Turbulent Transport and Heating of Trace Heavy Ions in Hot Magnetized Plasmas

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Turbulent transport and heating of trace heavy ions in hot, magnetized plasmas

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Scaling laws for the transport and heating of trace heavy ions in low-frequency, magnetized plasma turbulence are derived and compared with direct numerical simulations. The predicted dependences of turbulent fluxes and heating on ion charge and mass number are found to agree with numerical results for both stationary and differentially rotating plasmas. Heavy ion momentum transport is found to increase with mass, and heavy ions are found to be preferentially heated, implying a mass-dependent ion temperature for very weakly collisional plasmas and for partially-ionized heavy ions in strongly rotating plasmas.

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Introduction. Heavy ions are present in hot, magnetized plasmas both in laboratory experiments and in nature. These heavy ions are often trace, i.e., their densities are small enough that they have only a small direct effect on the bulk plasma dynamics. Nonetheless, trace heavy ions are important in numerous contexts: main ion properties are often inferred from heavy ion measurements because heavy ions radiate more readily [1]; accumulation of heavy ions leads to dilution and increased radiative energy losses in magnetic confinement fusion [2, 3]; and temperature measurements of minority ions in space and astrophysical plasmas indicate the existence of a novel heating mechanism [4–6].

Considerable effort has gone into understanding the particle transport of trace heavy ions, or impurities, in the context of magnetized toroidal plasmas for fusion. In particular, the scaling with charge number Z and mass number A of the impurity particle flux were predicted with a quasilinear fluid model and found to be in relatively good agreement with numerical and experimental results [7, 8]. However, little to no work has been done on impurity momentum and energy fluxes or for turbulent heating of impurities. The latter may play a role not only in fusion plasmas, but also in the context of astrophysical plasmas, where the temperature of minority ions has been observed to increase with increasing ion mass [4–6]. Cyclotron heating [9] and stochastic heating via large-amplitude fluctuations [10] have been proposed as possible explanations for this mass dependence. The turbulent heating mechanism described here provides an alternative explanation for the mass dependence of the minority ion temperature that is present even for low frequency, low amplitude fluctuations.

In this Letter we use local, nonlinear, δf-gyrokinetic theory [11–13] to provide scaling predictions for trace heavy ion particle, momentum, and energy fluxes, as well as turbulent heating in hot, magnetized plasmas. This approach has proven successful in determining scalings of temperature-gradient driven turbulence in tokamaks [14]. We consider an inhomogeneous, axisymmetric plasma rotating toroidally at angular frequency ω_ϕ, immersed in a curved, inhomogeneous magnetic field. To simplify our analysis, we restrict our attention to a region of plasma with rotation speed well below the ion sound speed but with a strong rotation gradient. We also consider only moderate values of $\beta = 8\pi p/B^2 \lesssim 1$, where p is the mean plasma pressure and B is the mean magnetic field magnitude. This is directly applicable to toroidal confinement experiments in magnetic confinement fusion, but the scaling laws we obtain are general: they do not change for a stationary, homogeneous plasma slab and therefore also pertain to various space and astrophysical plasmas.

Gyrokinetic turbulence. The δf-gyrokinetic theory is obtained by performing an asymptotic expansion in the small ratio of the Larmor radius, ρ, to system size, L, and averaging over the fast Larmor motion of particles. It is valid for low-amplitude turbulence with sub-Larmor frequencies, and spatial scales comparable to ρ and L in the directions across and along the mean magnetic field, respectively. Initially developed for magnetic confinement fusion plasmas, δf-gyrokinetics can also be applied to small-scale turbulence in the solar wind, solar corona, accretion disks, and galaxy clusters [15, 16].

We use $(\mathbf{R}, \mu, \varepsilon)$ as our coordinate system, where $\mathbf{R}$ is the position of the center of a particle's Larmor orbit, $\varepsilon = mv^2/2$ its kinetic energy, and $\mu = mv_\perp^2/2B$ its magnetic moment, with m its mass and v its speed. The subscripts $\perp$ and $\parallel$ are used to denote the components perpendicular and parallel to the mean magnetic field, respectively, with the magnetic field magnitude given by B. With this choice of coordinates, the electromagnetic gyrokinetic equation governing the evolution of the fluctuating piece of the distribution function, δf_s, is

$$\frac{Dg_s}{Dt} + \dot{\mathbf{R}}_s \cdot \nabla \left(g_s + \frac{Z_s e \langle \chi \rangle_s}{T_s} F_{M,s} \right) - \langle C[\delta f_s] \rangle_s = -\langle \mathbf{v}_\chi \rangle_s \cdot \left(\nabla F_{M,s} + R \nabla \omega_\phi \frac{m_s v_\parallel}{T_s} F_{M,s} \right), \tag{1}$$

where $g_s = \delta f_s + Z_s e F_{M,s}(\Phi - \langle \chi \rangle_s)/T_s$, $\langle . \rangle_s$ is an average over Larmor angle at fixed $\mathbf{R}_s$, $\langle \chi \rangle_s =$

$\left\langle \Phi - v_\parallel \delta A_\parallel/c + \int_0^{\mu_s} d\mu_s' \delta B_\parallel/Z_s e \right\rangle_s$, Φ is the fluctuating electrostatic potential, $\delta A_\parallel$ and $\delta B_\parallel$ the parallel components of the fluctuating magnetic vector potential and magnetic field, respectively, Z_s the charge number, e the proton charge, c the speed of light, T_s the mean temperature, $F_{M,s}$ a stationary Maxwellian distribution of velocities in the frame rotating with velocity $\mathbf{u} = R^2\omega_\phi \nabla\phi$, ϕ the toroidal angle, R the plasma major radius, $D/Dt = \partial/\partial t + \mathbf{u}\cdot\nabla$, $\dot{\mathbf{R}}_s = \mathbf{v}_\parallel + \mathbf{v}_{M,s} + \langle \mathbf{v}_\chi \rangle_s$, with $\mathbf{v}_{M,s} = \mathbf{b}/\Omega_s \times (v_\parallel^2 \mathbf{b}\cdot\nabla\mathbf{b} + v_\perp^2 \nabla B/2B)$ the drift velocity due to a curved, inhomogeneous mean magnetic field and $\langle \mathbf{v}_\chi \rangle_s = c\mathbf{b}\times\nabla\langle\chi\rangle_s/B$ the drift due to the fluctuating fields, $\mathbf{b}$ the unit vector along the mean magnetic field, $\Omega_s = Z_s eB/m_s c$, and C describes two-particle Coulomb interactions. Plasma species is indicated by the subscript s, which we henceforth drop where possible.

Note that the fields Φ, $\delta A_\parallel$, and $\delta B_\parallel$ are independent of Larmor angle at fixed particle position, $\mathbf{r}$, but not at fixed $\mathbf{R} = \mathbf{r} + \mathbf{v}_\perp \times \mathbf{b}/\Omega$. Thus care must be taken to specify which spatial coordinate is held fixed for velocity integration. The μ-integral contained in $\langle\chi\rangle$ is performed at fixed $\mathbf{R}$, but all other velocity integrals in this Letter are performed at fixed $\mathbf{r}$.

By definition, the trace ions considered here do not contribute to the fields. They are instead determined solely by the electron and main ion dynamics through the low-frequency Maxwell's equations, supplemented by the quasineutrality constraint:

$$0 = \sum_s Z_s \int d^3\mathbf{v}\ \delta f_s, \tag{2}$$

$$\nabla_\perp^2 \delta A_\parallel = -\frac{4\pi}{c}\sum_s Z_s e \int d^3\mathbf{v}\ v_\parallel \delta f_s, \tag{3}$$

$$\nabla_\perp \delta B_\parallel = \frac{4\pi}{c}\sum_s Z_s e \int d^3\mathbf{v}\,(\mathbf{b}\times\mathbf{v}_\perp)\,\delta f_s, \tag{4}$$

where Φ enters Eqs. (2-4) through the definition for δf given below Eq. (1).

With g and $\{\Phi, \delta A_\parallel, \delta B_\parallel\}$ specified by Eqs. (1-4), one can evaluate the turbulent heating and fluxes,

$$H = Ze\left\langle \chi \left((\mathbf{v}_\parallel + \mathbf{v}_M)\cdot\nabla g - \langle C[\delta f]\rangle \right)\right\rangle_\Lambda - \Pi\frac{\partial\omega_\phi}{\partial r}, \tag{5}$$

$$\Gamma = \left\langle \delta f \langle \mathbf{v}_\chi\rangle\cdot\nabla r \right\rangle_\Lambda, \tag{6}$$

$$Q = \left\langle \varepsilon\ \delta f \langle \mathbf{v}_\chi\rangle\cdot\nabla r \right\rangle_\Lambda, \tag{7}$$

$$\begin{aligned}\Pi = &\left\langle mR^2 \delta f (\mathbf{v}\cdot\nabla\phi)\langle\mathbf{v}_\chi\rangle\cdot\nabla r\right\rangle_\Lambda \\ &- \left\langle \frac{Ze}{c} R^2 (\mathbf{b}\cdot\nabla\phi)\delta f (\mathbf{v}_\perp\cdot\nabla r)\delta A_\parallel \right\rangle_\Lambda,\end{aligned} \tag{8}$$

where r labels surfaces of constant mean pressure, $\langle a\rangle_\Lambda = \int d^3\mathbf{r}\int d^3\mathbf{v}\ a / \int d^3\mathbf{r}$ is an integral over all velocity space and over a volume of width w ($\rho \ll w \ll L$) encompassing the mean magnetic field line of interest, H is the heating, and Γ, Π, and Q are the particle, toroidal angular momentum, and energy fluxes, respectively. Note that the momentum flux defined in Eq. (8) does not include each species' contribution to the Maxwell stress.

TABLE I: Scalings, S, for turbulent fluxes and heating

	$\left\lvert\frac{d\omega_\phi}{dr}\right\rvert \sim \frac{Z}{A}\frac{v_{ti}}{R^2}$	$\left\lvert\frac{d\omega_\phi}{dr}\right\rvert \ll \frac{Z}{A}\frac{v_{ti}}{R^2}$	$\left\lvert\frac{d\omega_\phi}{dr}\right\rvert \gg \frac{Z}{A}\frac{v_{ti}}{R^2}$
g_0	$A^{1/2}$ or $Z/A^{1/2}$	$Z/A^{1/2}$	$A^{1/2}$
g_1	1 or Z/A	1 or Z/A	1
Γ	1 or Z/A	1 or Z/A	1
Q	1 or Z/A	1 or Z/A	1
Π	A or Z	Z	A
H	Z^2/A, A, or Z	Z^2/A	A or Z

Expansion in $A^{1/2}$. To obtain scaling laws for the turbulent fluxes and heating of trace heavy ions, we take $Z \sim A \gg 1$, $d\omega_\phi/dr \sim v_{ti}/R^2$, and expand $g = g_0 + g_1 + ...$ in powers of $A^{1/2}$. Here A is the heavy ion to proton mass ratio and v_{ti} is the main ion thermal speed. We assume the ratio of the ion-ion collision time, τ_{ii}, to the fluctuation time, τ_t, is sufficiently long ($\tau_{ii}/\tau_t \gg Z^2/A^{1/2}$) that collisions may be neglected in our analysis. In what follows, we keep Z and A dependences separate so that we can consider the subsidiary expansion $A^{1/2} \ll Z \ll A$.

Because the heavy ions are trace, their space and time scales are those of the bulk plasma turbulence. Thus, Z and A only enter Eq. (1) through explicit factors of m, $v \sim v_t$, and Z, as well as through g itself. In what follows, we assume the ratio of the heavy ion to proton temperature is much smaller than A, giving $v_t \sim A^{-1/2}$. The two lowest order equations in our expansion are thus

$$\begin{aligned}\frac{Dg_0}{Dt} + \langle\mathbf{v}_E\rangle\cdot\nabla g_0 = &-\frac{Ze}{T}F_M \mathbf{v}_\parallel\cdot\nabla\langle\Phi\rangle \\ &- \frac{mv_\parallel}{T}F_M\langle\mathbf{v}_E\rangle\cdot R\nabla\omega_\phi,\end{aligned} \tag{9}$$

$$\begin{aligned}&\frac{Dg_1}{Dt} + \langle\mathbf{v}_E\rangle\cdot\nabla g_1 + v_\parallel(\mathbf{b}+\mathbf{b}_1)\cdot\nabla g_0 \\ &= -\frac{ZeF_M}{T}\left(\mathbf{v}_M\cdot\nabla\langle\Phi\rangle + \frac{v_\parallel}{c}\mathbf{b}\cdot\nabla\left(v_\parallel\langle\delta A_\parallel\rangle\right)\right) \\ &\quad - \frac{mv_\parallel^2}{T}F_M\langle\mathbf{b}_1\rangle\cdot R\nabla\omega_\phi - \langle\mathbf{v}_E\rangle\cdot\nabla F_M,\end{aligned} \tag{10}$$

where $\mathbf{v}_E = c\mathbf{b}\times\nabla\Phi/B$ and $\mathbf{b}_1 = \mathbf{b}\times\nabla\delta A_\parallel/B$.

There are two possible scalings for both g_0 and g_1 due to a competition between terms with different A and Z dependences in Eqs. (9) and (10). In particular, $g_0 \propto Z/A^{1/2}$ or $A^{1/2}$, with the two scalings coming from the parallel electric field and rotation gradient source terms in Eq. (9), respectively. Using these scalings in Eq. (10) gives $g_1 \propto 1$ or Z/A. The number of such possible scalings is reduced by considering the cases

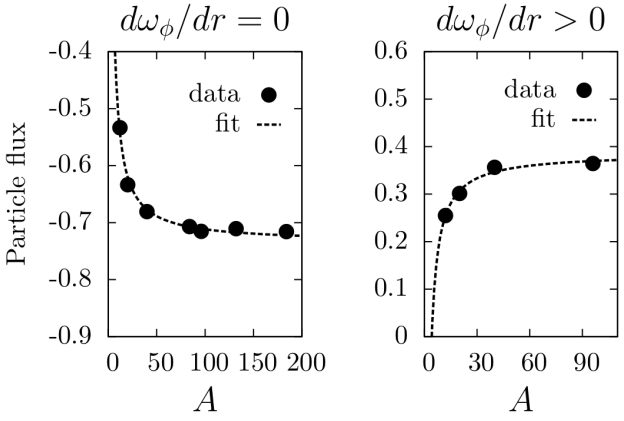

FIG. 1: Normalized particle flux, $(\Gamma_s/n_s v_{ti})(L/\rho_i)^2$, vs. mass number, A, for cases with and without differential rotation, ω_ϕ. The dashed line is a least-squares fit using our scaling predictions, given by $-0.7+2.3/A$ and $0.4-1.6/x$ for the left and right plots, respectively.

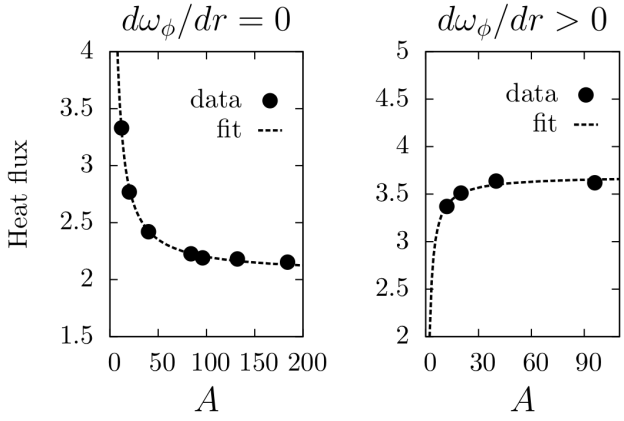

FIG. 2: Normalized heat flux, $(Q_s/n_s T_i v_{ti})(L/\rho_i)^2$, vs. mass number, A, for cases with and without differential rotation, ω_ϕ. The dashed line is a least-squares fit using our scaling predictions, given by $2.0+15/A$ and $3.7-3.7/A$ for the left and right plots, respectively.

where either the parallel electric field or rotation gradient source terms dominate in Eq. (9), corresponding to $|d\omega_\phi/dr| \ll (Z/A)v_{ti}/R^2$ and $|d\omega_\phi/dr| \gg (Z/A)v_{ti}/R^2$. The A- and Z-scalings for g_0 and g_1 in these limits, as well as for the general case, are summarized in Table I.

Flux and heating scalings. If $g_0(v_\parallel)$ is a solution to Eq. (9), then $-g_0(-v_\parallel)$ is also a solution. Thus, $\overline{\int_{-\infty}^{\infty} dv_\parallel g_0\{\Phi, \delta A_\parallel, \delta B_\parallel\}} = 0$, where the overline denotes a statistical average. As a result, g_0 does not contribute to the lowest order heating or particle and heat fluxes, Eqs. (5)-(7), whose integrands are otherwise even functions of $v_\parallel$. Conversely, the lowest order momentum flux integrand has a component proportional to $mv_\parallel$, so $\Pi \sim mv_t g_0 \propto A^{1/2} g_0$. Using our scalings for g_0, we see that Π has competing terms scaling as Z and A.

Note that Eq. (10) has a $v_\parallel$-symmetry opposite that of Eq. (9): if $g_1(v_\parallel)$ is a solution, then $g_1(-v_\parallel)$ is also a solution. For all higher order equations, one can show that the symmetry in $v_\parallel$ alternates between that of Eqs. (9) and (10). As a result, the only components of g that contribute to the particle and heat fluxes and heating are g_1, g_3, etc. Using Eqs. (6) and (7), we have $\{\Gamma, Q\} \sim g_1$, which in the general case has competing terms scaling as Z/A and 1 (no Z or A dependence), respectively.

The first term in the heating expression, (5), is the Joule heating and is scaled up by an explicit factor of Z (arising from the current), while the second term is viscous heating. At lowest order, the Joule heating term gives $H \propto (Zv_\parallel g_0,\ Zg_1) \propto (Z,\ Z^2/A)$. The viscous heating is proportional to $\Pi \propto (Z,\ A)$. H thus has competing terms scaling as Z^2/A, Z, and A. The scalings of the fluxes and heating are summarized in Table I.

Minority ion temperature. Integrating Eq. (5) by parts in time and using Eq. (1), the heating can be expressed as [13, 17, 18]

$$H_s = -\left\langle \frac{T_s \delta f_s}{F_{M,s}}\, C[\delta f_s] \right\rangle_\Lambda + \left(Q_s - \frac{3}{2}\Gamma_s\right)\frac{\partial \ln T_s}{\partial r} + \Gamma_s \frac{\partial \ln n_s}{\partial r}. \tag{11}$$

Our scalings indicate that H_s increases in magnitude with A or Z, but Γ_s and Q_s do not. The first term in Eq. (11) must thus dominate for A or Z large. This term, which we identify as the collisional entropy generation, is positive definite when summed over species. We argue that it is also positive definite species by species for the low collisionalities considered here.

The collision operator, C, consists of a test-particle piece, which is a diffusion operator in velocity space, and a field-particle piece, which is an integral operator [19]. Both contributions are inversely proportional to the collisional mean free path and thus small, except at small scales in the velocity space where large derivatives in the test-particle operator compensate [16, 20, 21]. The test-particle operator should thus dominate in weakly collisional plasmas, and its diffusive nature ensures that its contribution to entropy generation is positive-definite.

Consequently, trace heavy ions must be heated by turbulence instead of cooled. For this heating process to subside, the trace ion temperature must become large enough to interfere with our large A expansion. This happens when the heavy ion temperature exceeds the main ion temperature by a factor of $A \sim Z$. In this limit, the turbulent heating H becomes comparable to the heat flux Q so that H is no longer required to be positive definite. Our theory thus predicts that heavy ions will be hotter than light ions by a factor of $A \sim Z$ – but only if turbulent heating is larger than collisional temperature equilibration.

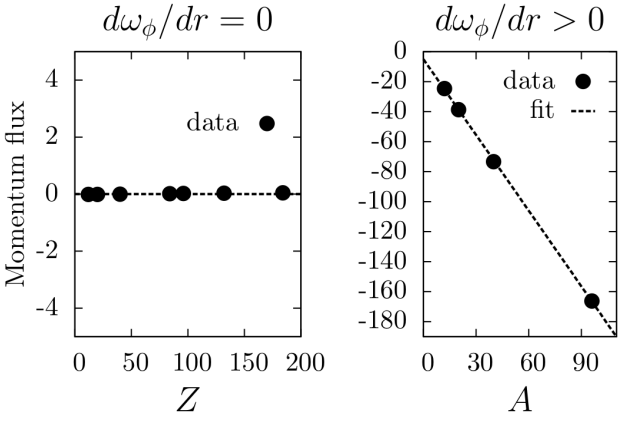

FIG. 3: Normalized toroidal angular momentum flux, $(\Pi_s/m_i n_s L v_{ti}^2)(L/\rho_i)^2$, vs. charge number, Z, and mass number, A, for cases with and without differential rotation, ω_ϕ. The dashed lines are least-squares fits using our scaling predictions, given by 0 and $-1.7A - 5.1$ for the left and right plots, respectively. The fact that $\Pi = 0$ for the case with $d\omega_\phi/dr = 0$ is a consequence of a symmetry property of the gyrokinetic equation [25].

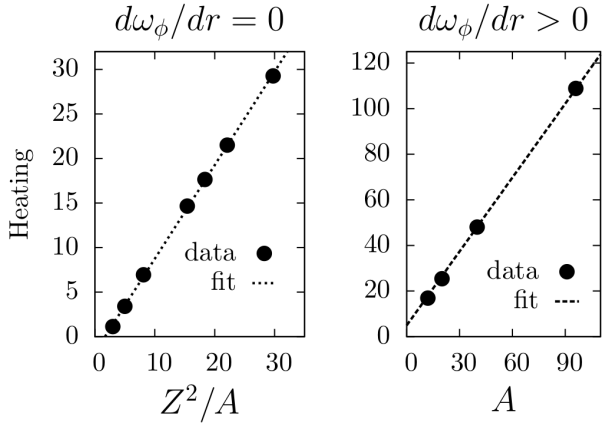

FIG. 4: Normalized heating, $H_s(L/n_s T_i v_{ti})(L/\rho_i)^2$, vs. charge number, Z, and mass number, A, for cases with and without differential rotation, ω_ϕ. The dashed lines are least-squares fits using our scaling predictions, given by $1.1Z^2/A - 1.8$ and $1.1A + 5.0$ for the left and right plots, respectively.

The collisional temperature equilibration of the main ions, i, and a trace heavy ion species, s, is $\mathcal{E}_s \equiv 2\sqrt{2}(Z_s^2/A_s)n_s\Delta T_s/\tau_{ii}$, where $\Delta T_s = T_s - T_i$. From Eq. (5), we estimate $H_s \sim S n_s T_i (\delta n_i/n_i)^2/\tau_t$, where S is the scaling of H with A and Z given in Table I, and we have assumed $e\Phi/T_i \sim \delta n_i/n_i \sim \delta n_s/n_s$. Balancing H_s and $\mathcal{E}_s$ gives $\Delta T_s/T_i \sim S(A/Z^2)(\tau_{ii}/\tau_t)(\delta n_i/n_i)^2 \sim S(A/Z^2)\tau_{ii}/\tau_E$, with τ_E the characteristic time scale over which the equilibrium density and temperature vary.

Numerical results. To test our predictions for the scalings of turbulent transport and heating, we employ the local, δf-gyrokinetic code `GS2` [22]. We consider an axisymmetric system with sheared magnetic field lines mapping out nested toroidal surfaces with circular cross sections (the Cyclone Base Case [23], parametrized with the Miller local equilibrium model [24]). Each simulation is electrostatic and includes kinetic electrons, as well as kinetic main and trace heavy ions with a wide range of Z and A values. The turbulence is driven by gradients in the mean ion and electron densities and temperatures, with $R_0(d\ln n/dr) = 2.2$ for the electrons and main ions, and $R_0(d\ln T/dr) = 6.9$ for all species, with R_0 the major radius at the center of the constant pressure surface. The collision frequency is chosen small, $\tau_{ii}^{-1}(a/v_{t,i}) = 0.001$, so that heavy ion collisions do not affect our scalings.

Two sets of simulations were carried out: one with a stationary plasma ($d\omega_\phi/dr = 0$) and one with a differentially rotating plasma ($d\omega_\phi/dr = 4.67 v_{ti}/R^2$). The simulation results are shown in Figs. (1)-(4). Data points for fluxes and heating at various Z and A values are plotted as solid circles and fit using a least-squares analysis with the predicted lowest order Z and A dependences, as well as the first order correction. In each case, the predicted scalings fit the data well. Note that the momentum flux for $d\omega_\phi/dr = 0$ is zero for all species due to a fundamental symmetry of the δf-gyrokinetic equation [25].

Discussion. We now discuss the implications of the trace heavy ion scalings derived in this Letter. First, the preferential heating of heavy ions should lead to large temperature disparities between different ion species in nearly collisionless plasmas. In many space and astrophysical plasmas, turbulent heating dominates over collisional equilibration because collisions are rare ($\tau_t/\tau_{ii} \ll \delta n_i^2/n_i^2$), and preferential heating of heavy ions is indeed observed [4, 5]. However, for such low collisionalities the equilibrium can deviate strongly from the isotropic Maxwellian assumed in our analysis, which cannot consequently address the large $T_\perp/T_\parallel$ values observed in coronal holes and the fast solar wind [6].

Using typical parameters for current fusion experiments ($n \sim 3\times10^{19}/m^3$, $T \sim 5\ keV$, and $\tau_E \sim 0.1s$) [26], we estimate turbulent heating to produce the impurity-ion temperature difference $\Delta T_s/T_i \sim 0.1S(A/Z^2)$. For rotating plasmas with $S \sim A$, our results indicate that the temperature of fully ionized impurities could differ from the main ions by several tens of percent; heavy ions such as tungsten will be only partially ionized at fusion temperatures, possibly leading to order unity temperature differences [27]. The increased densities and global confinement times expected for future fusion devices should reduce the average temperature difference to $\Delta T_s/T_i \sim 0.01S(A/Z^2)$, though this ratio may be larger in localized regions of the plasma.

Because the momentum transport of heavy ions is enhanced by A, heavy ions could significantly alter plasma momentum transport for densities as small as n_i/A. At this density level impurities cannot be modeled as a trace

species, but the trace scalings presented here suggest that non-trace impurities may play an important role in determining bulk plasma rotation.

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