rm max,0}$. In contrast, $\langle E\rangle_{\rm grad} = 0.5E_{\rm max,0}$ for a stationary density gradient. Accordingly, $\tilde{G}$ for PWFA in uniform plasma with and without compression is given by, $\tilde{G} \simeq 1.5L_{dp}/L_d \simeq (3\pi/8)[(N_b/\gamma_d)(n/n_b)]_0$, where $L_d \simeq (2/\pi)\gamma_d^2\lambda_p$ [3]. Thus, $\tilde{G}$ is large for long driver pulses, low driver energies, and modest beam densities. Even when compared to an axial density gradient, compression still offers a threefold gain improvement due to the enhanced wakefield amplitude, $\langle E\rangle_{\rm comp}/\langle E\rangle_{\rm grad} \approx 3$.

In LWFA, overall performance could be impacted negatively as compression alters the plasma response to a laser pulse of fixed dimensions. However, while the wakefield amplitude is maximized at a particular laser pulse length, $L_\ell \propto \lambda_p$, the density profile prescribed by Eq. (9) can be utilized in a way that minimizes the effect of detuning. For example, a circularly polarized Gaussian pulse produces a linear wakefield response, $E_{\rm max} \propto$

$E_0 a_0^2 k_p L_\ell \exp(-k_p^2 L_\ell^2/4)$ [3], where $E_0 = cm_e\omega_p/e$, $k_p \approx \omega_p/c$, and $E_{\max}$ is greatest when $k_p L_\ell = \sqrt{2}$. In terms of $x \equiv k_p L_\ell \propto \omega_p$, $E_{\max} \propto x^2 \exp(-x^2/4)$. By choosing ω_{p0} to be about 85% the optimal plasma frequency, $\omega_{p,\mathrm{opt}} = \sqrt{2}L_\ell/c$ [found by maximizing the integral, $(1/a)\int_a^{a+1.7} x^2 \exp(-x^2/4)\,dx$, where 1.7 is the approximate increase in $k_p L_\ell$ prescribed by Eq. (9)], an accelerating electron will, on average, experience $\langle E\rangle_{\mathrm{comp}} \approx 0.9E_{\max}$ for fixed L_ℓ. Thus, the gain improvement over a stationary, uniform plasma is simply $\tilde{G} \simeq 0.9L_{pd}/L_d \simeq 0.9(\omega_{p0}^3/\langle\omega_p\rangle^3)(1/a_0^2)$, where L_d and L_{pd} were given in the previous section, and $a_0^2 \ll 1$ is assumed. For the compression profile in Fig. 1, $\langle\omega_p\rangle^3/\omega_{p0}^3 \approx 4.3$, implying that $\tilde{G} > 1$ for all $a_0 < 0.46$. The focusing forces provided by relativistic optical guiding and density channel guiding scale like n and n^2, respectively [3]. Thus, magnetic compression also could enhance the suppression of laser pulse diffraction, improving further on the benefits established in channel formation studies employing a static, axial magnetic field [17].

Discussion.— In order that variations in $B(t)$ translate to proportional changes in the plasma density, we require that both plasma species be magnetized, i.e., $\omega_{cj}/2\pi\nu_j \gtrsim 1$ for species $j : \{e, i\}$, where $\omega_{cj} = q_j B/m_j c$ is the cyclotron frequency, and ν_j is the collision frequency, assuming electrons and ions are initially in thermal equilibrium and isotropic. The minimum B required is that which marginally magnetizes the ions, or $\omega_{ci}/2\pi\nu_i \approx 1$. For instance, assuming hydrogen plasma, the initial parameters $B = 5\times10^4$ G, $n = 10^{16}$ cm^{-3}, and $T = 20$ eV, where $T = T_e = T_i$ is the plasma temperature, lead to $\omega_{ci}/2\pi\nu_i \approx 1$, and $\omega_{ce}/2\pi\nu_e \approx 30$. As $B(t)$ evolves, the induced azimuthal electric field, $E_\phi(r) = -r\dot{B}/2c$, causes a radial drift of both electrons and ions such that the density $n \propto B$. Since this drift is a gyro-averaged phenomenon, averaging over the continuum of particle gyrophases will lead to uniform densification of the plasma, even on time scales short compared to $1/\omega_{ci}$.

There still can remain a separation of timescales between that of electron space charge oscillations, ω_p, and that of magnetic gyration, ω_{ce}. The parallel electrostatic plasma response is unaffected by the magnetic field, whereas the perpendicular electrostatic response is characterized by the upper hybrid frequency, $\omega_{\mathrm{uh}} = \sqrt{\omega_p^2 + \omega_{ce}^2} \approx \omega_p(1.0 + 0.5\,\omega_{ce}^2/\omega_p^2)$. The example parameters from the previous paragraph give $\omega_{\mathrm{uh}} \approx \omega_p$ to within about 1%. For the ordering $\omega_d \gg \omega_p \gg \omega_{ce}$, the laser ponderomotive force is also unchanged by the magnetic field [24]. Thus, wave excitation is virtually unaffected by $\mathbf{B}$ on such short timescales (of $\mathcal{O}(\omega_p^{-1})$ duration), as Ref. [16] confirmed in LWFA simulations using even stronger (1.2 MG) magnetic fields. While a strong axial magnetic field enhances electron self-injection in the nonlinear *blowout* regime by suppressing transverse electron motion [16], the processes of electron capture and trapping in the linear regimes considered here should not be affected significantly by the presence of more modest magnetic fields on such short ($\mathcal{O}(\omega_p^{-1})$) timescales.

With magnetic compression, some amount of perpendicular heating can be expected, leading to potential anisotropy-driven instabilities. The fastest-growing unstable modes, excited by the electron whistler instability when $T_\perp > T_\parallel$, exhibit growth rates $\Gamma \lesssim 0.01\omega_{ce}$ for the parameters considered here [25]. For the sample parameters listed above, $\nu_e/\Gamma \approx 1$, so the instability is suppressed to some extent by collisional isotropization. In addition, the instability is resonant with bulk electrons, thus interfering minimally with the dynamics of the ultrarelativistic Langmuir wave and accelerating electrons.

Practical realization of these methods might employ Helmholtz coils surrounding the acceleration stage of an existing plasma-based accelerator configuration. For our example parameters, one has $\lambda_p \approx 330$ μm, $L_d \approx 2$ cm, and $L_{dp} \approx 17$ cm, assuming a PWFA configuration with $\gamma_d = 10$, $n_{b0}/n_0 = 0.1$, and $N_b = 10$. The potential gain improvement is $\tilde{G} \sim \mathcal{O}(10)$. A 10 cm-radius coil arrangement, each coupled to a 10 kΩ resistive load, can exhibit sufficiently short L/R times ($\mathcal{O}(10^{-11}$ s)) to produce significant variations in $\mathbf{B}$ on timescales comparable to the beam transit time ($\mathcal{O}(10^{-10}$ s)).

The primary advantage of magnetic compression is that the solution to the dephasing problem is reduced to the task of shaping the magnetic coil current profile, which is easier technologically than controlling a spatially varying density profile. Also, shot-to-shot tailoring only requires reprogramming the current source. In contrast, Ref. [13] notes that optimal stationary density profiles may be difficult to realize experimentally, while shot-to-shot adjustments may require significant physical manipulation of the gas injection components. The compression itself also increases the wave amplitude [21], increasing gain further. Finally, besides some radial focusing of accelerating electrons on-axis, no other cross-beam electron motion is introduced, unlike the surfatron [7].

In summary, a new method is proposed to mitigate dephasing in leading plasma-based acceleration techniques, where the modulation of an axial, uniform magnetic field leads to plasma densification through magnetic compression, enabling direct, time-resolved control of the plasma wave. Optimized compression profiles and resulting energy gain enhancements are calculated for PBWA, PWFA, and LWFA. The benefits of compression are compared to other proposed techniques to control dephasing.

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