Steady-State Many-Body Entanglement of Hot Reactive Fermions

Michael Foss-Feig, Andrew J. Daley, James K. Thompson, and Ana Maria Rey

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Steady-state many-body entanglement of hot reactive fermions

Michael Foss-Feig,[1] Andrew J. Daley,[2] James K. Thompson,[1] and Ana Maria Rey[1]

[1] *JILA, NIST, and Department of Physics, University of Colorado, Boulder, CO 80309-0440, USA*
[2] *Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA*

Entanglement is typically created via systematic intervention in the time evolution of an initially unentangled state, which can be achieved by coherent control, carefully tailored non-demolition measurements, or dissipation in the presence of properly engineered reservoirs. In this paper we show that two-component Fermi gases at $\sim \mu$K temperatures naturally evolve, in the presence of *reactive* two-body collisions, into states with highly entangled (Dicke-type) spin wavefunctions. The entanglement is a steady-state property that emerges—without any intervention—from uncorrelated initial states, and could be used to improve the accuracy of spectroscopy in experiments with fermionic alkaline earth atoms or fermionic groundstate molecules.

Many-body entangled states are known to be useful for quantum computing, quantum teleportation and cryptography protocols [1], and precision metrology [2]. With these applications as motivation, the physics community has invested tremendous effort in preparing, stabilizing, and measuring entangled systems. Much of this effort has relied on coherent (Hamiltonian) dynamics to arrive at entangled states starting from less exotic states with only classical correlations. However, these approaches typically suffer from the necessity to either carefully engineer interactions between particles or to prepare extremely pure and specific initial states (or both). A bottom up implementation of coherent control has yielded entangled states of up to 14 atoms with relatively high fidelity [3] (in ion experiments), and a top down approach has yielded weakly entangled states in a Bose Einstein condensate of $\sim 10^4$ neutral atoms [4]. A promising alternative to coherent control is the collective-nondemolition measurement of some observable with a finite variance in an initially classical state. Such approaches have been used to generate entanglement (in the form of spin squeezing) amongst as many as 10^6 cold thermal atoms [5]. However, collective and coherence preserving measurements are generically difficult to make, and the induced nonclassical correlations are typically weak.

In this paper, we show that two-component nondegenerate fermionic gases can be driven by reactive s-wave two-body collisions into steady-state spin configurations that, for a given value of the saturated particle number, are pure and highly entangled. The entanglement comes in the form of Dicke states [6], in which the spin-wavefunction is fully symmetric under interchange of the particles (with the burden of fermionic antisymmetry being taken up entirely by the motional degrees of freedom). Such states have been sought in experiments with ultracold bosons for use in Heisenberg-limited phase measurements [7], however these approaches typically suffer from the necessity to reach extremely cold temperatures (for the validity of a two-mode approximation in a double-well potential) or to employ Feshbach resonances [8] (to enhance spin exchange interactions for two-component Bose Einstein condensates). The only requirements to achieve such entanglement in the steady-state of lossy non-degenerate fermions are to have an SU(2) invariant single-particle Hamiltonian (in the pseudo-spin degrees of freedom) and a significant separation of timescales between s-wave and p-wave collisions, with the second requirement typically being satisfied for temperatures in or below the μK range.

Because the Dicke type of spin-entanglement persists in the steady-state of dissipative dynamics, we do not rely on the highly controlled coherent manipulation that is typical of spin-squeezing experiments with bosons. Driven, dissipative preparation of nontrivial steady-states has been considered before in the context of many-body atomic systems [9–11], and has been achieved recently in [12, 13]. In contrast to these examples, the mechanism described here is intrinsic and generic to a variety of interesting and experimentally relevant systems, such as fermionic alkaline-earth atoms (AEAs) and fermionic dipolar molecules, and does not require any special engineering of the system-reservoir coupling. After presenting calculations in support of our claims, we discuss the possible realization of such steady states in an experiment. In particular, we will propose a simple proof of principle experiment in which the steady-state entanglement can be revealed via Ramsey spectroscopy of the ${}^1S_0$ to ${}^3P_0$ clock transition of an AEA [14, 15]. In this case, we will see that the interferometric precision stays relatively constant even as most of the particles are lost (all but $\sim \sqrt{\mathcal{N}}$ in the long time limit), signaling the development of quantum correlations and the pursuant violation of the standard quantum-limit. The total loss of precision (due to particle loss) exactly cancels the gain due to the growth of entanglement. However, a persistent precision under loss of particles can provide enhanced spectroscopic accuracy. In particular, interrogation of an atomic clock with fewer atoms will reduce $p-$wave elastic[16] and inelastic[17] collisions, which are a leading source of inaccuracy for states immune to s-wave interactions [18] (e.g. spin-polarized fermions).

Our description of spin-$\frac{1}{2}$ fermions with two-body reactive collisions relies on the formalism detailed in Refs. [19–21], generalized for fermions, where we assume the

temperature to be sufficiently low that losses are dominantly in the s-wave channel. As in Ref. [19], large kinetic energy of fermions in the outgoing channels (which for reactive molecules can correspond to temperatures in the 10K range) guarantees they will be rapidly lost from any typical atom trap, justifying a Born Markov approximation. Given a density matrix ϱ for the system (fermions, Hilbert space $\mathscr{S}$) plus reservoir (outgoing channels of the inelastic collisions, Hilbert space $\mathscr{R}$), the Born-Markov approximation leads to a master equation for the system reduced density matrix $\rho = \mathrm{Tr}_{\mathscr{R}}[\varrho]$ [20]:

$$\hbar\dot{\rho} = i[\rho, \mathcal{H}] - \frac{\kappa}{2}\int d^3\boldsymbol{r}\left(\mathcal{J}^\dagger\mathcal{J}\rho + \rho\mathcal{J}^\dagger\mathcal{J} - 2\mathcal{J}\rho\mathcal{J}^\dagger\right). \quad (1)$$

The system Hamiltonian $\mathcal{H} = \mathcal{H}_0 + g\int d^3\boldsymbol{r}\mathcal{J}^\dagger\mathcal{J}$ is composed of an unspecified single-particle term $\mathcal{H}_0$ and an interaction term with coupling constant $g = 4\pi\hbar^2 a_R/m$, where m is the particle mass and $a = a_R + ia_I$ ($a_I < 0$) is the complex s-wave scattering length. The jump operators are defined by $\mathcal{J}(\boldsymbol{r}) = \psi_\uparrow(\boldsymbol{r})\psi_\downarrow(\boldsymbol{r})$ (their explicit $\boldsymbol{r}$ dependence is suppressed in the integrals above), where $\psi_\sigma(\boldsymbol{r})$ annihilates a fermion located at position $\boldsymbol{r}$ in internal state $\sigma \in \{\uparrow,\downarrow\}$, and $\kappa = -4\pi\hbar^2 a_I/m$. Assuming without loss of generality that the initial number of particles $\mathcal{N}$ is even, the relevant system Hilbert space can be written as a direct sum over spaces with well defined particle number, $\mathscr{S} = \mathscr{S}^{\mathcal{N}} \oplus \mathscr{S}^{\mathcal{N}-2} \oplus \cdots \oplus \mathscr{S}^0$, between which coherence never develops. Hence, the density matrix can be decomposed into a sum of density matrices in each particle-number sector, any one of which we label by ρ^n once normalized. Furthermore, any Hilbert space $\mathscr{S}^n$ can be decomposed into a direct product between motional (m) and spin (s) degrees of freedom, $\mathscr{S}^n = \mathscr{S}^n_m \otimes \mathscr{S}^n_s$, and we can define a reduced spin density matrix by $\rho^n_s = \mathrm{Tr}_{\mathscr{S}_m}[\rho^n]$. For what follows, it will be useful to define a fidelity in a given Dicke state of the spin degrees of freedom of n particles, $|S = n/2, S^z\rangle$, given by the population of ρ^n_s in the Dicke state $\mathcal{F}_{S,S^z} \equiv \langle S, S^z|\rho^n_s|S, S^z\rangle$. Here S and S^z are quantum numbers for the total spin and its projection along the z-axis, respectively.

Two particles. To make the physics clear in a simple context, we begin by considering two fermions in a single double well potential (which could be formed in an optical super-lattice [22, 23]). We consider a single wavefunction $\varphi_\alpha(\boldsymbol{r})$ in each well ($\alpha \in \{L, R\}$), denote the creation operator for a fermion in spin state σ and wavefunction φ_α by $\psi^\dagger_{\sigma\alpha}$, and choose an initial state $\psi^\dagger_{\uparrow L}\psi^\dagger_{\downarrow R}|vac\rangle$ without spin correlations. Within a tight binding model for these two wavefunctions, the Hamiltonian is

$$\mathcal{H} = -J\sum_\sigma(\psi^\dagger_{\sigma L}\psi_{\sigma R} + \psi^\dagger_{\sigma R}\psi_{\sigma L}) + U\sum_{\alpha=L,R}\mathcal{J}^\dagger_\alpha\mathcal{J}_\alpha, \quad (2)$$

where J is the inter-well hopping, $\mathcal{J}_\alpha = \psi_{\uparrow\alpha}\psi_{\downarrow\alpha}$ are the jump operators [Eq. (1)], $U = g\int d^3\boldsymbol{r}|\varphi_\alpha(\boldsymbol{r})|^4$ is the onsite interaction energy, and $\gamma = \kappa\int d^3\boldsymbol{r}|\varphi_\alpha(\boldsymbol{r})|^4$ is the onsite loss rate. The initial state can be decomposed into an evenly weighted superposition of triplet and singlet $[(\psi^\dagger_{\uparrow L}\psi^\dagger_{\downarrow R} \pm \psi^\dagger_{\downarrow L}\psi^\dagger_{\uparrow R})|vac\rangle$, with plus for the triplet], and the spin wave function of the triplet is the entangled Dicke state $|1,0\rangle$. The triplet, $|t\rangle$, having a spin wavefunction that is symmetric under exchange, has an orbital wavefunction that is antisymmetric under exchange, and therefore it is "dark" to s-wave losses (i.e. $\mathcal{J}_L|t\rangle = \mathcal{J}_R|t\rangle = 0$). It also happens to be an eigenstate of $\mathcal{H}$, and so it is stationary under propagation of the master equation. On the other hand, there are no dark eigenstates in the singlet sector, and as a result ρ^2_s is pure at long times and satisfies $\mathcal{F}_{1,0} = 1$. In other words the steady state of our system, when restricted to the subspace with two particles, is the entangled Dicke state $|1,0\rangle$. There is also a 50% probability of obtaining the vacuum, and hence in an array of double wells the entanglement fidelity is only unity after post-selection of the non-vacant wells. In this simple example we see an important general feature of the physics we will discuss, that even purely *local* (intra-well) dissipation, when coexisting with Hamiltonian dynamics that delocalizes the particles, generates *non-local* (inter-well) spin correlations in the steady state.

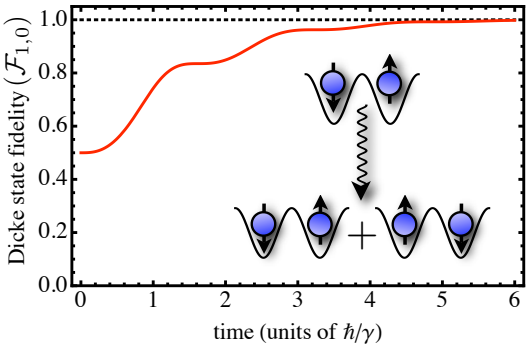

FIG. 1: (Color online). The fidelity of the spin density matrix (solid red line, after post-selection for a non-vacant well) with respect to the $S_z = 0$ Dicke state approaches one (black dotted line) at times long compared to γ^{-1}. The oscillations are due to inter-well hopping.

Many particles. Solving Eq. (1) for initial states with $\mathcal{N} > 2$ initial particles quickly becomes impossible, but strong statements can nevertheless be made regarding the steady state. It is crucial to observe that the jump operators only remove spin singlets from the system, which follows from Fermi statistics combined with the even exchange symmetry of the spatial part of any two-particle wave function susceptible to s-wave scattering. Intuitively, this suggests that losses do not decrease the expectation value of the total spin, $\boldsymbol{S} = \frac{1}{2}\int d^3\boldsymbol{r}\psi^\dagger_\sigma(\boldsymbol{r})\boldsymbol{\tau}_{\sigma\sigma'}\psi_{\sigma'}(\boldsymbol{r})$ ($\boldsymbol{\tau}$ being a vector whose components are the Pauli matrices). Mathematically, we say that $\frac{d}{dt}\langle\boldsymbol{S}\cdot\boldsymbol{S}\rangle = \mathrm{Tr}[\rho\boldsymbol{S}\cdot\boldsymbol{S}] = 0$, which can be verified

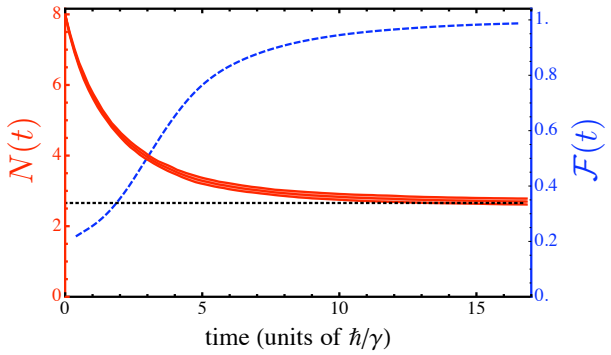

FIG. 2: (Color online). Particle number [$N(t)$, solid red line] and average Dicke state fidelity [$\mathcal{F}(t)$, dashed blue line] for an 8-site Hubbard chain. For $N(t)$, the shaded region is an estimate of the statistical error from sampling of a finite number of trajectories. The black dotted line is the analytic bound in Eq. (3).

in the case when $\mathcal{H}$ is SU(2) invariant by checking that $[\boldsymbol{S}\cdot\boldsymbol{S}, \mathcal{J}(\boldsymbol{r})] = 0$. A stronger consequence of the commutation of all $\mathcal{J}(\boldsymbol{r})$ with $\boldsymbol{S}\cdot\boldsymbol{S}$ is that population in any sector of total spin, $\mathcal{P}_S$, is also conserved. Because any state with well defined total spin S must have $\langle\hat{N}\rangle \geq 2S$ particles (where $\hat{N} = \int d^3\boldsymbol{r}\psi^\dagger_\sigma\psi_\sigma$ is the total number operator), an immediate consequence is that the loss of particles can only yield the vacuum deterministically at long times if the initial state is a total spin-singlet. For an uncorrelated spin state, such as a non-degenerate thermal distribution of $\mathcal{N}$ fermions in a balanced incoherent mixture of $\uparrow$ and $\downarrow$, it can be shown that [24]

$$N(t) \equiv \mathrm{Tr}[\rho\hat{N}] \geq \sum_S 2S\mathcal{P}_S = \frac{\pi^{1/2}\Gamma\left[\frac{\mathcal{N}}{2}+1\right]}{\Gamma[\frac{\mathcal{N}}{2}+\frac{1}{2}]} - 1, \quad (3)$$

which places a lower bound on the steady-state expectation value for the number of particles $N(t)$. This expectation value determines the particle number in a typical steady-state configuration, and is achieved (on average) *without* any post selection, but variations of the steady-state particle number will occur from shot to shot. Stirling's approximation for large $\mathcal{N}$ yields an approximate bound $N(t) \gtrsim \sqrt{\pi\mathcal{N}/2}$. For the chosen restriction on the initial state, the validity of Eq. (3) depends only on the SU(2) invariance of $\mathcal{H}$, and not on its precise form. Whether the bound (3) is saturated in the steady-state, however, is an important and delicate issue. The bound is only saturated when, in every sector of total spin S, there are *exactly* $\mathcal{N}_S = 2S$ particles. Because the latter condition is only satisfied by Dicke states, saturation of Eq. (3) guarantees that all of the ρ^n_s describe pure Dicke states in the steady-state. Demonstrating that this bound *is* saturated in certain experimentally relevant situations, namely a 1D harmonic trap and a 1D Hubbard chain, is a central technical result of this paper.

Saturation of the bound in Eq. (3) is guaranteed if, for any fixed value of n and S_z, the pure density matrix $|n/2, S_z\rangle\langle n/2, S_z|$ is the *unique* steady-state reduced spin density matrix. This uniqueness, in turn, is equivalent to requiring that any dark-state with quantum numbers n and S_z has a well defined spin wavefunction given by the Dicke state $|n/2, S_z\rangle$. In the supplement we prove this to be true for a 1D harmonic oscillator potential, and we have verified it numerically for a 1D Hubbard chain (see below). It is worth noting at this point that, while the equivalence of dark-states with the Dicke states is intuitive, there are natural Hamiltonians for which this intuition is incorrect. In particular, all Hamiltonians in $D > 1$ that are separable in cartesian coordinates do have dark-states with $\mathcal{F}_{S,S_z} < 1$.

In order to verify the above statements numerically, we have performed quantum trajectories simulations for an 8-site Hubbard chain with open boundary conditions, an initial filling of one particle per site, and zero polarization ($\mathcal{N} = 8$ and $S_z = 0$). In Fig. 2 we show the calculated particle number and average Dicke state fidelity, $\mathcal{F}(t) = \frac{1}{4}\sum_{S=1}^{4}\mathcal{F}_{S,0}$, and one can see that the former saturates the bound Eq. (3) while the latter approach unity at long times. For this calculation we solve for $\mathcal{O}(10^4)$ trajectories with no approximation.

Experimental realization. Dicke states are known to be useful for a variety of quantum information protocols, including but not limited to quantum secret sharing [25], teleportation [26], and sub shot-noise limited precision spectroscopy [7]. Here we give a brief description of how the proposed Dicke state preparation could be used in precision spectroscopy of the clock transition in alkaline-earth atoms. For a fixed interrogation time, spectroscopy on $\mathcal{N}$ uncorrelated atoms has a phase sensitivity $\delta\varphi \gtrsim 1/\sqrt{\mathcal{N}}$, a bound known as the standard quantum limit (SQL). On the other hand, spectroscopy on a Dicke state of $\mathcal{N}$ particles with spin $S_z = 0$ has the potential to approach the Heisenberg limit (HL) of phase sensitivity, $\delta\varphi \sim 1/\mathcal{N}$ [7, 27]. It is important to realize that the production of Dicke states with $\sqrt{\mathcal{N}}$ fermions via two-particle loss does not actually enhance the phase sensitivity relative to the initial state with $\mathcal{N}$ fermions; the enhancement in phase sensitivity between the SQL and HL exactly compensates the reduction of particle number. However, the reduced particle number in the Dicke state and darkness to *real* s-wave interactions (which if present generate clock shifts), can render the accuracy of the final Dicke state superior to that of the initial $\mathcal{N}$ fermion uncorrelated state.

Rather than allowing losses amongst a macroscopic sample of atoms, for which the approach to the steady state could be quite slow, we imagine an array of $\mathcal{T}$ 1D tubes created by a 2D optical lattice. Although there will be variations in the atom number from tube to tube, for simplicity we take each tube to have exactly $\mathcal{N}$ fermionic AEAs in the $^1S_0$ electronic state and $I^z = I$ nuclear-spin state, denoted $|^1S_0, I\rangle$. For the analysis in this paper to

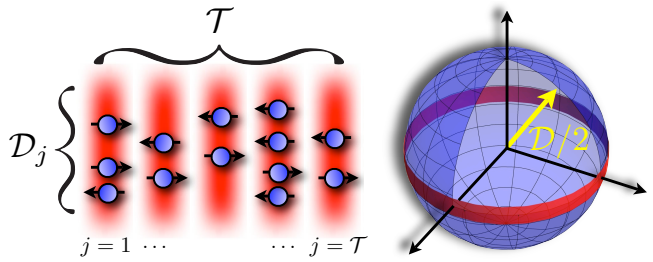

FIG. 3: (Color online). (a) An array of $\mathcal{T}$ 1D tubes, each having $\mathcal{D}_j$ atoms in a Dicke state. (b) Bloch sphere representation of a Dicke state in a particular tube.

be valid, the temperature should be small compared to the vibrational level-spacing in the transverse tube direction, and also low enough that only the harmonic part of the trapping potential along the tube axis is sampled by the atoms. A $\pi/2$-pulse on the spin degrees of freedom $[|^1S_0, I\rangle \rightarrow \frac{1}{\sqrt{2}}(|^1S_0, I\rangle + |^1S_0, I-1\rangle)]$, followed by single particle dephasing, generates a statistical mixture of the two spin states ($I^z = I$ and $I^z = I-1$). Losses can be initiated by applying a π-pulse on the clock transition ($|^1S_0, I^z\rangle \rightarrow |^3P_0, I^z\rangle$). We estimate that this π-pulse can be achieved on the $\lesssim 100\mu$s timescale without exciting transverse excitations in the tubes (which, if present, violate the assumption of a 1D geometry and destroy the uniqueness of the steady-state). Thus the transfer into $^3P_0$ is sufficiently fast that it can be considered instantaneous on the initial timescale of reactive collisions—which, based on universal considerations for a Lieb-Liniger gas, we estimate to be $\gtrsim$ 1ms for experimentally relevant 1D densities [19]—such that it suddenly initiates strong 2-body s-wave losses.

The steady state of the system is a statistical mixture of Dicke states in the different tubes, each having some value of $\mathcal{D}_j$ particles (centered around $\mathcal{D}_0 \approx \sqrt{\mathcal{N}}$) and spin projection S_j^z (centered around zero). Spin selective transfer of $|^3P_0, I-1\rangle$ into $|^1S_0, I\rangle$ maps the spin degree of freedom onto the clock states, leaving a spin-polarized sample, and Ramsey spectroscopy on the clock transition can then be performed [27]. Despite the fluctuation of both $\mathcal{D}_j$ and S_j^z from one tube to another, accurate knowledge of the initial value of *total* $S^z = \sum_j S_j^z$ guarantees that the minimum resolvable rotation angle in a Ramsey experiment scales as

$$\delta\varphi_{\min} \sim 1/\mathcal{D}_0\sqrt{\mathcal{T}}. \quad (4)$$

This result is derived in the supplement, and can be interpreted as the existence of Heisenberg scaling ($\sim 1/\mathcal{D}_0$) of phase-sensitivity for each tube, which is then combined between tubes in a statistically independent manner (hence the $1/\sqrt{\mathcal{T}}$). As mentioned above, in order to utilize this phase sensitivity the initial value of S^z must be accurately known. Because S^z is conserved by the losses, it can be measured before transfer to the $^3P_0$ state, and hence the measurement does not need to preserve any inter-particle correlations (since these develop during the losses). Accurate measurements of this type and precision for ~ 100 atoms in an optical cavity have recently been demonstrated [28].

The primary limitations on the final state fidelity achievable in experiments are likely to be a combination of finite p-wave losses (which the Dicke states are not dark to) and magnetic field gradients. At sub μK temperatures, the s-wave losses in a spin mixture of ^{87}Sr are expected to be about an order of magnitude faster than the p-wave losses [18]. For reactive molecules (or ^{171}Yb), where the inelastic collisions are expected to more fully saturate the unitarity bound [17], this separation of rates will most likely be even larger. Magnetic field gradients couple sectors of different total S, all of which are separated from the Dicke manifold by a gap for finite systems and nonzero a_R, so in principle their adverse effects can be suppressed to first order [29]. Furthermore, if the two components of the Fermi gas are two nuclear spin states of an AEA, they will be extremely insensitive to magnetic field gradients: We estimate that typical gradients (1mG/cm) will cause spin dephasing on a 100s timescale for a linear system size of 100μm. This timescale is several orders of magnitude longer than the initial two-body loss rate in tightly confined 1D tubes, which we estimate to be on the order of 10ms for ^{87}Sr (assuming a $50E_{\rm R}$ 2D lattice and scaling the density dependent loss rate from Ref. [18]), and even faster for ^{171}Yb [17]. A more quantitative analysis of the effects of both magnetic field imperfections and finite p-wave losses requires numerics beyond the scope of this work, and is left for future study.

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