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Radiative natural SUSY with a 125 GeV Higgs boson

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It has been argued that requiring low electroweak fine-tuning (EWFT) along with a (partial) decoupling solution to the SUSY flavor and CP problems leads to a sparticle mass spectra characterized by light Higgsinos at 100-300 GeV, sub-TeV third generation scalars, gluinos at a few TeV and multi-TeV first/second generation scalars (natural SUSY). We show that by starting with multi-TeV first/second and third generation scalars and trilinear soft breaking terms, the natural SUSY spectrum can be generated radiatively via renormalization group running effects. Using the complete 1-loop effective potential to calculate EWFT, significantly heavier third generation squarks can be allowed even with low EWFT. The large negative trilinear term and heavier top squarks allow for a light Higgs scalar in the \(\sim 125\) GeV regime.

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Over 11 fb\(^{-1}\) of data has now been collected at the CERN LHC, and evidence at the 5\(\sigma\) level has emerged for the existence of a Higgs-like boson with mass \(m_h \approx 125\) GeV[1, 2]. While the Standard Model (SM) allows for a Higgs scalar anywhere within the range \(115-800\) GeV\(^1\) the minimal supersymmetric Standard Model (MSSM) requires that \(m_h \approx 135\) GeV[5]. That the Higgs boson mass value falls within the narrow MSSM window may be regarded as at least as supportive evidence for the existence of weak scale supersymmetry[6]. However, during the same data taking run of LHC, no signal for SUSY has emerged[7–9], leading to mass limits of \(m_{\tilde{g}} > 1.4\) TeV for \(m_{\tilde{q}} \sim m_{\tilde{g}}\), and \(m_{\tilde{g}} \approx 0.85\) TeV when \(m_{\tilde{q}} \gg m_{\tilde{g}}\) within the popular minimal supergravity (mSUGRA or CMSSM) model[10]. These strong new sparticle mass limits from LHC push models such as mSUGRA into rather severe conflict with electroweak fine-tuning (EWFT) calculations[11], leading many physicists to consider alternative SUSY models which allow for much lower EWFT[12–20].

The EWFT arising in SUSY models can be gleaned most easily from the Higgs portion of the scalar potential, which in the MSSM is given by

\[
V_{\text{Higgs}} = V_{\text{tree}} + \Delta V,
\]

where the tree level portion is given by

\[
V_{\text{tree}} = (m_{H_u}^2 + \mu^2) |h_u^0|^2 + (m_{H_d}^2 + \mu^2) |h_d^0|^2
- B\mu(h_u^0 h_d^0 + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)(|h_u^0|^2 - |h_d^0|^2)^2
\]

and the radiative corrections (in the effective potential approximation) by

\[
\Delta V = \sum_i \frac{(-1)^{2n_i}}{64\pi^2} Tr \left( (M_i M_i^\dagger)^2 \left[ \log \frac{M_i M_i^\dagger}{Q^2} \frac{3}{2} \right] \right),
\]

where the sum over \(i\) runs over all fields that couple to Higgs fields, \(M_i^2\) is the Higgs field dependent mass squared matrix (defined as the second derivative of the tree level Lagrangian) of each of these fields, and the trace is over the internal as well as any spin indices. Minimization of the scalar potential in the \(h_u^0\) and \(h_d^0\) directions allows one to compute the gauge boson masses in terms of the Higgs field vacuum expectation values \(v_u\) and \(v_d\), and leads to the well-known condition that

\[
\frac{m_Z^2}{2} = \frac{(m_{H_u}^2 + \Sigma_d^d) - (m_{H_d}^2 + \Sigma_u^u) \tan^2 \beta}{(\tan^2 \beta - 1)} - \mu^2,
\]

where the \(\Sigma_u^u\) and \(\Sigma_d^d\) terms arise from derivatives of \(\Delta V\) evaluated at the potential minimum and \(\tan \beta \equiv \frac{v_u}{v_d}\). At the one-loop level, \(\Sigma_u^u\) contains 18 and \(\Sigma_d^d\) contains 19 separate contributions from various particles/sparticles[11]. This minimization condition relates the \(Z\)-boson mass scale to the soft SUSY breaking terms and the superpotential higgsino mass \(\mu\).

In order for the model to enjoy electroweak naturalness\(^2\) we adopt a fine-tuning measure which requires that

\^1\ The lower end of this mass range comes from previous Higgs searches at the LEP2 collider[3], while the upper value comes from the classic unitarity limits[4].

\^2\ Our definition of electroweak naturalness derives directly from the relation Eq. 4, which only involves SUSY parameters at the electroweak scale. Alternatively, one may apply fine-tuning considerations to how likely it is to generate specific weak scale parameter sets from high scale model parameters, or on how sensitive \(M_Z\) is to GUT scale parameters. The hyperbolic branch/focus point (HB/FP) region of the mSUGRA model is not fine-tuned with respect to the \(\mu\)-parameter, but the presence of heavy third generation scalars requires large cancellations between \(m_{H_u}^2\) and \(\Sigma_u^u\) terms in Eq. (4).
partial decoupling solution to the SUSY flavor and CP problems[24–27]. Thus, it is also possible that

- $m_{\tilde{q}_{1,2}}$, $m_{\tilde{t}_{1,2}} \sim 10 - 20$ TeV,

which is well beyond LHC search limits.

Numerous recent papers have been published examining aspects of natural SUSY. Regarding collider searches for natural SUSY, the light higgsinos can be produced at LHC at appreciable rates, but their small mass gaps $m_{\tilde{W}_{1}} - m_{\tilde{Z}_{1}} \sim m_{\tilde{Z}_{2}} - m_{\tilde{Z}_{1}} \sim 10 - 20$ GeV lead to very soft visible energy release which is hard to detect above SM background at LHC[12]. The light third generation squarks, gluinos and heavier electroweak-inos may not be accessible to LHC searches depending on their masses and decay modes. A definitive test of natural SUSY may have to await searches for the light higgsino-like charginos and neutralinos at an International Linear $e^+ e^-$ Collider (ILC), which in this case would be a higgsino factory, in addition to a Higgs factory[12, 16, 28, 29].

While the advantages of natural SUSY are clear (low EWFT, decoupling solution to SUSY flavor and CP problems), some apparent problems seem to arise. First among these is that the sub-TeV spectrum of top squarks feed into the calculation of $m_b$, usually leading to $m_b$ in the 115-120 GeV range, rather than $m_b \approx 125$ GeV. Put more simply, a value $m_b \sim 125$ GeV favors top squark masses in excess of $1$ TeV[30], while natural SUSY expects top squark masses below the TeV scale. A separate issue is the apparent disparity between the TeV third generation scale and the 10-20 TeV first/second generation mass scale; we will illustrate that it is possible to generate this radiatively. Several papers have appeared which attempt to reconcile the large value of $m_h$ with naturalness by adding extra singlet fields to the theory, which provide extra contributions to $m_h$, thereby lifting it into its measured range[17, 19, 31]. This is what occurs in the NMSSM[32]. This solution may not be as appealing as it sounds in that additional singlets can destabilize the gauge hierarchy via tadpole effects[33], and may lead to cosmological problems via domain walls[34]. In this paper, we reconcile a large value of $m_h \sim 123 - 127$ GeV with low EWFT, and at the same time avoid at least a gross disparity between the soft breaking matter scalar mass scales, all the while avoiding the introduction of extra gauge singlets or any other sort of exotic matter.

To begin with, we return to our measure of EWFT: $\Delta = C_{\text{max}}/(m_{Z}^{2}/2)$. We calculate the complete 1-loop effective potential contributions to the quantities $\Sigma_{u}^{d}$ and $\Sigma_{u}^{u}$ in Eq. (4). We include contributions from $W^{\pm}$, $Z$, $\tilde{t}_{1,2}, \tilde{b}_{1,2, \tilde{q}_{1,2}}, \tilde{\tau}_{1,2}, W_{1,2, 1,2, 1,3, 4}$, $t$, $b$, $\tau$, $h$, $H$, and $H^{\pm}$. We adopt a scale choice $Q^{2} = m_{\tilde{t}_{1}} m_{\tilde{t}_{2}}$ to minimize the largest of the logarithms. Typically, the largest contributions to $\Sigma_{u}^{d}$ come from the top squarks, where we find

$$\Sigma_{u}^{d}(\tilde{t}_{1,2}) = \frac{3}{16\pi^{2}} F(m_{\tilde{t}_{1,2}}^{2}) \times$$

3 Barbieri and Giudice[22] define a fine tuning measure $\Delta_{BG} = \text{max}|a_{i} / M_{Z}^{2} \partial M_{Z}^{2} / \partial a_{i}|$ for input parameters $a$. If we apply this to weak scale parameters $\mu^{2}$ or $m_{H}^{2}$ in Eq. (4), our EWFT measure coincides with theirs at tree-level but differs when radiative corrections embodied in the $\Sigma$ terms are included. In models defined at the high scale there are additional contributions to finetuning from corrections involving large logarithms that show up in $\Delta_{BG}$ applied to $m_{\tilde{t}_{1,2}}(M_{GUT})$. Details will be presented in a future publication[23].
where $\Delta_i = (m_{Li}^2 - m_{Li,0}^2)/2 + m_Z^2 \cos 2\beta (\frac{1}{2} - \frac{2}{3}x_W)$, $g_Z^2 = (g_1^2 + g_2^2)/8$ and $x_W \equiv \sin^2 \theta_W$. This equation is somewhat more complicated than the naive expression mentioned earlier, and contains contributions from the $A_t$ parameter. For the case of the $t_\tilde{t}$ contribution, as $|A_t|$ gets large there is a suppression of $\Sigma_u^q(\tilde{t}_i)$ due to a cancellation between terms in the square brackets of Eq. (5).

For the $t_2$ contribution, the large splitting between $m_{\tilde{t}_2}$ and $m_{\tilde{t}_1}$ yields a large cancellation within $F(m_{\tilde{t}_2}^2)$ \( \log(m_{\tilde{t}_2}^2/Q^2) \rightarrow \log(m_{\tilde{t}_2}/m_{\tilde{t}_1}) \rightarrow 1 \), leading also to suppression. So while large $|A_t|$ values suppress both top squark contributions to $\Sigma_u$, at the same time they also lift up the value of $m_{\tilde{t}}$, which is near maximal for large, negative $A_t$. Combining all effects, there exists the possibility that the same mechanism responsible for boosting the value of $m_{\tilde{t}}$ into accord with LHC measurements can also suppress EWFT, leading to a model with electroweak naturalness.

To illustrate these ideas, we adopt a simple benchmark point from the 2-parameter non-universal Higgs mass SUSY model NUHM2,[35] but with split generations, where $m_0(3) < m_0(1,2)$. In Fig. 1, we take $m_0(3) = 5$ TeV, $m_0(1,2) = 10$ TeV, $m_{1/2} = 700$ GeV, $\tan \beta = 10$ with $\mu = 150$ GeV and $m_A = 173.2$ GeV. We allow the GUT scale parameter $A_0$ to vary, and calculate the sparticle mass spectrum using Isajet 7.83,[36] which includes the new EWFT measure. In frame a), we plot the value of $m_{\tilde{t}}$ versus $A_0$. While for $A_0 \sim 0$ the value of $m_{\tilde{t}} \sim 120$ GeV, as $A_0$ moves towards $-2m_0(3)$, the top squark radiative contributions to $m_{\tilde{t}}$ increase, pushing its value up to 125 GeV. (There is an expected theory error of $\pm 2$ GeV in our RGE-improved effective potential calculation of $m_{\tilde{t}}$, which includes leading 2-loop effects.[37]) At the same time, in frame b), we see the values of $m_{\tilde{t}_{1,2}}$ versus $A_0$. In this case, large values of $A_t$ suppress the soft terms $m_{\tilde{t}_i}^2$ and $m_{\tilde{t}_i}^3$ via RGE running. But also large weak scale values of $A_t$ provide large mixing in the top squark mass matrix which suppresses $m_{\tilde{t}_i}$ and leads to an increased splitting between the two mass eigenstates which suppresses the top squark radiative corrections $\Sigma_u$. The EWFT measure $\Delta$ is shown in frame c), where we see that while $\Delta \sim 50$ for $A_0 = 0$, when $A_0$ becomes large, then $\Delta$ drops to 10, or $\Delta^{-1} = 10\%$ EWFT. In frame d), we show the weak scale value of $A_t$ versus $A_0$ variation. While the EWFT is quite low- in the range expected for natural SUSY models- we note that the top squark masses remain above the TeV level, and in particular $m_{\tilde{t}_1,2} \sim 3.5$ TeV, in contrast to previous natural SUSY expectations.

The sparticle mass spectrum for this radiative NS benchmark point (RNS1) is shown in Table I for $A_0 = -7300$ GeV. The heavier spectrum of top and bottom squarks seem likely outside of any near-term LHC reach, although in this case gluino[38] and possibly heavy electroweakino[39] pair production may be accessible to LHC14. Dialing the $A_0$ parameter up to $-8$ TeV allows for $m_{\tilde{t}} = 125.2$ GeV but increases EWFT to $\Delta = 29.5$, or 3.4% fine-tuning. Alternatively, pushing $m_{\tilde{t}}$ up to 174.4 GeV increases $m_{\tilde{t}}$ to 124.5 GeV with 6.2% fine-tuning; increasing $\tan \beta$ to 20 increases $m_{\tilde{t}}$ to 124.6 GeV with 3.3% fine-tuning. We show a second point RNS2 with $m_0(1,2) = m_0(3) = 7.0$ TeV and $\Delta = 11.5$ with $m_{\tilde{t}} = 125$ GeV; note the common sfermion mass parameter at the high scale. For comparison, we also show in Table I the NS2 benchmark from Ref. [16]; in this case, a more conventional light spectra of top squarks is generated leading to $m_{\tilde{t}} = 121.1$ GeV, but the model- with $\Delta = 23.7$- has higher EWFT than RNS1 or RNS2.

To illustrate how low EWFT comes about even with rather heavy top squarks, we show in Fig. 2 the various third generation contributions to $\Sigma_u^q$ where the lighter mass eigenstates are shown as solid curves, while heavier eigenstates are dashed. The sum of all contributions to $\Sigma_u^q$ is shown by the black curve marked total. From the figure we see that for $A_0 \sim 0$, indeed both top squark contributions to $\Sigma_u^q$ are large and negative, leading to a large value of $\Sigma_u^q(total)$, which will require large fine-tuning in Eq. (4). As $A_0$ gets large negative, both top squark contributions to $\Sigma_u^q$ are suppressed, and $\Sigma_u^q(\tilde{t}_1)$ even changes sign, leading to cancellations amongst the various $\Sigma_u^q$ contributions.

The overall effect on EWFT is exhibited in Fig. 3.
for cancellation to maintain a small value of $\mu$

Eq. (4) versus $A_0$

with

Radiative natural SUSY benchmark points and one NS point

TABLE I: Input parameters and masses in GeV units for two

C

quiring a large value of $C$

tation

where we plot several contributions $C_i$ to the RHS of

Eq. (4) versus $A_0$. Since $\mu$ is chosen close to $m_Z$, $C_{\mu} = (150 \text{ GeV})^2$ is already quite small. The contribution $C_{\Sigma_2} \equiv -\Sigma_2^\mu \tan^2 \beta/(\tan^2 \beta - 1)$ is large at $A_0 \sim 0$, requiring a large value of $C_{H_\mu} \equiv -m_{H_\mu} \tan^2 \beta/(\tan^2 \beta - 1)$ for cancellation to maintain a small value of $\mu$. As $A_0$ becomes large negative, $C_{\Sigma_2}$ drops towards zero, so that only small values of $C_{H_\mu}$ are needed to maintain $\mu = 150 \text{ GeV}$.

**Summary:** Models of Natural SUSY are attractive in that they enjoy low levels of EWFT, which arise from a low value of $\mu$ and possibly a sub-TeV spectrum of top squarks and $b_1$. In the context of the MSSM, such light top squarks are difficult to reconcile with the LHC Higgs boson discovery which favors $m_h \sim 125 \text{ GeV}$. Models with a large negative trilinear soft-breaking parameter $A_t$ can maximize the value of $m_h$ into the 125 GeV range without recourse to adding exotic matter into the theory. The large value of $A_t$ also suppresses top squark contributions to the scalar potential minimization condition leading to models with low EWFT and a light Higgs scalar consistent with LHC measurements. (More details on the allowable parameter space of RNS will be presented in Ref. [23].) The large negative $A_t$ parameter can arise from large negative $A_0$ at the GUT scale. In this case, large $A_0$ acts via 1-loop renormalization group equations (RGEs) and large $m_0(1, 2)$ acts through 2-loop RGEs [25, 40] to squeeze multi-TeV third generation masses down into the few TeV range, thus generating

\[
\Omega_{\text{tot}}^{\mu \beta} 
\]

\[
BF(b \rightarrow s\gamma) \times 10^4 
\]

\[
BF(B_s \rightarrow \mu^+\mu^-) \times 10^9 
\]

\[
\sigma^{SI}(\bar{Z}_1p) (\text{pb}) 
\]

\[
\Delta
\]

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\[
\Omega_{\text{tot}}^{\mu \beta} 
\]

\[
BF(b \rightarrow s\gamma) \times 10^4 
\]

\[
BF(B_s \rightarrow \mu^+\mu^-) \times 10^9 
\]

\[
\sigma^{SI}(\bar{Z}_1p) (\text{pb}) 
\]

\[
\Delta
\]

![FIG. 2: Plot of third generation contributions to $\Sigma_0^2$ versus $A_0$ for benchmark point RNS1 where solid curves come form the lighter mass eigenstate and dashed curves from the heavier. The black solid curve is $\Sigma_0^2$ which has summed over all contributions.](image2)

![FIG. 3: Various $C_i$ contributions to Eq. (4) versus $A_0$ for benchmark point RNS1.](image3)
the natural SUSY model radiatively. While RNS may be
difficult to detect at LHC unless gluinos, third generation
squarks or the heavier electroweak-inos are fortuitously
light, a linear $e^+e^-$ collider with $\sqrt{s} \sim 2|\mu|$ would have
enough energy to produce the hallmark light higgsinos
which are expected in this class of models.

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was supported in part by the US Department of Energy,
[6] For reviews of SUSY, see H. Baer and X. Tata, Weak Scale Supersymmetry: From Superfields to Scattering Events, (Cambridge University Press, 2006);
[9] For a recent review, see A. Parker, talk at 36th ICHEP, Melbourne, Australia, July 2012.