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Radiative natural SUSY with a 125 GeV Higgs boson

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It has been argued that requiring low electroweak fine-tuning (EWFT) along with a (partial) decoupling solution to the SUSY flavor and CP problems leads to a sparticle mass spectra characterized by light Higgsinos at 100-300 GeV, sub-TeV third generation scalars, gluinos at a few TeV and multi-TeV first/second generation scalars (natural SUSY). We show that by starting with multi-TeV first/second and third generation scalars and trilinear soft breaking terms, the natural SUSY spectrum can be generated radiatively via renormalization group running effects. Using the complete 1-loop effective potential to calculate EWFT, significantly heavier third generation squarks can be allowed even with low EWFT. The large negative trilinear term and heavier top squarks allow for a light Higgs scalar in the ~ 125 GeV regime.

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Over 11 fb^{-1} of data has now been collected at the CERN LHC, and evidence at the 5σ level has emerged for the existence of a Higgs-like boson with mass $m_h \simeq 125$ GeV[1, 2]. While the Standard Model (SM) allows for a Higgs scalar anywhere within the range $\sim 115-800$ GeV[1] the minimal supersymmetric Standard Model (MSSM) requires that $m_h \lesssim 135$ GeV[5]. That the Higgs boson mass value falls within the narrow MSSM window may be regarded at least as supportive evidence for the existence of weak scale supersymmetry[6]. However, during the same data taking run of LHC, no signal for SUSY has emerged[7–9], leading to mass limits of $m_{\tilde g} > 1.4$ TeV for $m_{\tilde q} \sim m_{\tilde g}$, and $m_{\tilde g} \gtrsim 0.85$ TeV when $m_{\tilde q} \gg m_{\tilde g}$ within the popular minimal supergravity (mSUGRA or CMSSM) model[10]. These strong new sparticle mass limits from LHC push models such as mSUGRA into rather severe conflict with electroweak fine-tuning (EWFT) calculations[11], leading many physicists to consider alternative SUSY models which allow for much lower EWFT[12–20].

The EWFT arising in SUSY models can be gleaned most easily from the Higgs portion of the scalar potential, which in the MSSM is given by

$$V_{Higgs} = V_{tree} + \Delta V, \tag{1}$$

where the tree level portion is given by

$$\begin{aligned} V_{tree} &= (m_{H_u}^2 + \mu^2)|h_u^0|^2 + (m_{H_d}^2 + \mu^2)|h_d^0|^2 \\ &-B\mu(h_u^0 h_d^0 + h.c.) + \frac{1}{8}(g^2 + g'^2)(|h_u^0|^2 - |h_d^0|^2)^2 \end{aligned} \tag{2}$$

and the radiative corrections (in the effective potential approximation) by

$$\Delta V = \sum_i \frac{(-1)^{2s_i}}{64\pi^2} Tr\left((\mathcal{M}_i \mathcal{M}_i^\dagger)^2 \left[\log \frac{\mathcal{M}_i \mathcal{M}_i^\dagger}{Q^2} - \frac{3}{2} \right] \right), \tag{3}$$

where the sum over i runs over all fields that couple to Higgs fields, $\mathcal{M}_i^2$ is the *Higgs field dependent* mass squared matrix (defined as the second derivative of the tree level Lagrangian) of each of these fields, and the trace is over the internal as well as any spin indices. Minimization of the scalar potential in the h_u^0 and h_d^0 directions allows one to compute the gauge boson masses in terms of the Higgs field vacuum expectation values v_u and v_d, and leads to the well-known condition that

$$\frac{m_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{(\tan^2\beta - 1)} - \mu^2, \tag{4}$$

where the Σ_u^u and Σ_d^d terms arise from derivatives of ΔV evaluated at the potential minimum and $\tan\beta \equiv \frac{v_u}{v_d}$. At the one-loop level, Σ_u^u contains 18 and Σ_d^d contains 19 separate contributions from various particles/sparticles[11]. This minimization condition relates the Z-boson mass scale to the soft SUSY breaking terms and the superpotential higgsino mass μ.

In order for the model to enjoy *electroweak naturalness*[2] we adopt a fine-tuning measure which requires that

[1] The lower end of this mass range comes from previous Higgs searches at the LEP2 collider[3], while the upper value comes from the classic unitarity limits[4].

[2] Our definition of electroweak naturalness derives directly from the relation Eq. 4, which only involves SUSY parameters at the electroweak scale. Alternatively, one may apply fine-tuning considerations to how likely it is to generate specific weak scale parameter sets from high scale model parameters, or on how sensitive M_Z is to GUT scale parameters. The hyperbolic branch/focus point (HB/FP) region of the mSUGRA model is not fine-tuned with respect to the μ-parameter, but the presence of heavy third generation scalars requires large cancellations between $m_{H_u}^2$ and Σ_u^u terms in Eq. (4).

each of the 40 terms on the right-hand-side (RHS) of Eq. (4) should be of order $\sim m_Z^2/2$. Labeling each term as C_i (with $i = H_d,\ H_u,\ \mu,\ \Sigma_d^d(\tilde{t}_1),\ \Sigma_u^u(\tilde{t}_1),\ etc.$), we may require $C_{max} \equiv max|C_i| < \Lambda_{max}^2$, where $\Lambda_{max} \sim 100-300$ GeV, depending on how much EW fine-tuning one is willing to tolerate. This measure of fine-tuning is similar to (but not exactly the same as) Kitano-Nomura[21] but different from Barbieri-Giudice[22] beyond the tree-level. [3] We will define the new fine-tuning parameter $\Delta = C_{max}/(m_Z^2/2)$, where lower values of Δ correspond to less fine-tuning, and *e.g.* $\Delta = 20$ would correspond to $\Delta^{-1} = 5\%$ fine-tuning.

If we now require $\Lambda_{max} = 200$ GeV (*i.e.* $\Delta \lesssim 10$ or $\Delta^{-1} > 10\%$ fine-tuning), we find that $|\mu| < 200$ GeV, leading to a SUSY spectrum with light higgsinos in the range 100-200 GeV. Since the terms in Eq. (4) involving $m_{H_d}^2$ and Σ_d^d are suppressed by $\tan^2\beta$, $m_{H_d}^2$, and hence m_A^2, can be large without violating our fine tuning criterion; in this case, $m_A \lesssim \Lambda_{\max}\tan\beta$. The largest of the radiative corrections in Σ_u^u is expected to come from top squarks: $\Sigma_u^u \sim \frac{3}{16\pi^2}f_t^2 F(m_{\tilde{t}_{1,2}}^2)$, where $F(m^2) = m^2(\log\frac{m^2}{Q^2}-1)$ and f_t is the top quark Yukawa coupling. Requiring $\Sigma_u^u \lesssim \Lambda_{max}^2 = (200\ \mathrm{GeV})^2$ and assuming $F(m^2) \sim m^2$ then seemingly implies a spectrum of light top squarks $m_{\tilde{t}_{1,2}} \lesssim 1.5$ TeV and by $SU(2)$ symmetry, $m_{\tilde{b}_L} \lesssim 1.5$ TeV. Since the gluino loop contribution to the top squark mass goes like $\delta m_{\tilde{t}_i}^2 \sim \frac{2g_s^2}{3\pi^2}m_{\tilde{g}}^2 \times log$, where the $log \sim 1$, we also get a bound that $m_{\tilde{g}} \lesssim 3m_{\tilde{t}_i} \lesssim 4.5$ TeV. Thus, the sparticle mass spectra, here known as *natural SUSY*[13, 14, 16, 21], is characterized by (in the case where $\Lambda_{max} = 200$ GeV)

- higgsino-like charginos $\widetilde{W}_1$ and neutralinos $\widetilde{Z}_{1,2}$ with mass $\lesssim 200$ GeV,
- third generation squarks $m_{\tilde{t}_{1,2}},\ m_{\tilde{b}_1} \lesssim 1.5$ TeV,
- $m_{\tilde{g}} \lesssim 3-4.5$ TeV, depending on $m_{\tilde{t}_1}$.

Since first/second generation Yukawa couplings are tiny, the first/second generation squarks and sleptons enter Σ_u^u with only tiny contributions, so that their masses can be pushed into the multi-TeV regime, offering at least a partial decoupling solution to the SUSY flavor and CP problems[24–27]. Thus, it is also possible that

- $m_{\tilde{q}_{1,2}},\ m_{\tilde{\ell}_{1,2}} \sim 10-20$ TeV,

which is well beyond LHC search limits.

Numerous recent papers have been published examining aspects of natural SUSY. Regarding collider searches for natural SUSY, the light higgsinos can be produced at LHC at appreciable rates, but their small mass gaps $m_{\widetilde{W}_1} - m_{\widetilde{Z}_1} \sim m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1} \sim 10-20$ GeV lead to very soft visible energy release which is hard to detect above SM background at LHC[12]. The light third generation squarks, gluinos and heavier electroweak-inos may not be accessible to LHC searches depending on their masses and decay modes. A definitive test of natural SUSY may have to await searches for the light higgsino-like charginos and neutralinos at an International Linear e^+e^- Collider (ILC), which in this case would be a *higgsino factory*, in addition to a Higgs factory[12, 16, 28, 29].

While the advantages of natural SUSY are clear (low EWFT, decoupling solution to SUSY flavor and CP problems), some apparent problems seem to arise. First among these is that the sub-TeV spectrum of top squarks feed into the calculation of m_h, usually leading to m_h in the 115-120 GeV range, rather than $m_h \simeq 125$ GeV. Put more simply, a value $m_h \sim 125$ GeV favors top squark masses in excess of 1 TeV[30], while natural SUSY expects top squark masses below the TeV scale. A separate issue is the apparent disparity between the TeV third generation scale and the 10-20 TeV first/second generation mass scale; we will illustrate that it is possible to generate this radiatively. Several papers have appeared which attempt to reconcile the large value of m_h with naturalness by adding extra singlet fields to the theory, which provide extra contributions to m_h, thereby lifting it into its measured range[17, 19, 31]. This is what occurs in the NMSSM[32]. This solution may not be as appealing as it sounds in that additional singlets can destabilize the gauge hierarchy via tadpole effects[33], and may lead to cosmological problems via domain walls[34]. In this paper, we reconcile a large value of $m_h \sim 123-127$ GeV with low EWFT, and at the same time avoid at least a gross disparity between the soft breaking matter scalar mass scales, all the while avoiding the introduction of extra gauge singlets or any other sort of exotic matter.

To begin with, we return to our measure of EWFT: $\Delta = C_{max}/(m_Z^2/2)$. We calculate the complete 1-loop effective potential contributions to the quantities Σ_d^d and Σ_u^u in Eq. (4). We include contributions from $W^\pm$, Z, $\tilde{t}_{1,2}$, $\tilde{b}_{1,2}$, $\tilde{\tau}_{1,2}$, $\widetilde{W}_{1,2}$, $\widetilde{Z}_{1,2,3,4}$, t, b and τ, h, H and $H^\pm$. We adopt a scale choice $Q^2 = m_{\tilde{t}_1}m_{\tilde{t}_2}$ to minimize the largest of the logarithms. Typically, the largest contributions to Σ_u^u come from the top squarks, where we find

$$\Sigma_u^u(\tilde{t}_{1,2}) \;=\; \frac{3}{16\pi^2}F(m_{\tilde{t}_{1,2}}^2)\times$$

[3] Barbieri and Giudice[22] define a fine tuning measure $\Delta_{BG} = max|(a_i/M_Z^2)\partial M_Z^2/\partial a_i|$ for input parameters a_i. If we apply this to weak scale parameters μ^2 or $m_{H_u}^2$ in Eq. (4), our EWFT measure coincides with theirs at tree-level but differs when radiative corrections embodied in the Σ terms are included. In models defined at the high scale there are additional contributions to finetuning from corrections involving large logarithms that show up in $\Delta_{\rm BG}$ applied to $m_{H_u}^2(M_{GUT})$. Details will be presented in a future publication[23].

$$\left[f_t^2 - g_Z^2 \ \mp \ \frac{f_t^2 A_t^2 - 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_W)\Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}\right] \qquad (5)$$

where $\Delta_t = (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)/2 + m_Z^2\cos 2\beta(\frac{1}{4} - \frac{2}{3}x_W)$, $g_Z^2 = (g^2 + g'^2)/8$ and $x_W \equiv \sin^2\theta_W$. This equation is somewhat more complicated than the naive expression mentioned earlier, and contains contributions from the A_t parameter. For the case of the $\tilde{t}_1$ contribution, as $|A_t|$ gets large there is a suppression of $\Sigma_u^u(\tilde{t}_1)$ due to a cancellation between terms in the square brackets of Eq. (5). For the $\tilde{t}_2$ contribution, the large splitting between $m_{\tilde{t}_2}$ and $m_{\tilde{t}_1}$ yields a large cancellation within $F(m_{\tilde{t}_2}^2)$ $\left(\log(m_{\tilde{t}_2}^2/Q^2) \to \log(m_{\tilde{t}_2}/m_{\tilde{t}_1}) \to 1\right)$, leading also to suppression. So while large $|A_t|$ values suppress both top squark contributions to Σ_u^u, at the same time they also lift up the value of m_h, which is near maximal for large, negative A_t. Combining all effects, there exists the possibility that the same mechanism responsible for boosting the value of m_h into accord with LHC measurements can also suppress EWFT, leading to a model with electroweak naturalness.

To illustrate these ideas, we adopt a simple benchmark point from the 2-parameter non-universal Higgs mass SUSY model NUHM2[35], but with split generations, where $m_0(3) < m_0(1,2)$. In Fig. 1, we take $m_0(3) = 5$ TeV, $m_0(1,2) = 10$ TeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$ with $\mu = 150$ GeV, $m_A = 1000$ GeV and $m_t = 173.2$ GeV. We allow the GUT scale parameter A_0 to vary, and calculate the sparticle mass spectrum using Isajet 7.83[36], which includes the new EWFT measure. In frame *a*)., we plot the value of m_h versus A_0. While for $A_0 \sim 0$ the value of $m_h \sim 120$ GeV, as A_0 moves towards $-2m_0(3)$, the top squark radiative contributions to m_h increase, pushing its value up to 125 GeV. (There is an expected theory error of ± 2 GeV in our RGE-improved effective potential calculation of m_h, which includes leading 2-loop effects.[37]) At the same time, in frame *b*)., we see the values of $m_{\tilde{t}_{1,2}}$ versus A_0. In this case, large values of A_0 suppress the soft terms $m_{Q_3}^2$ and $m_{U_3}^2$ via RGE running. But also large weak scale values of A_t provide large mixing in the top squark mass matrix which suppresses $m_{\tilde{t}_1}$ and leads to an increased splitting between the two mass eigenstates which suppresses the top squark radiative corrections Σ_u^u. The EWFT measure Δ is shown in frame *c*)., where we see that while $\Delta \sim 50$ for $A_0 = 0$, when A_0 becomes large, then Δ drops to 10, or $\Delta^{-1} = 10\%$ EWFT. In frame *d*)., we show the weak scale value of A_t versus A_0 variation. While the EWFT is quite low– in the range expected for natural SUSY models– we note that the top squark masses remain above the TeV level, and in particular $m_{\tilde{t}_2} \sim 3.5$ TeV, in contrast to previous natural SUSY expectations.

The sparticle mass spectrum for this radiative NS benchmark point (RNS1) is shown in Table I for $A_0 = -7300$ GeV. The heavier spectrum of top and bot-

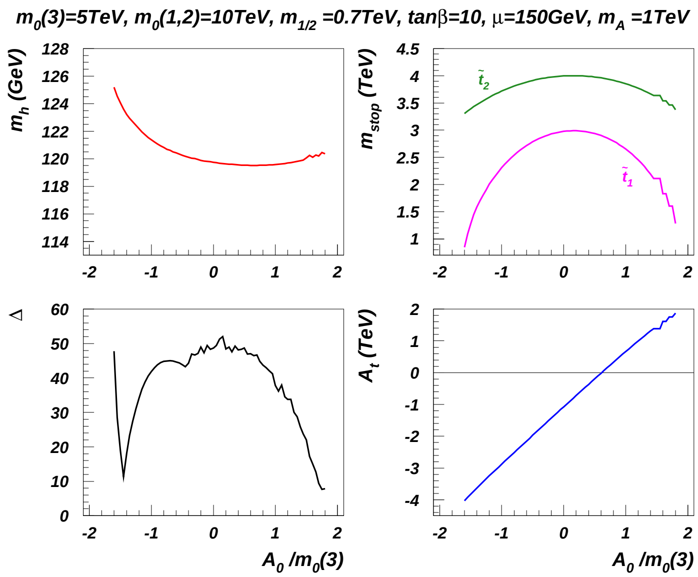

FIG. 1: Plot of *a*). m_h, *b*). $m_{\tilde{t}_{1,2}}$, *c*). Δ and *d*). A_t versus variation in A_0 for a model with $m_0(1,2) = 10$ TeV, $m_0(3) = 5$ TeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$ and $\mu = 150$ GeV and $m_A = 1$ TeV.

tom squarks seem likely outside of any near-term LHC reach, although in this case gluino[38] and possibly heavy electroweak-ino[39] pair production may be accessible to LHC14. Dialing the A_0 parameter up to -8 TeV allows for $m_h = 125.2$ GeV but increases EWFT to $\Delta = 29.5$, or 3.4% fine-tuning. Alternatively, pushing m_t up to 174.4 GeV increases m_h to 124.5 GeV with 6.2% fine-tuning; increasing $\tan\beta$ to 20 increases m_h to 124.6 GeV with 3.3% fine-tuning. We show a second point RNS2 with $m_0(1,2) = m_0(3) = 7.0$ TeV and $\Delta = 11.5$ with $m_h = 125$ GeV; note the common sfermion mass parameter at the high scale. For comparison, we also show in Table I the NS2 benchmark from Ref. [16]; in this case, a more conventional light spectra of top squarks is generated leading to $m_h = 121.1$ GeV, but the model– with $\Delta = 23.7$– has higher EWFT than RNS1 or RNS2.

To illustrate how low EWFT comes about even with rather heavy top squarks, we show in Fig. 2 the various third generation contributions to Σ_u^u, where the lighter mass eigenstates are shown as solid curves, while heavier eigenstates are dashed. The sum of all contributions to Σ_u^u is shown by the black curve marked total. From the figure we see that for $A_0 \sim 0$, indeed both top squark contributions to Σ_u^u are large and negative, leading to a large value of $\Sigma_u^u(total)$, which will require large fine-tuning in Eq. (4). As A_0 gets large negative, both top squark contributions to Σ_u^u are suppressed, and $\Sigma_u^u(\tilde{t}_1)$ even changes sign, leading to cancellations amongst the various Σ_u^u contributions.

The overall effect on EWFT is exhibited in Fig. 3

parameter	RNS1	RNS2	NS2
$m_0(1,2)$	10000	7025.0	19542.2
$m_0(3)$	5000	7025.0	2430.6
$m_{1/2}$	700	568.3	1549.3
A_0	-7300	-11426.6	873.2
$\tan\beta$	10	8.55	22.1
μ	150	150	150
m_A	1000	1000	1652.7
$m_{\tilde{g}}$	1859.0	1562.8	3696.8
$m_{\tilde{u}_L}$	10050.9	7020.9	19736.2
$m_{\tilde{u}_R}$	10141.6	7256.2	19762.6
$m_{\tilde{e}_R}$	9909.9	6755.4	19537.2
$m_{\tilde{t}_1}$	1415.9	1843.4	572.0
$m_{\tilde{t}_2}$	3424.8	4921.4	715.4
$m_{\tilde{b}_1}$	3450.1	4962.6	497.3
$m_{\tilde{b}_2}$	4823.6	6914.9	1723.8
$m_{\tilde{\tau}_1}$	4737.5	6679.4	2084.7
$m_{\tilde{\tau}_2}$	5020.7	7116.9	2189.1
$m_{\tilde{\nu}_\tau}$	5000.1	7128.3	2061.8
$m_{\widetilde{W}_2}$	621.3	513.9	1341.2
$m_{\widetilde{W}_1}$	154.2	152.7	156.1
$m_{\widetilde{Z}_4}$	631.2	525.2	1340.4
$m_{\widetilde{Z}_3}$	323.3	268.8	698.8
$m_{\widetilde{Z}_2}$	158.5	159.2	156.2
$m_{\widetilde{Z}_1}$	140.0	135.4	149.2
m_h	123.7	125.0	121.1
$\Omega^{std}_{\widetilde{Z}_1}h^2$	0.009	0.01	0.006
$BF(b\to s\gamma)\times 10^4$	3.3	3.3	3.6
$BF(B_s\to \mu^+\mu^-)\times 10^9$	3.8	3.8	4.0
$\sigma^{SI}(\widetilde{Z}_1 p)$ (pb)	1.1×10^{-8}	1.7×10^{-8}	1.8×10^{-9}
Δ	9.7	11.5	23.7

TABLE I: Input parameters and masses in GeV units for two Radiative natural SUSY benchmark points and one NS point with $\mu = 150$ GeV and $m_t = 173.2$ GeV.

where we plot several contributions C_i to the RHS of Eq. (4) versus A_0. Since μ is chosen close to m_Z, $C_\mu = (150\ \mathrm{GeV})^2$ is already quite small. The contribution $C_{\Sigma_u^u} \equiv -\Sigma_u^u \tan^2\beta/(\tan^2\beta - 1)$ is large at $A_0 \sim 0$, requiring a large value of $C_{H_u} \equiv -m_{H_u}^2 \tan^2\beta/(\tan^2\beta-1)$ for cancellation to maintain a small value of μ. As A_0 becomes large negative, $C_{\Sigma_u^u}$ drops towards zero, so that only small values of C_{H_u} are needed to maintain $\mu = 150$ GeV.

Summary: Models of Natural SUSY are attractive in that they enjoy low levels of EWFT, which arise from a low value of μ and possibly a sub-TeV spectrum of top squarks and $\tilde{b}_1$. In the context of the MSSM, such light top squarks are difficult to reconcile with the LHC Higgs boson discovery which favors $m_h \sim 125$ GeV. Models with a large negative trilinear soft-breaking parameter

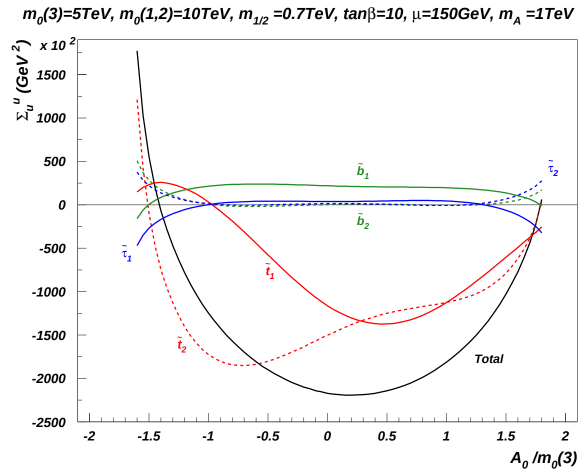

FIG. 2: Plot of third generation contributions to Σ_u^u versus A_0 for benchmark point RNS1 where solid curves come form the lighter mass eigenstate and dashed curves from the heavier. The black solid curve is Σ_u^u which has summed over all contributions.

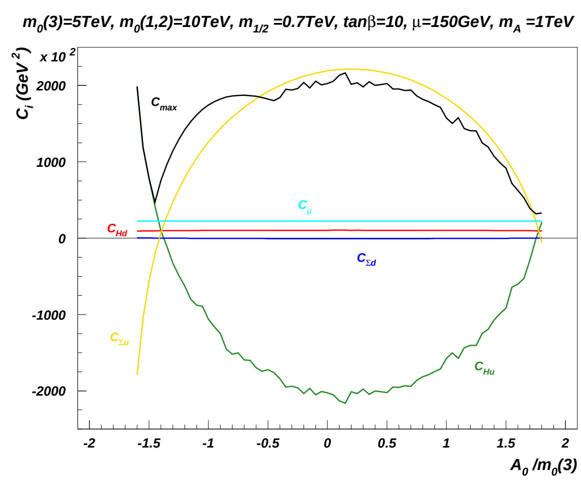

FIG. 3: Various C_i contributions to Eq. (4) versus A_0 for benchmark point RNS1.

A_t can maximize the value of m_h into the 125 GeV range without recourse to adding exotic matter into the theory. The large value of A_t also suppresses top squark contributions to the scalar potential minimization condition leading to models with low EWFT and a light Higgs scalar consistent with LHC measurements. (More details on the allowable parameter space of RNS will be presented in Ref. [23].) The large negative A_t parameter can arise from large negative A_0 at the GUT scale. In this case, large A_0 acts via 1-loop renormalization group equations (RGEs) and large $m_0(1,2)$ acts through 2-loop RGEs[25, 40] to squeeze multi-TeV third generation masses down into the few TeV range, thus generating the natural SUSY model radiatively. While RNS may be difficult to detect at LHC unless gluinos, third generation squarks or the heavier electroweak-inos are fortituously light, a linear e^+e^- collider with $\sqrt{s} \gtrsim 2|\mu|$ would have enough energy to produce the hallmark light higgsinos which are expected in this class of models.

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