Magnetic Control of the Pair Creation in Spatially Localized Supercritical Fields

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Magnetic control of the pair creation in spatially localized supercritical fields

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We examine the impact of a perpendicular magnetic field on the creation mechanism of electron-positron pairs in a supercritical static electric field, where both fields are localized along the direction of the electric field. In the case where the spatial extent of the magnetic field exceeds that of the electric field, quantum field theoretical simulations based on the Dirac equation predict a suppression of pair creation even if the electric field is supercritical. Furthermore, an arbitrarily small magnetic field outside the interaction zone can bring the creation process even to complete halt, if it is sufficiently extended. The mechanism for this magnetically-induced complete shutoff can be associated with a re-opening of the mass gap and the emergence of electrically-dressed Landau levels.

The possibility to break down the electrodynamical vacuum by an external supercritical field to create electron-positron pairs has been one of the most astonishing theoretical predictions of quantum electrodynamics [1]. Recent experimental efforts aim at the development of highly powered laser systems with the ultimate goal to be able to focus the beam on a minute spot where the fields are sufficiently intense to spark the vacuum. While an experimental observation is still challenging, it is clear that the interaction zone will be rather localized.

Theoretically laser-triggered pair creation has become a hot research topic [2] and many works have examined how one could use additional external electric or magnetic fields to control the pair creation process [3-8]. The first studies date back to Sauter [9] and Schwinger [10]. In the limiting case of an infinitely extended electric field the long-time pair creation rate per volume Γ_S is given by the Schwinger expression (in atomic units):

$$\Gamma_S = \frac{E^{3/2}}{2\pi^2 c^{1/2}} \exp\left(-\frac{\pi c^3}{E}\right) \tag{1}$$

The typical spontaneous pair production requires strong electric fields of amplitude $E \geq 1.3\times10^{16}$ V/cm or $E \geq c^3$ in atomic units. We focus in this note on how the pair-creation process can be controlled by a static magnetic field that is perpendicular to the supercritical electric field. For infinitely extend fields [11], it is possible to Lorentz transform to a reference frame, in which the magnetic field vanishes and the electric field is given by $E_{eff}(\infty) = \sqrt{[E^2 - B^2]}$. In order to maintain supercriticality in this case, we require that $\sqrt{[E^2 - B^2]} \geq c^3$. This means that the magnetic field has to have at least the amplitude $B \geq \sqrt{[E^2 - c^6]}$ in order to shut off the pair-creation mechanism.

In this Letter we stress the importance of the finite extension of the interaction zone. It turns out that due to this finiteness the pair-creation process can be controlled for magnetic field strengths that are much smaller than previously assumed or suggested by the above Lorentz transformation based argument. In fact, we will show that in contrast to the predictions above (for homogenous fields) *any* magnetic field of *arbitrarily small* strength can bring the pair-creation process even to a complete halt, if its spatial extent W_B is just sufficiently large. This follows from the generalized condition for the onset of supercriticality $\sqrt{[E^2 W_E^2 - B^2 W_B^2]} \geq c^2$, which we will derive below, where W_E (or W_B) denotes the width of the electric (or magnetic) field. This shutoff scenario is dynamical in the sense that the pair-creation probability reveals an oscillatory behavior as a function of time. Space-time resolved quantum field theoretical simulations permit us to relate the shutoff to a re-opening of the mass gap of the Dirac energy spectrum and to associate the oscillations with electrically-dressed Landau levels.

If the electric field is spatially localized, the Schwinger expression [Eq. (1)] using $E_{eff}(\infty)$ becomes invalid as a description solely in terms of the field strength is no longer sufficient and one has to incorporate also the corresponding spatial information, or equivalently, choose scalar and vector potentials. Localized interaction zones are characterized by a sharp threshold condition for supercriticality that can be expressed only via a potential (such as $V>2c^2$, e.g.) and not in terms of the field alone. According to Eq. (1) any static electric field is capable of creating pairs, even with an infinitesmal amplitude. It is therefore not clear that we obtain a meaningful rate when the peak amplitude of a localized field is simply inserted in this formula.

To obtain a better estimate for the long-time creation rate we can use an expression, which was originally proposed by Hund [12] for the purpose of electric field alone:

$$\Gamma_H = 1/(2\pi) \int T(E)\, dE \qquad (2)$$

Here T(E) denotes the quantum mechanical transmission coefficient for an incoming electron with energy E that scatters off the same supercritical field configuration described by the electric and magnetic fields. In the special case of an infinite interaction zone this approach reproduces Eq. (1). Hund's formula can be generalized if a magnetic field is present. It turns out that in the limit of equal width for electric and magnetic fields, one can describe the scattering in this quantum mechanical framework by an effective scalar potential $V_{eff} = \sqrt{[V^2 - M^2]}$ that is related to the peak scalar and vector potentials V and M.

Quantum field theoretical (QFT) simulations with space-time resolution for external field configurations with arbitrary temporal and spatial characteristics are notoriously difficult as the requirements on CPU time and memory are presently still exorbitantly high. However, a recent work [13] has introduced a computational algorithm that permits us to study the decay process of the vacuum for the case where the magnetic field is perpendicular to the electric field and both fields have arbitrary spatial extensions along this polarization direction. It was shown that the three dimensional dynamics can be reduced to a quasi one-dimensional set of equations. In this particular configuration the canonical momentum along the E×B-direction is conserved and just has to be integrated over to obtain 3D data for the total pair creation.

While these high-performance computations still take several days, they permit us to compute time-dependent pair creation probabilities and spatial densities of the created electrons and positrons. In technical terms, a set of coupled time-dependent Dirac equations have to be solved repeatedly on a numerical space-time lattice. The Dirac Hamiltonian is given by $H = c\,\alpha\,[p-\mathbf{A}(\mathbf{r})/c] + c^2\,\beta + V(\mathbf{r},t)$, where α and β are the usual 4×4 Dirac matrices, and the three components of the vector potential are given by $A(\mathbf{r})=(0,M\,(\tanh(x/W_B)+1)/2,0)$ and the scalar potential is $V(\mathbf{r},t) = V\,(\tanh(x/W_E)+1)/2\,f(t)$. These assignments correspond to an E- and B-field pointing in the x- and z-direction, respectively, and both fields vary along x within a range of about 2W around x=0. The temporal pulse shape of the electric field is denoted by f(t). The maximum field strengths (at x=0) are given by $E= -dV(x)/dx \sim V/(2W_E)$ and $B = dA_y(x)/dx \sim M/(2W_B)$ so V and M are measured in c^2, W in units of 1/c, and E and B in c^3.

In order to compute the time-evolution of the electron-positron field operator, the Dirac equation has to be solved for each energy eigenstate of the entire Hilbert space associated with the Hamiltonian with V=0, but $A\neq 0$. The latter step is based on the development of sufficiently efficient split-operator algorithms, for more details see [14]. In these simulations, we have assumed that the magnetic field given is present all the time, while the supercritical electric field is turned on and off smoothly via f(t). The data presented below correspond to the pair-creation probability after the supercritical field has been turned off.

Let us now present the results of these simulations. In Figure 1 we show the temporal growth of the number of created electron-positron pairs N(t) as a function of the interaction time t for eight different sizes W_B of the magnetic field. All the other parameters, such as the strengths of both fields ($E=12.5c^3$, $B=0.6c^3$) and the size of the electric field ($W_E=0.1/c$) are the same. All curves show an identical short-time behavior, whose details are solely governed by the turn-on shape of the supercritical electric field. The associated created pairs are not necessarily induced by supercriticality, but are due to the high frequencies contained in the Fourier spectrum of the temporal turn-on pulse. In principle, these could be minimized by an adiabatic pulse, which due to the long interaction time is not practical from a computational point of view.

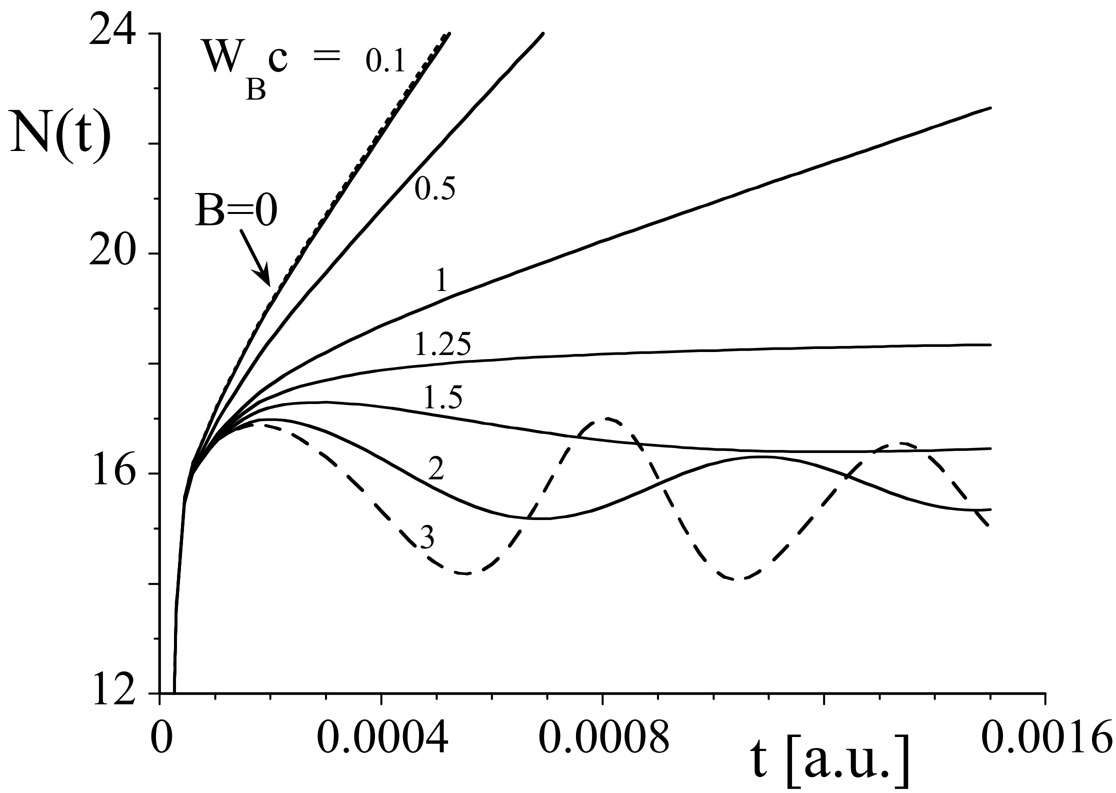
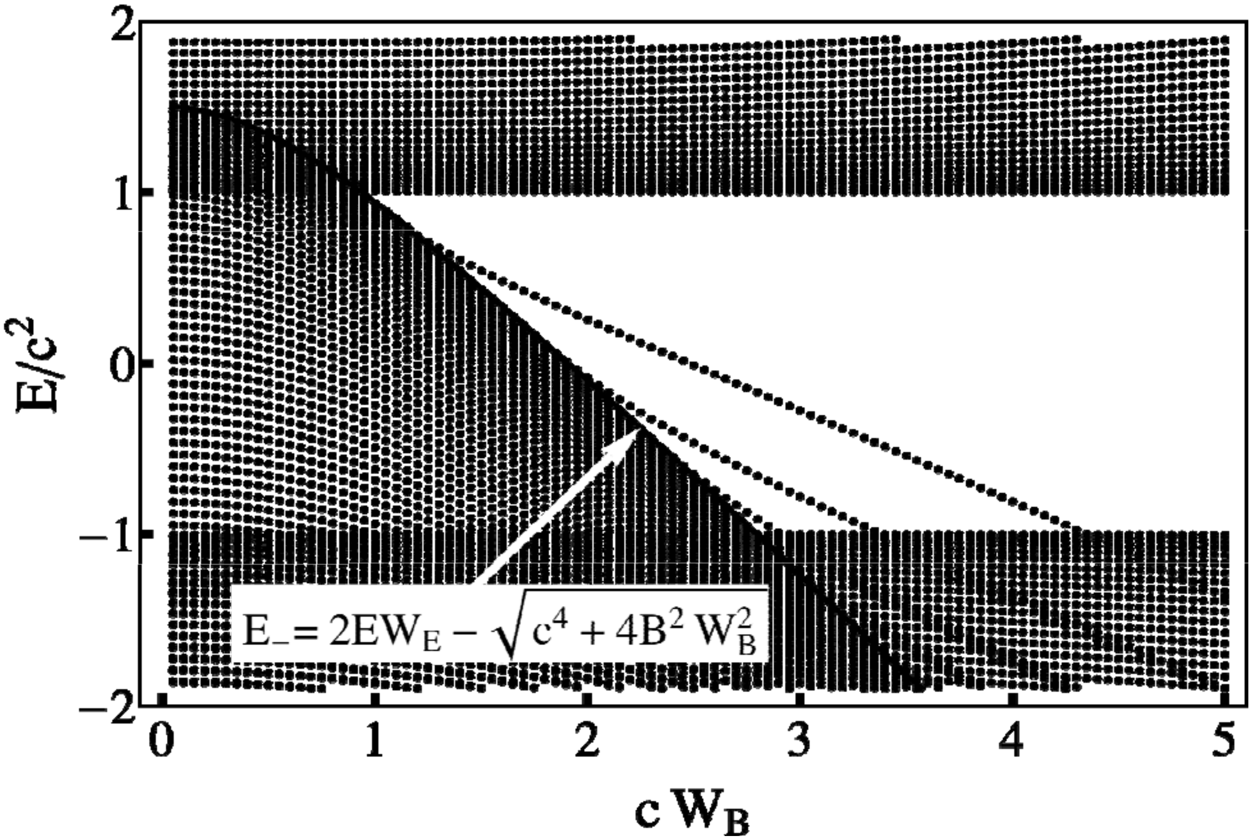


Fig.1 The final number of created electron-positron pairs as a function of the interaction time for eight different widths of the magnetic field. Parameters used include $E=12.5c^3$, $W_E=0.1/c$, $B=0.6c^3$. B=0 case is the upper dashed line.

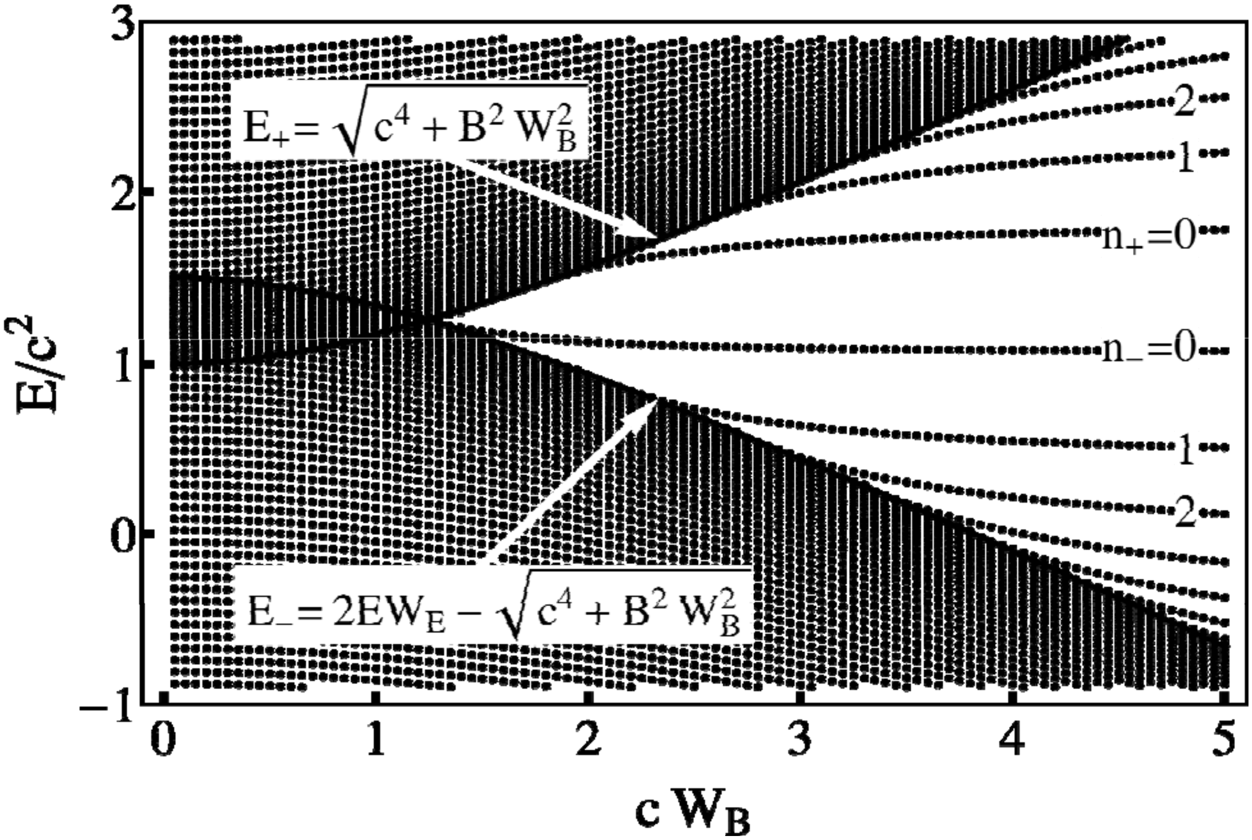
To set the scale for our discussion of the long-time behavior, for comparison we have also included (dashed curve) the data for B=0. Due to the fields' finite extensions the Schwinger rate of Eq. (1) does not give an accurate estimation of pair creation rate, but the slope still follows Hund's formula [Eq. (2)]. For example, the slope measured from Fig. 1 according to QFT is 1.461×10^4 while $\Gamma_H=1.419\times10^4$, an agreement within 2.9%. Please note that in the other eight curves the B-field was chosen so small ($E/B \approx 21$), such that for $W_B=W_E$ the asymptotic slope is nearly identical to the case for B=0. This is also consistent with the fact that in this case the effective electric field $\sqrt{[E^2-B^2]}$ ($=12.49c^3$) is only slightly smaller than E itself ($=12.5c^3$).

As the simulations are repeated for larger spatial extentions of the magnetic field, the situation changes drastically. The slope (long-term rate of pair creation Γ) of N(t) decreases rapidly as we increase W_B. For $W_B=5W_E$ the slope Γ decreased by 26% from 1.461×10^4 to 1.075×10^4, while for $W_B=10W_E$ it is only $\Gamma=3.420\times10^3$, a 76% reduction. For comparison, in this range the QFT creation rates Γ are amazingly well described by Hund's approach, which predicts Γ_H=1.0458×10^4 and 3.267×10^3 for W_B =5W_E and 10W_E. This corresponds to a mismatch of only 2.8% and 4.4%, respectively. If W_B≈12.5W_E the suppression reaches 100%, corresponding to Γ=0 and a complete shutoff of pair creation due the magnetic field. This shut-off might generalize to more complicated geometries where the electric field is also localized in the other two spatial directions, as long as it is encompassed by the magnetic field. It is equally remarkable that for even larger values of W_B the pair creation starts to exhibit an oscillatory behavior, moving around a constant amount. For this range of W_B >1.25/c even the rate predicted by the Hund formula Eq. (2) begins to become inapplicable.

Let us now illuminate these findings from a spectral perspective. This will give us a physical picture for the mechanisms leading to the shutoff and the associated oscillations and it will also provide us with analytical estimates for how the shutoff value for W_B and the frequency the oscillation depend on the characteristics of the two fields. To do so we analyze the energy eigen-spectrum of the Dirac Hamiltonian where the electric field is turned on. Prior studies of the supercritical breakdown of the vacuum triggered by a supercritical Coulomb field have associated the onset condition for supercriticality with the "diving" [1] of the ground state into the continuum of negative energy eigenstates. In our case, a supercritical electric field leads to the complete overlap of the positive and negative energy continua.



2
1
0
−1
−2
E/c²
0
1
2
3
4
5
c W_B
E₋= 2EW_E − √(c⁴ + 4B² W_B²)



3
2
1
0
−1
E/c²
0
1
2
3
4
5
c W_B
E₊= √(c⁴ + B² W_B²)
E₋= 2EW_E − √(c⁴ + B² W_B²)
2
1
n₊=0
n₋=0
1
2

Fig.2 The energy spectrum of the total Hamiltonian as a function of the spatial size of the magnetic field W_B. All parameters are the same as in Fig. 1 (except for numerical box length is 0.6, instead of 1.0 as used in Fig. 1). The top (bottom) figure shows the spectra for $p_y=0$ ($p_y=-BW_B/c$). It is apparent that when $W_B > 1.25/c$ the two continua begin to separate from each other. In the band gap area new discrete energy levels emerge.

In Figure 2 we have displayed the numerically obtained energy eigenvalues of H with for 100 different sizes W_B. We graph only those that are dynamically important for the initial vacuum state. All parameters are identical to those used in Figure 1. The top figure is for the simpler case of $p_y=0$ and gives us a first qualitative insight. For small magnetic field widths W_B the black regions show the complete overlap between the positive and negative energy states, consistent with the fact that the system is supercritical and pairs are created continuously as the positive slope Γ in Fig. 1 indicated. As W_B is increased the previously overlapping positive and negative energy states start to separate from each other such that the mass gap opens again and also new discrete states emerge.

A more quantitative picture is obtained for the dynamically most relevant momentum $p_y= -BW_B/c$. As a result, the continuum edge E_+ (= c^2 for $p_y=0$) curves upwards following $E_+=\sqrt{[c^4+B^2 W_B^2]}$. At $W_B=1.25/c$, the mass gap opens. This is precisely where in Fig.1 the system changed from supercritical to subcritical and Γ vanished.

The lower continuum edge is described by $E_-= 2\, E\, W_E -\sqrt{[c^4+B^2 W_B^2]}$ as indicated by the dark curve superimposed on the spectra. The region of supercriticality for finite interaction regions is therefore characterized by $E_+< E_-$, or equivalently by $\sqrt{[E^2W_E^2-B^2W_B^2]} \geq c^2$, as we mentioned in the introduction. From this generalized condition for supercriticality we can easily set $\sqrt{[E^2W_E^2 - B^2W_B^2]} = c^2$ to determine the characteristic shutoff width for the magnetic field, denoted by $W_{B\dagger}$. It corresponds to the crossing point of both continua, $E_+=E_-$, and we obtain $W_{B\dagger}=B^{-1} \sqrt{(E^2W_E^2-c^4)}$. In other words, even the most minute magnetic field strength B is fully sufficient to turn the entire pair creation process off, if its spatial size W_B is chosen only sufficiently large, $W_B>W_{B\dagger}$.

In addition to the separation of the two continua, the data in Fig. 2 reveal a new set of discrete states that emerge as the width W_B is increased beyond $W_{B\dagger}$. While the two continuum edges E_+ and E_- are symmetric around energy E W_E, the energies of the dynamically relevant discrete states are not. This is because the positive and negative energy states with the same spin have different magnetic moments. From Fig. 2 we find the first bound level (with negative energy) occurs at $W_B \geq 1.498/c$ while the next several levels (with alternating signs of energy) require $W_B \geq 2.104/c$, 2.504/c, 2.891/c, 3.194/c, 3.495/c, 3.703/c, respectively. We have assigned these discrete states with the quantum numbers $n_\pm$ and the corresponding energies $E(n_\pm)$. While each continuum and discrete state in Fig. 2 is a complicated superposition of free energy eigenstates with positive and negative energies one could view the discrete states as electrically dressed Landau levels.

The actual energies of the discrete levels at each W_B value in Fig. 2 permit us also to estimate the frequencies for N(t) in Fig. 1. For example, for $W_B=3/c$ the energies of the ground state of the negative levels and the ground state of the positive levels would predict an oscillation period of $2\pi/(E(0_+)-E(0_-)) = 5.47\times10^{-4}$, which agrees (within 2%) with the observed period of 5.36×10^{-4} in Fig. 1. For $W_B \to\infty$, the period approaches $2\pi/(\sqrt{c^4+4Bc}-2.5c^2+\sqrt{c^4+2Bc})$, which for our value of B is 4.05×10^{-4}.

In the limit of $W_B\to\infty$, the energies become independent of W_B and approach asymptotically the values $E(n_+)=\varepsilon^+_{2n+1}$ and $E(n_-)=\varepsilon^-_{2n+1}+V_0$, where $\varepsilon^\pm_n = \pm\sqrt{c^4+(2n+1+\sigma)Bc}$ are the (positive and negative) energy Landau levels with $\sigma = +1$ or -1. Here we choose different values of σ for the positive and negative energies to keep the spin the same, as transition between the positive and negative energy manifolds does not change the spin. Here $E(n_\pm)$ denotes the n-th positive (n_+) or negative (n_-) discrete state in Fig. 2, while $\varepsilon^\pm_n$ represents the n-th positive (ε^+_n) or negative (ε^-_n) Landau level, respectively. The scalar potential can be regarded essentially as a step jump with magnitude V. As V is sufficiently large, the effective potential in the combined electric and magnetic fields becomes a half of a harmonic potential, which is closed on one end by a steep potential wall. In such a potential, the wave function must vanish at the potential wall. As a result the Landau levels with n = 0, 2, 4… are absent and only odd orders survive.

To see the validity of the above energy formula for large magnetic field widths we pick $n_+=0$ to find $E(0_+)=1.84c^2$. Compare this energy value with the energy for the corresponding positive state of the largest W_B in Fig.2, $1.78c^2$, the deviation amounts to only 3.37%. For $n_+=1$, the deviation between $E(1_+)=2.41c^2$ and the corresponding numerical energy of $2.23c^2$ is 3.59%. For the negative level of $n_-=0$, the deviation between $E(0_-)=1.02c^2$ and the corresponding numerical energy of $1.08c^2$ is 5.56%.

To summarize, we have shown that the pair-creation process in a supercritical electric field of finite extension can be remarkably sensitive to very small magnetic fields, if their direction is chosen perpendicular to the electric field. This finite size phenomenon cannot be predicted by Schwinger-like rate formulas in terms of traditional effective electric fields or Hund's generalizations. The complete shutoff is related to a re-opening of the mass gap when the magnetic field's width exceeds the cyclotron radius. This condition should be realized rather easily experimentally once the required supercritical electric fields become available in the lab. For instance, a magnetic field with $W_B \approx$1cm can inhibit pair creation, if its magnitude is only 0.2T, which is 200 times smaller than the magnetic fields available in the labs [15]. While the numerical values that we used in this note were used for computational convenience only and serve as a proof of principle, the explicit analytical estimates are scalable to the parameters for the expected experiments.

This work raises also several interesting questions. For example, the onset of the shutoff is related to the corresponding classical gyration radius and one could therefore conjecture that some aspects of the suppression mechanism is related to the fermionic Pauli-blocking, where the magnetically induced-trapping permits the electrons and positrons to return to the supercritical interaction zone. If Pauli-blocking were a key mechanism, then a simulation based on the corresponding bosonic system should result in an exponential enhancement [16] of pair-creation for these field configurations. Could this found sensitivity due to finite size also be used to enhance the electron-positron pair creation? Prior works have shown that if the magnetic field direction is other than perpendicular or even time-dependent, the rates could be enhanced.

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