Binary Black-Hole Mergers in Magnetized Disks: Simulations in Full General Relativity

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Binary black hole mergers in magnetized disks: simulations in full general relativity

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We present results from the first fully general relativistic, magnetohydrodynamic (GRMHD) simulations of an equal-mass black hole binary (BHBH) in a magnetized, circumbinary accretion disk. We simulate both the pre and post-decoupling phases of a BHBH-disk system and both "cooling" and "no-cooling" gas flows. Prior to decoupling, the competition between the binary tidal torques and the effective viscous torques due to MHD turbulence depletes the disk interior to the binary orbit. However, it also induces a two-stream accretion flow and mildly relativistic polar outflows from the BHs. Following decoupling, but before gas fills the low-density "hollow" surrounding the remnant, the accretion rate is reduced, while there is a prompt EM luminosity enhancement following merger due to shock heating and accretion onto the spinning BH remnant. This investigation, though preliminary, previews more detailed GRMHD simulations we plan to perform in anticipation of future, simultaneous detections of gravitational and electromagnetic radiation from a merging BHBH-disk system.

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When galaxies merge, massive black hole binaries (BHBHs) are likely to form in gas rich environments [1]. These systems constitute unique multi-messenger sources. The GW signals can be detected by future space-based laser interferometers like eLISA [2] and Pulsar-Timing-Arrays [3, 4]. The EM signals provide information about MHD accretion onto black holes and can serve as precursors to GW observations [5].

If BHBHs are embedded in a gas with negligible angular momentum, the accretion flow will resemble the Bondi or Bondi-Hoyle-Lyttleton accretion solutions [6–8]. When the gas has intrinsic angular momentum, it will form a circumbinary disk that accretes via angular momentum transport induced by an effective viscosity. The BHBH-disk problem has been studied extensively in Newtonian gravitation, both analytically [9–13], where the emphasis has been on low mass-ratio binaries, and numerically [9, 14, 15] where equal-mass systems have been the focus. The goal is to compute observable EM "precursor" and "aftermath" radiation that will accompany the GW signal. While the vacuum BHBH [16] and accretion onto a single BH problems in GR are now well developed, simulations of disk accretion onto BHBHs are still in their infancy [6–8, 17, 18]. Vacuum calculations offer an accurate description of the spacetime and the GWs emitted close to merger in typical cases. Determining the EM signatures requires a GRMHD computation in this dynamical BHBH spacetime.

Here we report the first fully GRMHD simulation of a magnetized, circumbinary BHBH accretion disk. The effective viscosity driving accretion arises from MHD turbulence triggered by the magnetorotational instability (MRI) [19]. This effective viscosity competes with the tidal torques exerted by the binary, so that a quasi-stationary state is reached prior to binary-disk decoupling [10, 11]. This state has been simulated both in Newtonian [10] and in Post-Newtonian [20] gravitation. Typically, the computational domain excludes the region near the BHs and artificial inner boundary conditions are imposed. Recent Newtonian studies [21] make clear the importance of imposing the correct boundary conditions on the flow inside the central disk hollow and near the BHs, further motivating a treatment in full, dynamical GR whereby BH horizons can be modeled reliably.

The basic evolution of the system is as follows: For large binary separations a, the inspiral time due to GW emission is much longer than the viscous time ($t_{\rm GW} \gtrsim t_{\rm vis}$), so that the disk settles into a quasi-stationary state. For equal-mass BHs, the binary tidal torques carve out a partial hollow in the disk [9, 10, 12, 14] of radius $\sim 2a$ and excite spiral density waves throughout the disk, that dissipate and heat the gas. However, gas can penetrate the hollow in response to the time-varying tidal torque [12, 14, 20, 22]. At sufficiently small separations $t_{\rm GW} \lesssim t_{\rm vis}$, and the BHBH *decouples* from the disk. The disk structure at decoupling crucially determines its subsequent evolution and the EM emission. GW emission close to merger leads to mass loss [23, 24] and may induce remnant BH recoil [25], which give rise to further characteristic EM signatures. Here we simulate the system in two different epochs: (I) The pre-decoupling phase ($t_{\rm GW} > t_{\rm vis}$) and (II) the post-decoupling phase ($t_{\rm GW} < t_{\rm vis}$), including the inspiral and merger of the BHBH. We consider equal-mass, nonspinning binaries. While the BH mass scales out, we are primarily interested in total (ADM) masses $M \gtrsim 10^6 M_\odot$ and low density disks for which the tidally-induced binary inspiral and the disk self-gravity are negligible.

We use the Illinois numerical relativity code to carry

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out our simulations. The code has been extensively tested [26, 27] and used in our earlier BHBH simulations in gaseous media [6, 22]. For details and equations see [26–28]. The main new feature concerns our vector potential ($\mathcal{A}_\mu = \Phi n_\mu + A_\mu$) formulation for the magnetic induction equation, where n^μ is the future-directed timelike unit vector normal to a t =const. slice and $n^\mu A_\mu = 0$. We introduce a new generalized Lorenz gauge condition $\nabla_\mu \mathcal{A}^\mu = \xi n_\mu \mathcal{A}^\mu$, where ξ is a parameter (typically $\xi = 4/M$) and M is the total BHBH (ADM) mass. This modification results in *damped*, traveling EM gauge modes, preventing spurious B-fields from appearing on refinement boundaries more strongly than the original, undamped Lorenz gauge condition [29].

The disk initial data represent an equilibrium disk orbiting a single Schwarzschild BH [22, 30] with an inner disk edge at $R_{\rm in} = 18M = 1.8a$, where the specific angular momentum $\ell_{\rm in} = 5.15M$ at $R_{\rm in}$, and a nearly Keplerian rotation profile parameter $q = 1.7$. We adopt a Γ-law equation of state with $\Gamma = 5/3$, appropriate for a disk composed of an ideal, nonrelativistic gas. Magnetic fields are poorly constrained by observations. Thus, we choose to seed the disk with a weak poloidal B-field as described in [28]. Such initial poloidal B-field configurations are widely used in single BH MHD accretion studies (e.g. [31]), because they facilitate the study of MRI-induced turbulence. The maximum relative strength of the initial B-field in the equatorial plane is $(P_M/P)_{\rm max} = 0.025$. Here $P_M \equiv B^2/8\pi$ is magnetic pressure, P is gas pressure, and B^μ is the magnetic field measured in the comoving frame of the fluid. The B-field strength is chosen such that it is dynamically unimportant initially, but sufficiently large to capture MRI.

Prior to decoupling we can neglect the slow BHBH inspiral. We model the spacetime during this epoch by adopting the BHBH metric derived in the conformal thin-sandwich (CTS) formalism [32], whereby the spacetime is stationary in the corotating frame (see [22] for details). The inner part of the disk settles into a quasiequilibrium state on a "viscous" time scale

$$\frac{t_{\rm vis}}{M} = \frac{2R_{\rm in}^2}{3\nu M} \sim 6500 \left(\frac{R_{\rm in}}{18M}\right)^{3/2} \left(\frac{\alpha}{0.13}\right)^{-1} \left(\frac{H/R}{0.3}\right)^{-2}, \quad (1)$$

where ν is the effective viscosity induced by MHD turbulence [19]. This viscosity can be fit (approximately) to an 'α-disk' law for purposes of analytic estimates. Here R is the disk radius, $\nu(R) \equiv (2/3)\alpha(P/\rho_0)\Omega_K^{-1} \approx (2/3)\alpha(R/M)^{1/2}(H/R)^2 M$, H is the disk scale height, and we have assumed vertical hydrostatic equilibrium to derive an approximate relationship between P/ρ_0 and H/R (see [33]). Equating the viscous time scale and the GW inspiral time scale yields the decoupling separation

$$\frac{a_d}{M} \approx 13 \left(\frac{\alpha}{0.13}\right)^{-2/5} \left(\frac{H/R}{0.3}\right)^{-4/5}, \quad (2)$$

where the normalizations give the typical parameters our simulations obtain for the relaxed state. Note, that for the geometrically thick, magnetic disks we treat, the expected decoupling radius is an order of magnitude smaller than typical thin-disk cases [10, 34]. We thus set our initial binary separation at $a/M = 10$ (orbital period $2\pi/\Omega = 225M$).

We evolve the system using the CTS spacetime for ~ 45 binary orbits (10,000 M) to allow the inner parts of the disk to settle into a quasistationary state. This epoch (1) models the pre-decoupling phase and (2) provides realistic, relaxed disk initial data for the post-decoupling inspiral phase. We model the post-decoupling phase by continuing the GRMHD evolution in the dynamical spacetime of the inspiraling and merging BHBH binary. We treat two extreme opposite limiting cases: "no-cooling", which allows for gas heating via shocks induced by tidal torques and MHD turbulence, and "cooling", which removes all the heat generated via an effective local emissivity Λ of the form $T^{\mu\nu}{}_{;\nu} = -\Lambda u^\mu$ as in [35]. Our "cooling" case, though artificial, provides a representative example of the effects of cooling and has been adopted in previous work (e.g. [20, 36]). The particular "cooling" prescription we use drives the gas to isentropic behavior, i.e. $P/\rho_0^\Gamma = const$. The cooling timescale is set to the local, Keplerian orbital period. Our simulations resolve the BH horizons and we impose *no inner boundary conditions*. In the pre-decoupling phase our grid consists of a hierarchy of 6 refinement levels with (coarsest, finest) resolution of ($5.33M$, $0.16M$) and outer boundary at $250M$. We resolve the wavelength of the fastest-growing MRI mode ($\lambda_{\rm MRI}$) by at least 10 grid points in the bulk of the inner disk. Note that resolving $\lambda_{\rm MRI}$ by 10 grid points is sufficient to capture the main effects of MRI [37]. We add two extra levels centered on each BH in the post-decoupling phase, increasing the (coarsest, finest) resolution to ($4M$, $M/32$). After merger and ringdown we freeze the spacetime evolution, but continue to evolve the plasma. Equatorial symmetry is imposed throughout. We normalize results to those for a single BH that we evolved with the same initial magnetized disk and BH mass equal to M.

The initial disk (see Fig. 1) is not in equilibrium around the binary as it is perturbed by the binary torques. The torques lead to spiral density waves in the disk that dissipate and heat the gas, puffing up the disk. The gas gains angular momentum and the surface density profile moves slightly outward. Magnetic winding converts the poloidal field into one with a large toroidal component. MRI is induced, resulting in turbulent flow. After about 20 binary orbits ($\sim 4-5$ disk orbits at the pressure maximum) the MRI saturates, driving disk accretion onto the BHBH. In the relaxed disk prior to decoupling we measure a time-averaged Maxwell-stress as in [38] at $20M < R < 30M$, and find $\alpha = 0.13$ for the "no-cooling" (see Fig. 1) and $\alpha = 0.2$ for the "cooling" case. The magnetic-to-gas-

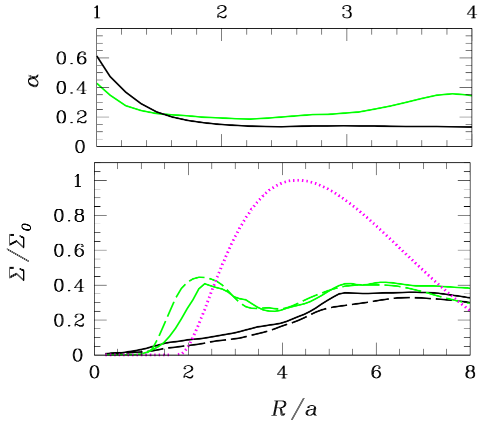

FIG. 1. The disk α parameter (upper panel) and surface-density Σ (lower panel) profiles. Σ_0 is the maximum surface density at $t = 0$. Dotted magenta (gray in grayscale) line is the initial data, solid lines are at decoupling, and dashed lines are at merger. Black lines are from the "no-cooling" case, and green (light gray in grayscale) lines are from the "cooling" case.

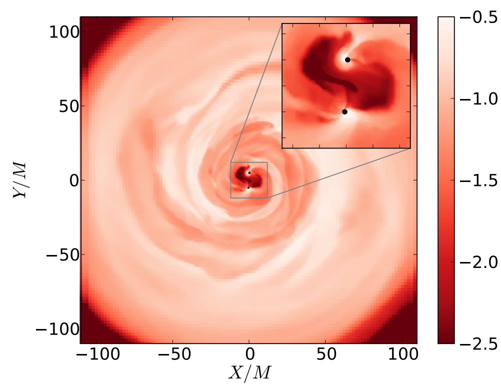

FIG. 2. Orbital plane snapshot of rest-mass density $\log(\rho_0/\rho_{0,max})$ from the "no-cooling" simulation at $t \sim 10000M$ in the relaxed disk, prior to decoupling. The inset zooms in on the region close to the BHs.

pressure ratio $1/\beta$ ranges from 0.1 to 5 in the bulk of the disk (where no numerical fixes or caps are applied).

Cooling influences the global disk structure. In particular, we observe matter pile-up near the inner disk edge only when there is cooling (see Fig. 1), as has previously been found in [11, 14, 15, 20]. The binary maintains a partial hollow in the disk (see Fig. 1) by exerting torques on the plasma, while the MRI-induced effective viscosity drives matter inward. Cooling leads to smaller scale height and lower effective ν, which explains the enhanced pile-up at R_{in}. We confirm the result in [14, 15, 20, 22] that non-negligible amounts of gas are present inside the cavity.

Accretion occurs predominantly via two spiral density

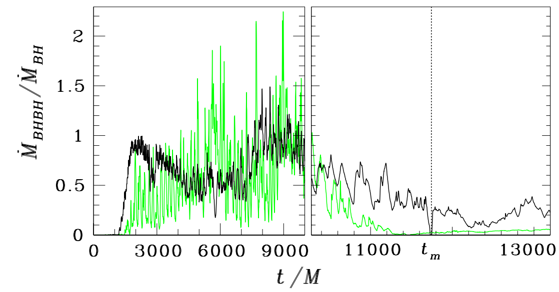

FIG. 3. Time-averaged binary accretion rate $\dot{M}_{BHBH}$, normalized to the average value for a single BH $\dot{M}_{BH}$, versus time. Colors have the same meaning as in Fig. 1. Left panel: pre-decoupling (a =const) phase. Right panel: post-decoupling (inspiral) phase. $\dot{M}_{\rm BH} = 0.45 n_{12} M_8^2 M_\odot {\rm yr}^{-1}$ where $M_8 \equiv M/10^8 M_\odot$ and $n_{12} \equiv n/10^{12}{\rm cm}^{-3}$ is the initial maximum gas particle number density. Merger occurs at time $t_m = 11743M$.

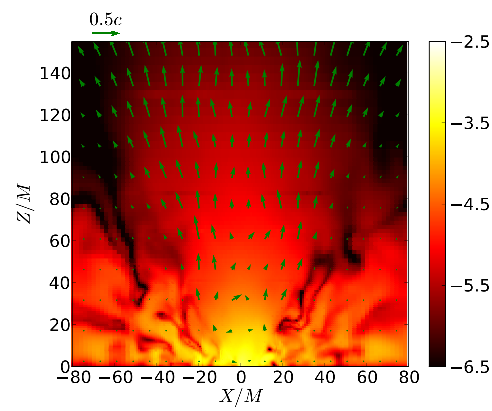

FIG. 4. Meridional snapshot of magnetic pressure $\log(P_M/\rho_{0,max})$ and fluid velocity vectors at $t \sim 10000M$ for the "no-cooling" case.

streams inside the cavity (see Fig. 2). We find that accretion exhibits an alternating pattern by accreting primarily on one of the BHs for about half a binary orbit. This is similar to flow features observed in [15]. The behavior has been attributed to a gradual increase of disk-eccentricity [14], which weakens one of the two streams when the BHs pass near the disk-apocenter and strengthens the other stream at pericenter.

Prior to decoupling, but after an initial transient phase ($5000M \lesssim t \lesssim t_{\rm vis}(R_{\rm in})$), the accretion rate $\dot{M}_{\rm BHBH}$ settles to values comparable to those onto a single BH of mass M (see Fig. 3). We perform a Fourier analysis of $\dot{M}_{BHBH}$ and find that the strongest contributions arise at $f \sim 2/3\Omega$ in both of our cases. This is likely associated with the dominant (2,3) Lindblad resonance [14]. During the early inspiral the inward drift of the disk edge lags behind the binary orbital decay, decreasing $\dot{M}_{BHBH}$. In contrast to the magnetic-free case [22], the accretion

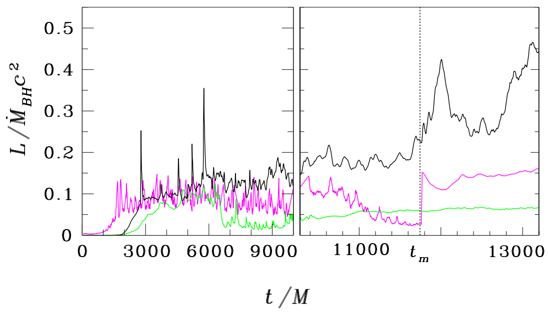

FIG. 5. The Poynting luminosity $L_{\rm EM}$ measured at $R = 100M$ as a function of time [green (light gray in grayscale) line for "cooling", black line for "no-cooling"]. The cooling luminosity L_{cool} from the "cooling" case [magenta (gray in grayscale) line]. $\dot{M}_{\rm BH}c^2 = 2.58 \cdot 10^{46} {\rm erg s}^{-1} n_{12} M_8^2$.

streams *remain present* until merger at t_m for the "no-cooling" case and up until $t - t_m \gtrsim -400M$ for the "cooling" case. At merger $\dot{M}_{\rm BHBH}$ decreases gradually to about 30% of the single, quasi-stationary BH accretion rate, $\dot{M}_{\rm BH}$ (see Fig. 3).

The remnant BH settles down via quasi-normal mode ringing to a Kerr-like BH with mass $M_f \sim 0.95M$ and dimensionless spin $s = J_f/M_f^2 = 0.68$. In the "cooling" case the inner disk edge reaches $R_{\rm in}(t_m) \sim 10M$ at merger, while the edge is more dispersed in the "no-cooling" case (see Fig. 1).

Prior to decoupling we detect persistent, magnetized, mildly relativistic ($v \gtrsim 0.5c$) collimated outflows in the polar regions (see Fig. 4) in both cases. After merger there is an increase in the velocities of these outflows. In the "no-cooling" case the outflows accelerate within a time of $\sim 400M$ to Lorentz factors of $\Gamma_L \lesssim 4$. In the "cooling" case this transition takes $\sim 800M$ and the outflow velocities are smaller, $\Gamma_L \lesssim 2$. The fast outflows persist throughout the postmerger evolution in both cases. After merger the effective, turbulent viscous torque will cause the gas to refill the cavity and accrete on the merger remnant. Thus, brightening crucially depends on the surface density at decoupling. If there is only a small pile-up and the majority of gas lies at radii $R \gtrsim 40M$, then a significant brightening will take $t_{\rm vis}(R \gtrsim 40M) > \mathcal{O}(10^4 M)$ following merger. Using Eq. (1) we estimate $t_{\rm vis}(R = 40M) \gtrsim 22000M$ ($\alpha \sim 0.13, H/R \sim 0.3$) for the "no-cooling" case and $t_{\rm vis}(R = 14M) \gtrsim 6800M$ ($\alpha \sim 0.27, H/R = 0.17$) for the "cooling" case, respectively. We observe the surface density profile diffusing inward, but we have not followed the evolution for this long.

According to [39] an initially quasi-equilibrium, hollowed out disk around a non-recoiling remnant BH whose mass is suddenly reduced due to energy radiated away in GWs will be shock heated as the inner disk edge retracts supersonically, leading to prompt EM emission. However, pseudo-Newtonian MHD and HD simulations [23] find that shock heating from BH mass loss occurs only if the energy loss in GWs $\Delta E_{\rm GW}/M$ is less than the disk half-thickness H/R. This limits this process to thin disks only. Moreover, [23] shows that for thick disks instead, the total luminosity should *decrease* as the disk inner edge retracts (a result found to apply to GRHD thick disks in [40] and to Newtonian thin disks in [24]). Based on this picture the luminosity arising from our thick disk ($H/R \sim 0.3$) should not increase, because $\Delta E_{\rm GW}/M$ 0.05 $\ll H/R$. Instead, we should observe a decline in the total luminosity according to this criterion.

By contrast, we find (Fig. 5) a sudden *increase* in the total luminosity [Poynting luminosity ($L_{\rm EM} \equiv -\int T^{r\,({\rm EM})}_{\;t}\sqrt{-g}dS$) plus luminosity from disk cooling ($L_{cool} \equiv \int \Lambda u_t \sqrt{-g} d^3x$)] after merger. This enhancement originates from shocked gas in the immediate vicinity of the binary and from accretion onto the spinning BH remnant, which taps the BH rotational energy, boosting jet outflows along the BH spin axis. This provides a new picture for prompt EM signals arising from *thick*, relativistic, circumbinary MHD disks following the merger. We find that the outward flux of kinetic energy is much smaller in both cases. The total energy efficiency $\epsilon \equiv L/\dot{M}_{BH}c^2$ increases to $\epsilon = 0.25$ at merger, before settling down to ~ 0.1 at late times [31].

The Poynting luminosity is presumably reprocessed at larger distance from the remnant. The characteristic frequencies of the total emitted EM radiation will depend on the BH masses, disk densities and dominant cooling mechanisms. Relaxing equatorial symmetry will allow advection through the equator to occur. This in turn will likely enhance the poloidal B-fields and thereby increase the wavelength of the fastest growing MRI mode, leading to improved modeling of MRI effects [41]. In addition, the MHD outflow structure may be sensitive to the initial B-field topology, although the accretion flows and field saturation levels are probably not [42]. We plan to investigate these issues, along with other precursor (e.g. twin jets) and afterglow effects, different mass ratios, and different BH spins more thoroughly in future work.

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