Quantized Casimir Force

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We investigate the Casimir effect between two-dimensional electron systems driven to the quantum Hall regime by a strong perpendicular magnetic field. In the large separation (d) limit where retardation effects are essential we find i) that the Casimir force is quantized in units of $3\hbar c\alpha^2/8\pi^2 d^4$, and ii) that the force is repulsive for mirrors with same type of carrier, and attractive for mirrors with opposite types of carrier. The sign of the Casimir force is therefore electrically tunable in ambipolar materials like graphene. The Casimir force is suppressed when one mirror is a charge-neutral graphene system in a filling factor $\nu = 0$ quantum Hall state.

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Introduction — The Casimir effect is an intriguing quantum electrodynamic phenomenon in which the vacuum energy of the electromagnetic field is altered by the presence of two closely-spaced mirrors coupled to the field, resulting in a measurable force between them. The force between two parallel metal plates, for example, is attractive because the plates restrict the number of vacuum electromagnetic modes present in the space between them. Growing interest in this facscinating topic has been driven by improving Casimir force measurement capabilities at small separations from 1 μm down to about 10 nm [1–4] and by new materials. When semiconductors like Si are used as mirrors instead of conventional dielectric materials and metals, for example, it is possible to control the Casimir force by optical or electrical carrier density modulation [5]. This property, combined with recent advances in micro-electromechanical systems (MEMS) and nano-electromechanical systems (NEMS), could enable new electromechanical applications.

At submicron distances, the Casimir force between materials usually dominates the gravitational force. Measurement of the Casimir force has therefore played an important role in the search for the non-Newtonian gravitational forces suggested by a number of unification theories [6]. Because the Newtonian gravitational law has been poorly tested below 10 μm, an accurate quantitative understanding of the Casimir effect is key to the identification of any new gravitational force law which might prevail at small length scales. Theories of the Casimir effect have in fact been employed [7, 8] in establishing constraints on the magnitude of non-Newtonian gravitational effects. In these studies measured force values are compared with known contributions from Newtonian gravity, and Casimir forces computed using a separate set of measurements of the detailed dielectric properties of the test materials [9]. The Casimir force can be sensitive to unintended variations in impurity density, degree of surface roughness, and sample thickness. The errors associated with Casimir force estimation are therefore difficult to control and this in turn crucially limits the precision of bounds placed on hypothetical gravitational forces. A way to suppress or even neutralize the Casimir effect at small length scales is therefore desirable, enabling a more sensitive direct measurement of gravity.

In this Letter, we address one possibility for suppressing the Casimir effect by developing a theory of the Casimir force between two-dimensional (2D) electron systems in the presence of strong perpendicular magnetic fields. The development of quantized Landau levels and the associated quantum Hall effect opens up a new regime for investigations of the Casimir effect. We show that in ambipolar materials like graphene the Casimir force is electrically tunable between attractive and repulsive values, and that in the large-separation relativistic regime, the Casimir force is quantized. Importantly, a strongly suppressed Casimir force can be achieved under a high magnetic field by using charge-neutral graphene sheets as mirrors.

Theory — When spatial dispersion in the mirrors is negligible, the Casimir effect is determined by their local (*i.e.*, $q = 0$) charge and current response functions. In this limit, the Casimir energy (per unit area) can be elegantly expressed in terms of the reflection matrices of the mirrors [10]

$$E = \frac{\hbar}{4\pi^2}\int_0^\infty \mathrm{d}q_\perp q_\perp \int_0^{q_\perp c} \mathrm{d}\omega \;\mathrm{tr}\ln\left[\mathbb{I} - r^{\mathrm{L}}(iq_\perp, i\omega) r^{\mathrm{R}}(iq_\perp, i\omega) e^{-2q_\perp d}\right], \quad (1)$$

where $r^{\mathrm{L}}, r^{\mathrm{R}}$ are the reflection matrices of the left (L) and right (R) mirrors separated at a distance d apart, $q_\perp = \sqrt{(\omega/c)^2 - q^2}$ is the component of photon momentum perpendicular to the mirror planes, and q the in-plane momentum component. It is worthwhile to emphasize that Eq. (1) includes contributions from both electromagnetic propagating and evanescent modes [10].

We apply Eq. (1) to mirrors made of ultrathin films that can be adequately modeled as a quasi-2D layer; this class of systems includes atomically thin materials like graphene and bilayer graphene [11]. Several groups have studied the Casimir force between graphene sheets in the absence of an external magnetic field [12]. When a field is present the optical characteristics of a 2D mirror depend on its longitudinal σ_{xx} and Hall σ_{xy} conductivities, where x corresponds to the direction parallel and y perpendicular to the plane of incidence. For general angle of incidence θ, straightforward calculations yield [13] the following expressions for the components of the 2×2 reflection matrix of a 2D mirror:

$$\begin{aligned} r_{xx} &= -2\pi \left\{ \bar{\sigma}_{xx}(\omega)/\lambda + 2\pi \left[\bar{\sigma}^2_{xx}(\omega) + \bar{\sigma}^2_{xy}(\omega) \right] \right\} / R, \\ r_{xy} &= -r_{yx} = -2\pi \bar{\sigma}_{xy}(\omega)/R, \\ r_{yy} &= -2\pi \left\{ \bar{\sigma}_{xx}(\omega)\lambda + 2\pi \left[\bar{\sigma}^2_{xx}(\omega) + \bar{\sigma}^2_{xy}(\omega) \right] \right\} / R, \quad (2) \end{aligned}$$

where $\lambda = \cos\theta = q_\perp/(\omega/c)$, $R = 1 + 2\pi\bar{\sigma}_{xx}(\lambda + 1/\lambda) + 4\pi^2[\bar{\sigma}^2_{xx} + \bar{\sigma}^2_{xy}]$, and $\bar{\sigma}_{xx,xy} = \sigma_{xx,xy}/c$ are *dimensionless* optical conductivities normalized by c.

We now apply a strong magnetic field normal to two parallel 2D mirrors that are separated by a distance d. When d/c is larger than characteristic electronic time scales the retardation effects captured by Eq. 1 are essential. In the quantum Hall regime characterized by well-resolved Landau levels $\Delta \gg \Gamma$ (where Δ is the characteristic energy for inter-Landau level transitions and $\Gamma \lesssim 1 - 10\,\mathrm{meV}$ the typical disorder broadening energy), the large d limit holds for $d/c \gtrsim \hbar/\Delta$. For graphene, $\Delta \approx \hbar v/l_B = 25.78\sqrt{B[T]}\,\mathrm{meV}$ where $v = 10^6\,\mathrm{ms}^{-1}$ is the band velocity in graphene and $l_B = \sqrt{\hbar/eB}$ the magnetic length, $d \gtrsim 7.71/\sqrt{B[T]}\,\mu\mathrm{m}$. Due to the exponential dependence on d in Eq. (1), electromagnetic correlations between mirrors separated by large distances are carried by long-wavelength virtual photons; the integral in Eq. (1) is therefore dominated by low-frequency contributions. The value of the Casimir energy is thus determined at large separations by the static longitudinal and Hall conductivities $\sigma_{xx,xy}(\omega = 0)$. If the applied magnetic field is strong enough that the mirrors sit on well-formed quantum Hall plateaus, then the low-frequency longitudinal conductivity vanishes $\bar{\sigma}^{\mathrm{L,R}}_{xx} \simeq 0$ and the Hall conductivity $\bar{\sigma}^{\mathrm{L,R}}_{xy} \simeq \nu_{\mathrm{L,R}}\alpha/2\pi$, where $\alpha = 1/137$ is the fine structure constant and $\nu_{\mathrm{L,R}} = 2\pi\hbar n_{\mathrm{L,R}}/(eB)$ are the Landau level filling factors of the left and right mirrors with carrier density $n_{\mathrm{L,R}}$. Performing the frequency and momentum integrations in Eq. (1), we obtain the following closed-form analytic result for the Casimir energy $E = -(\hbar c/8\pi^2 d^3)\mathrm{Re}\{\mathrm{Li}_4\{[-\nu_\mathrm{L}\nu_\mathrm{R}\alpha^2(1 - \nu_\mathrm{L}\nu_\mathrm{R}\alpha^2) + i|\nu_\mathrm{L}||\nu_\mathrm{R}||\nu_\mathrm{L} + \nu_\mathrm{R}|\alpha^3]/[(1 + \nu^2_\mathrm{L}\alpha^2)(1 + \nu^2_\mathrm{R}\alpha^2)]]\}\}$, with Re denoting the real part and $\mathrm{Li}_n(z)$ the polylogarithm function of order n. Expanding the expression in $\alpha^2 \ll 1$ we obtain the leading contribution to the Casimir force per unit area $F = -\partial E/\partial d$ between two quantum Hall mirrors

$$F = \frac{3\hbar c}{8\pi^2}\alpha^2 \frac{\nu_\mathrm{L}\nu_\mathrm{R}}{d^4}. \quad (3)$$

Eq. (3) is a central result of this Letter. The integer and fractional quantum Hall effects are characterized by remarkably flat plateaus in the Hall conductivity at discrete integer and small-denominator fractional multiples (*i.e.*, filling factor ν) of e^2/h. Therefore we conclude that under quantum Hall conditions, the large-separation asymptotic Casimir force will be approximately quantized in integer or fractional multiples of $3\hbar c\alpha^2/8\pi^2 d^4$. Higher order corrections to Eq. (3) are $\sim \mathcal{O}(\alpha^4)$ and negligible. Comparing with the well-known result between perfect metals $F_0 = -\hbar c\pi^2/240d^4$, one notices that the Casimir force is suppressed by a factor $\propto \alpha^2$.

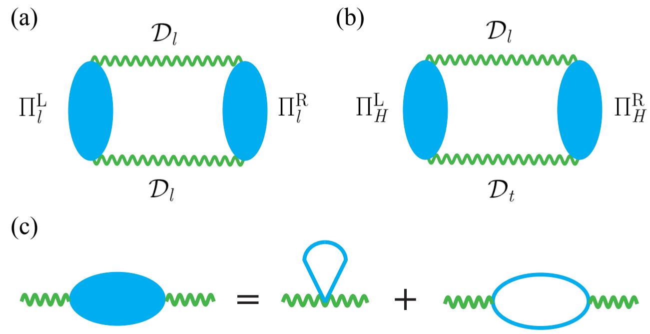

FIG. 1: (Color online). Feynman diagrams for the ground state correlation energy between mirrors, where green wavy lines represent the photon Green function $\mathcal{D}$ and filled blue/dark bubbles represent the current-current correlation function Π. (a). Leading-order diagram for the longitudinal contribution E_l to the ground state correlation energy. The transverse contribution E_t is given by replacing $\mathcal{D}_l, \Pi_l \rightarrow \mathcal{D}_t, \Pi_t$. (b) Leading-order diagram for the Hall contribution E_H, this diagram is to be counted twice because an equivalent diagram results from interchanging $\mathcal{D}_l$ and $\mathcal{D}_t$ and using $\Pi_H \equiv \Pi_{xy} = -\Pi_{yx}$. (c) Diagram for the current-current correlation function. The first diagram on the right represents the diamagnetic contribution whereas the second represents the paramagnetic contribution.

It is informative to also understand Eq. (3) from the perspective of quantum electrodynamic perturbation theory in which the Casimir effect arises from mutual electromagnetic correlations between two separated systems and therefore contributes to the ground-state [14–16] energy of the coupled system. In the following, we adopt the axial gauge with a zero electromagnetic scalar potential $\phi = 0$. The longitudinal (photon momentum parallel to vector potential) $\mathcal{D}_l$ and transverse (photon momentum perpendicular to vector potential) $\mathcal{D}_t$ interlayer photon propagators [17] are given as a function of mirror separation by $\mathcal{D}_l(q_\perp, \omega) = -(2\pi i c q_\perp/\omega^2)\exp(iq_\perp d)$ and $\mathcal{D}_t(q_\perp, \omega) = -[2\pi i/(q_\perp c)]\exp(iq_\perp d)$. In the absence of a magnetic field, the longitudinal contribution to the interaction energy is given to leading order in α by the diagram in Fig. 1a:

$$E_l = -\hbar \int \frac{\mathrm{d}\omega}{2\pi} \sum_q \Pi^\mathrm{L}_l \mathcal{D}_l \Pi^\mathrm{R}_l \mathcal{D}_l, \quad (4)$$

where $\Pi^{\mathrm{L,R}}_l$ is the longitudinal component of the current-current correlation tensor of the mirrors. By virtue of the continuity equation for charge density, the longitudinal current-current correlation function is related to the density-density correlation function χ by $\Pi_l(q,\omega) = (e\omega/q)^2\chi(q,\omega)$. Eq. (4) therefore describes the leading-order contribution to the Casimir effect due to fluctuations of charge density. Because $\Pi_l \propto \sigma_{xx}$ is independent of the sign of charge carriers, the Casimir force between two parallel-plate mirrors placed in vacuum is attractive for like or unlike carrier densites. The transverse interaction energy E_t gives a comparable contribution to Eq. (4) with integrand $\Pi_t^{\rm L}\mathcal{D}_t\Pi_t^{\rm R}\mathcal{D}_t$ where $\Pi_t^{\rm L,R}$ is the transverse component of the current-current correlation tensor.

When a magnetic field is present, the Casimir energy acquires a contribution that depends on the Hall current-current correlation function $\Pi_H(q,\omega)$, for which the leading-order diagram (Fig. 1b) yields

$$E_H = 2\hbar\int\frac{d\omega}{2\pi}\sum_q \Pi_H^{\rm L}\mathcal{D}_t\Pi_H^{\rm R}\mathcal{D}_l. \quad (5)$$

Expressed in terms of the Hall conductivity, $\Pi_H = -i\omega\sigma_{xy}$ in Eq. (5). Changing integration variable from the in-plane momentum component q to the perpendicular component $q_\perp$, and performing the complex plane rotations [10] $q_\perp \to iq_\perp$ and $\omega \to i\omega$, we obtain

$$E_H = \frac{2\hbar}{c^2}\int_0^\infty dq_\perp q_\perp \int_0^{q_\perp c} d\omega\sigma_{xy}^{\rm L}(iq_\perp, i\omega)\sigma_{xy}^{\rm R}(iq_\perp, i\omega)^{-2q_\perp d}, \quad (6)$$

showing that the Hall contribution E_H depends on the product of the Hall conductivities of the two mirrors, and is therefore sensitive to the signs of their charge carriers. To leading order in α, the Casimir energy in a magnetic field is given by the sum of these three contributions $E = E_l + E_t + E_H$.

When the magnetic field is strong, Landau quantization of the electronic energy spectrum of the mirror leads to energy gaps Δ. We confine our discussion to low temperatures $k_{\rm B}T \ll \hbar c/d \lesssim \Delta$ at which the quantum Hall effect is established and thermal contributions to the Casimir force can be ignored. In this limit density fluctuations and hence the longitudinal and transverse contributions to the Casimir energy are suppressed because the 2D quantum Hall systems are incompressible [18]. Hence, Hall-like quantum fluctuations of the current dominates. Evaluating the Hall energy in Eq. (6), we can rederive the Casimir force result Eq. (3) obtained earlier from macroscopic electrodynamic considerations.

Repulsive Casimir Effect — Repulsive Casimir forces are unusual and have been realized for the first time in recent experiments involving test bodies immersed in a liquid medium [19]. An important implication of Eq. (3) is that repulsive Casimir forces can be easily obtained in ambipolar 2D quantum Hall systems by gating them to like-sign carrier densities. We demonstrate this repulsive Casimir effect by considering the case of two graphene sheets, numerically evaluating the Casimir force $F = -\partial E/\partial d$ from the Casimir energy E given by Eqs. (1)-(2). We obtain the graphene sheet's optical conductivity tensor from the Kubo formula [20]. Fig. 2 shows the calculated Casimir force versus d between (1) an electron-doped graphene sheet and a hole-doped graphene sheet (grey/red curve), and (2) between two electron-doped graphene sheets (dark/black curve). The plot is normalized by the Casimir force between two perfect metals $F_0 = -\hbar c\pi^2/240d^4$. For small separations the Casimir force is attractive for both cases. For separations large enough that retardation effects are important however, contributions from charge density fluctuations are exponentially suppressed, Hall current fluctuations gain dominance, and repulsive forces can appear. Our numerical results for both cases settle onto the values predicted by the analytic result Eq. (3) beyond a threshold separation, $d_{\rm QH}$. For $d > d_{\rm QH}$ we find Casimir attraction for opposite-sign carrier densities and repulsion for like-sign carrier densities. The crossover length $d_{\rm QH}$ can be estimated by expanding Eq. (1) to one higher-order term in $1/d$ and α beyond the limit given by Eq. (3). We find [20] that $d_{\rm QH} \gtrsim l_B(c/v)f(\nu_{\rm L},\nu_{\rm R})$, where $f(\nu_{\rm L},\nu_{\rm R})$ is a monotonic increasing function of $\nu_{\rm L,R} \propto 1/B$ of order $\mathcal{O}(1)$, consistent with the estimate obtained from the criterion of the long-distance regime $d/c \gtrsim \hbar/\Delta$. In agreement with numerical results in Fig. 2, $d_{\rm QH}$ decreases with magnetic field.

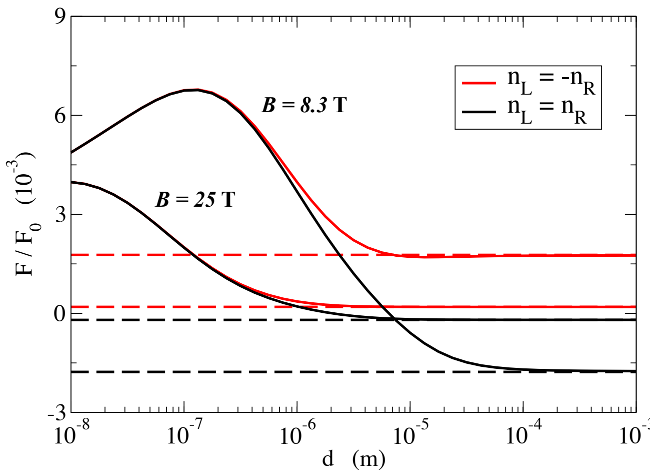

FIG. 2: (Color online). Casimir force F normalized by the perfect metal value $F_0 = -\hbar c\pi^2/240d^4$ as a function of separation d in a strong magnetic field. Solid lines correspond to numerical results and dashed lines to the analytic large d limits in Eq. (3). The case with opposite-sign carrier densities on the two graphene sheets $n_{\rm L} = 3\times10^{11}\,{\rm cm}^{-2} = -n_{\rm R}$ is depicted by the red (grey) curve whereas the case with the same-sign carrier density $n_{\rm L} = n_{\rm R} = 3\times10^{11}\,{\rm cm}^{-2}$ is depicted by the black (dark) curve. At these carrier densities, the magnetic fields $B = 25, 8.3\,{\rm T}$ correspond to filling factors $|\nu| = 2, 6$. These calculations are valid for $d \gg l_B$ where $l_B = 257/\sqrt{B[T]}\,\mathring{A}$ is the magnetic length and weak disorder.

Casimir Force Quenching — We have shown that

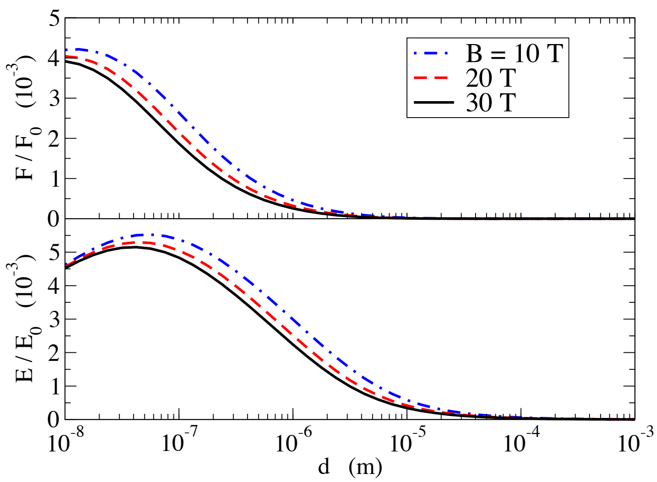

FIG. 3: (Color online). (a) Casimir force versus separation for magnetic field strengths $B = 10, 20, 30\,\mathrm{T}$ between two charge-neutral graphene sheets. (b) Casimir energy between a charge-neutral graphene sheet and a half-space composed of an insulator with dielectric constant ϵ, normalized by the perfect metal value $E_0 = -\hbar c\pi^2/720d^3$. In the large d regime where the suppression effect becomes prevalent, the insulator is characterized by its low-frequency dielectric behavior, and we take $\epsilon \simeq 4$ corresponding to SiO_2 or hexagonal BN.

the asymptotic Casimir force between 2D quantum Hall insulators follows the same power law as the Casimir force for ideal thick metals but is weaker by a factor of $\pi^4\nu_L\nu_R\alpha^2/90$ where α is the fine structure constant. At the same time it is qualitatively stronger than the Casimir force between ordinary 2D insulators which falls off especially rapidly (like d^{-6}) at large distances. For the special case of charge-neutral graphene in a $\nu = 0$ quantum Hall state [21], the leading Hall contribution Eq. (3) to the Casimir force vanishes, and the system behaves like an ordinary 2D insulator, with the Casimir force given asymptotically at large d by $F = -9\hbar c\,\sigma_{xx}^{\mathrm{L}\,\prime}(0)\sigma_{xx}^{\mathrm{R}\,\prime}(0)/2d^6$ where $\prime$ denotes a derivative with respect to frequency [20]. Fig. 3(a) shows the calculated Casimir forces between two parallel charge-neutral graphene sheets as a function of magnetic field strength. The force value is substantially suppressed for large separations $d \gtrsim 10\,\mu\mathrm{m}$ at a typical field strength of $10\,\mathrm{T}$, indicating that the relativistic-regime Casimir force is quenched by strong magnetic fields. In this regime, we find [20] that

$$F = -\frac{9\hbar c^3}{4\pi^2 d^6}\alpha^2\left(\frac{l_B}{v}\right)^2\left[\frac{1}{2} + h(1)\right]^2, \qquad (7)$$

where $h(n) = \sum_{m=n}^{\infty} 1/\left(\sqrt{m} + \sqrt{m+1}\right)^3$ and $h(1) \approx 0.25$. For $d \simeq 10 - 100\,\mu\mathrm{m}$, F is suppressed by $\sim 10^{-8} - 10^{-10}$ compared to the ideal metal value F_0.

Because of experimental difficulties in maintaining precise parallelism between two plates, it is common in Casimir force measurements to replace one of the plates by a sphere to mimic the parallel-plates geometry at the point of closest separation. It is therefore essential to ascertain whether the quenching effect of the Casimir force also survives under this geometry. When the sphere radius $R \gg d$, the proximity force approximation [22] provides an accurate determination of the Casimir force as $F = 2\pi RE$, commensurate with the Casimir energy E per unit area in the same geometry with $R \to \infty$ (*i.e.* a plate interacting with an infinite half-space). To capture the behavior of the Casimir force in the sphere-plate geometry we have therefore numerically computed the Casimir energy between a charge-neutral graphene sheet and a dielectric half-space as shown in Fig. 3(b). The Casimir energy again drops rapidly, and does so at a larger value of d due to the higher opacity of the dielectric region. The quenching effect of the Casimir force should therefore remain observable in a sphere-plate geometry with a charge-neutral graphene sheet in the $\nu = 0$ quantum Hall state.

The strong quantum Hall effects of graphene 2D electron systems might be attractive for circumstances where a weak Casimir force is desirable, such as in experimental tests of the short-distance gravitational law. The $\nu = 0$ state in particular has the exceptionally weak Casimir force characteristic of 2D insulators. The quantum Hall effect can also be used to gate the graphene sheet accurately to charge neutrality, ensuring that electrostatic forces are absent.

In conclusion, we find that the large-separation Casimir force asymptote is quantized in the quantum Hall regime, and that in ambipolar systems it can be tuned electrically between attractive and repulsive values. When one of the mirrors is charge neutral, strong suppression of the Casimir effect can be achieved. Our findings suggest that the use of two-dimensional electronically gapped materials offer a new strategy for control of the Casimir effect.

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