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Stimulation of the fluctuation superconductivity by the $\mathcal{PT}$-symmetry

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We discuss fluctuations near the second order phase transition where the free energy has an additional non-Hermitian term. The spectrum of the fluctuations changes when the odd-parity potential amplitude exceeds the critical value corresponding to the $\mathcal{PT}$-symmetry breakdown in the topological structure of the Hilbert space of the effective non-Hermitian Hamiltonian. We calculate the fluctuation contribution to the differential resistance of a superconducting weak link and find the manifestation of the $\mathcal{PT}$-symmetry breaking in its temperature evolution. We successfully validate our theory by carrying out measurements of far from equilibrium transport in mesoscale-patterned superconducting wires.

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An Hermitian character of the Hamiltonian expressed by the condition $H^\dagger = H$ is a cornerstone of quantum mechanics as it ensures that the energies of its stationary states are real. Yet it was discovered not long ago [1] that the weaker requirement $H^\ddagger = H$, where $\ddagger$ represents combined parity reflection and time reversal ($\mathcal{PT}$), introduces new classes of *complex Hamiltonians* [2] whose spectra are still real and positive [1, 3–5]. This generalization of Hermiticity opened a new field of research in quantum mechanics and beyond that had been enjoying ever since a rapid growth.

We focus here on the superconducting fluctuations above the superconductor – normal metal transition in quasi 1D superconducting wire of the finite length L driven far from equilibrium by an electric field $\mathcal{E}$, see Fig. 1. We show that either the presence or absence of $\mathcal{PT}$-symmetry in the Cooperon (fluctuation) propagator, which depends on the magnitude of $\mathcal{E}$, effects strongly the structure of fluctuations.

The $\mathcal{PT}$-symmetrical state corresponds to small drive, $|\mathcal{E}| < \mathcal{E}_c$, where $\mathcal{E}_c$ is of the order of the Thouless energy, $E_{\rm Th} = \hbar D/L^2$, the characteristic energy scale of the dirty quasi-one-dimensional conductor, see Fig. 1, where D is the electron diffusion coefficient in the wire and L is its length. This state is nonequilibrium but *stationary* where fluctuating Cooper pairs survive in the presence of the electric field. Breaking the $\mathcal{PT}$-symmetry at $|\mathcal{E}| = \mathcal{E}_c$ is the dynamic phase transition from the stationary to the nonstationary dynamic state where the electric field quickly destroys the Cooper-pairs. In this state Cooper pair wave function qualitatively is represented as a linear superposition of the Ivlev-Kopnin “kinks” located at the wire ends [6], having the phases that rotate with the opposite rate. We calculate the fluctuation correction to conductivity and show that for $|\mathcal{E}| > \mathcal{E}_c$ this correction is strongly suppressed by an electric field. It implies that

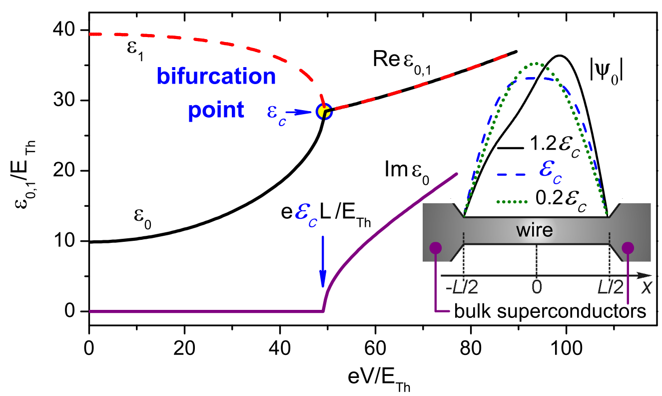

FIG. 1. (Color online) Evolution of the two lowest energy levels, ε_0 and ε_1 upon applying bias $V = \mathcal{E}L$. At $\mathcal{E} = \mathcal{E}_c$ the levels merge and form the complex conjugate pair at $\mathcal{E} > \mathcal{E}_c$. The inset shows the change of the eigenfunction $|\psi_0(x)|$, normalized to unity, upon variation of $\mathcal{E}$: $\psi_0(x)$ is symmetrical for $\mathcal{E} \le \mathcal{E}_c$ and is asymmetric at $\mathcal{E} > \mathcal{E}_c$. The asymmetry of $\psi_0(x)$ is the signature of the $\mathcal{PT}$-invariance breakdown.

$\mathcal{PT}$-symmetry effectively protects Cooper pairs from the detrimental effect of the electric field and stabilizes superconductivity.

The dynamics of the superconducting fluctuations is described by the retarded fluctuation propagator $\hat{L}_R(t, t'; x, x')$ [10, 11]:

$$\hat{L}_R^{-1} = \partial_{\rm t} + \mathcal{H}_{\rm eff}\,, \qquad (1)$$

where we use the units $k_{\rm B} = e = \hbar = 1$. The effective Hamiltonian, $\mathcal{H}_{\rm eff}[\mathcal{E}]$, describes the linearized Ginsburg-Landau (GL) field theory [7, 8]. In general, $\hat{L}_R$ can be expanded through the eigen functions ψ_n and the eigen values ε_n of $\mathcal{H}_{\rm eff}$:

$$\hat{L}_R(\omega; x, x') = \sum_n \frac{\psi_n(x)\psi_n^*(x')}{2i\omega - \varepsilon_n}. \quad (2)$$

We show below using the technique developed in Refs. [1, 5, 6] that $\mathcal{PT}$-symmetry of $\hat{L}_R$ holds at low drives. At large $\mathcal{E}$ exceeding the certain critical value, $\mathcal{E}_c$, the $\mathcal{PT}$-invariance breaks down and the two lowest energy states ε_0 and ε_1 of $H_{\rm eff}$ merge. At $\mathcal{E} > \mathcal{E}_c$ they form the complex conjugate pair (Fig.1). From the general viewpoint of the *catastrophe theory* [12] bifurcations of $H_{\rm eff}[\mathcal{E}]$ belong to the so-called fold catastrophe topological class. This class of the bifurcations is (topologically) protected with respect to small local perturbation of $\mathcal{H}_{\rm eff}$ preserving the symmetry of the system. Therefore, in order to establish the existence of the bifurcation and to find its type it would suffice to investigate the effective Hamiltonian, $\mathcal{H}_{\rm eff} = -D\nabla_x^2 - \tau^{-1} - 2i\varphi$. Here φ is the potential of the electric field responsible for the nonhermitivity of $\mathcal{H}_{\rm eff}$. In application to our problem, neglecting in $\mathcal{H}_{\rm eff}$ the decay of the mean-field superconducting order parameter from the reservoirs into the wire does not violate the catastrophe theory classification of bifurcation symmetries. For the same reason one may choose the boundary conditions in a form: $\psi(x = \pm L/2 \mp 0) = \psi(x \to \pm\infty) = 0$. Here τ is the GL-time and $D > 0$ is the material constant (e.g., electron diffusion coefficient in the dirty superconductor). We further discuss the case where $\varphi(x) = \mathcal{E}x$ and x is the coordinate along the wire.

The problem

$$\mathcal{H}_{\rm eff}\psi = \varepsilon\psi, \quad (3)$$

can be solved using the anzatz [6]:

$$\psi(x) = \alpha\,{\rm Ai}(Z) + \beta\,{\rm Bi}(Z), \quad (4)$$

$$Z(x) = \frac{\varepsilon + 2ix\mathcal{E}}{E_{\rm Th}}\left(\frac{E_{\rm Th}}{2V}\right)^{2/3}, \quad (5)$$

where Ai and Bi are the Airy-functions, and α, β are fixed by the boundary conditions. We absorbed τ^{-1} into the definition of ε. Then the equation determining the eigenvalues acquires the form:

$$F(\varepsilon, \mathcal{E}) \equiv {\rm Im}\left[{\rm Ai}(Z(L/2))\,{\rm Bi}(Z(-L/2))\right] = 0. \quad (6)$$

The critical field $\mathcal{E}_c$ is the field of emergence of the first *bifurcation* [12, 13] of Eq.(6) corresponding to merging of lowest levels and is given by the conditions

$$F(\varepsilon_c, \mathcal{E}_c) = 0\,, \quad \partial_\varepsilon F(\varepsilon_c, \mathcal{E}_c) = 0\,, \quad (7)$$

where ε_c is the value of the energy at the levels merging point. We find $\mathcal{E}_c \approx 49.25 E_{\rm Th}/L$, where $\varepsilon_c = \varepsilon_0 = \varepsilon_1 \approx 28.43 E_{\rm Th}$. The same conditions give the next bifurcations where higher pairs of levels merge pairwise, $\mathcal{E}_c^{(1)} \approx 4\mathcal{E}_{\rm c}$, $\mathcal{E}_c^{(2)} \approx 10\mathcal{E}_{\rm c}$, *etc...* As we have mentioned above, the bifurcations described here belong to the universality class of the "fold catastrophe" (A_2 in ADE classification). Then $\mathcal{E}_c$ is the *tipping point* of the catastrophe.

Expanding Eq.(6) near the bifurcation one finds $[\frac{1}{2}(\varepsilon - \varepsilon_c)^2\partial_\varepsilon^2 + (\mathcal{E} - \mathcal{E}_c)\partial_{\mathcal{E}_c}]F(\varepsilon, \mathcal{E})|_{\varepsilon\to\varepsilon_c, \mathcal{E}\to\mathcal{E}_c} = 0$, so,

$$\varepsilon_{0,1}(\mathcal{E}) \approx \varepsilon_c \mp E_{\rm Th}\sqrt{\eta\left[1 - \frac{\mathcal{E}^2}{\mathcal{E}_c^2}\right]}, \quad (8)$$

$$\eta = \mathcal{E}_c\frac{\partial_{\mathcal{E}_c}F(\varepsilon_c, \mathcal{E}_c)}{E_{\rm Th}^2\partial_\varepsilon^2 F(\varepsilon_c, \mathcal{E}_c)} \approx \frac{\pi^2}{\sqrt{2}}\frac{L\mathcal{E}_c}{E_{\rm Th}}.$$

The results of the numerical solution of the eigenvalue problem are shown in Fig.1 [14]. In the limiting case of the semi-infinite wire, $\psi_{0,1}$ for $\mathcal{E} > \mathcal{E}_c$ change with coordinates similarly to the solution for the order parameter found in Ref. [6].

Now we proceed with the analysis of the dynamics of the fluctuations in the wire using the following equation:

$$(\hat{L}^R)^{-1}\psi = 0. \quad (9)$$

As long as the field does not exceed the critical value, $\mathcal{E} < \mathcal{E}_c$, the stationary solution of Eq.(1) remains stable and is given by

$$\psi(x) \simeq \psi_0(x), \quad (10)$$

where we have taken $\tau^{-1} = \varepsilon_0$. This solution is $\mathcal{PT}$-invariant, i.e. $|\psi_0(x)| = |\psi_0(-x)|$, see Fig.1. The extremum of $|\psi_0(x)|$ is thus located at $x = 0$, at the center of the weak link. The effective field-dependent critical temperature for the superfluid correlations-induced superfluidity within the weak link is to be found from the relation $\tau^{-1} = \varepsilon_0$ and is given by $T_c^{\rm (eff)}(\mathcal{E}) = T_c - \pi\varepsilon_0(\mathcal{E})/8$. The $T_c^{\rm (eff)}(\mathcal{E})$ dependence become singular near the critical field $\mathcal{E}_c$, $dT_c^{\rm (eff)}/d\mathcal{E}|_{\mathcal{E}=\mathcal{E}_c} = \infty$; this singularity results in the anomalous behaviour of the nonlinear fluctuation corrections to the conductivity. As the field goes above the threshold, $\mathcal{E} > \mathcal{E}_c$, the stationary solution of Eq.(1) ceases to exist. The eigenvalues become complex conjugate, ${\rm Re}\,\varepsilon_0 = -\,{\rm Re}\,\varepsilon_1 = \tau^{-1}$ [see inset in Fig.1], and

$${\rm Im}\,\varepsilon_0(\mathcal{E}) = -\,{\rm Im}\,\varepsilon_1(\mathcal{E}) \approx E_{\rm Th}\sqrt{\eta\left[\frac{\mathcal{E}^2}{\mathcal{E}_c^2} - 1\right]}. \quad (11)$$

The eigenfunctions at $\mathcal{E} > \mathcal{E}_c$ are not $\mathcal{PT}$-invariant any more, $|\psi_i(x)| \neq |\psi_i(-x)|$, $i = 1, 2$. Thus

$$\psi \sim e^{-i\,{\rm Im}(\varepsilon_0 - \varepsilon_1)t/2}\psi_0(x) + e^{i\,{\rm Im}(\varepsilon_0 - \varepsilon_1)t/2}\psi_1(x). \quad (12)$$

This implies that the order parameter becomes two-component with the relative phase between the two components rotating with the Josephson frequency ${\rm Im}(\varepsilon_0 - \varepsilon_1)$. Since $|\psi_0(x)| = |\psi_1(-x)|$, the time averaged order parameter $\langle|\psi(x)|^2\rangle_{\rm time} \sim |\psi_0(x)|^2+|\psi_0(-x)|^2$ and develops a dip at $x=0$, increasing in amplitude with growing $\mathcal{E}$. This spot of the relatively suppressed superfluidity finally serves as a heating nucleus in the weak link.

Having calculated the eigenvalues ε_n and the eigenfunctions, ψ_n, $n=0,1,\ldots$, of $\mathcal{H}_{\rm eff}$, we proceed with the analysis of the superfluidity in the wire under the external drive. We will focus on the effect of superconducting fluctuations on the conductivity of the weak link. The most singular fluctuation contributions to the conductivity come from the Maki-Thompson and Aslamazov-Larkin mechanisms [10, 11, 20], and the corresponding currents read

$$j^{(\rm MT)} = 2DT^2\mathcal{E}\,{\rm Tr}\{(L^R_\omega)^\tau(C^R_\varepsilon)^\tau C^A_\varepsilon L^A_\omega\}\partial_\epsilon n_F(\tilde\epsilon), \quad (13)$$
$$j^{(\rm AL)} = DT^2\,{\rm Tr}\{(L^R_\omega)^\tau(C^R_\varepsilon)^\tau\nabla C^R_\varepsilon L^A_\omega + h.c.\}\delta n\,,$$

where T is the temperature, $\tilde\epsilon=\epsilon+\omega$, $\delta n = n_F(\tilde\epsilon+\varphi)-n_F(\tilde\epsilon-\varphi)$, $C^{R(A)}_\varepsilon = 4[D\nabla^2_{\rm x}\pm 2i\varepsilon+\gamma]^{-1}$ is the retarded (advanced) Cooperon propagator, γ is the inelastic relaxation rate, and trace 'Tr' means the integration over coordinates, ε and ω (the latter two with the weights $1/2\pi$). Writing Eq.(13) in terms of $L^{R(A)}$ and $C^{R(A)}$ eigenfunctions and eigenvalues yields the fluctuation correction to the resistance as (hereafter we restore the physical units and dimensions of the weak link)

$$\delta R \sim -\frac{E^2_{\rm Th}d}{e^2k_{\rm B}TL}\frac{1}{\sqrt{(\tau^{-1}-{\rm Re}\,\varepsilon_0(\mathcal{E})/\hbar)^2+(\Gamma(\mathcal{E})+\gamma)^2}}\,, \quad (14)$$

where d is the weak link thickness, $\Gamma = {\rm Im}\,\varepsilon_0$ and $\tau^{-1}=8(T_{\rm c}-T)/\pi$ while $T_{\rm c}$ is the critical temperature in the bulk. The resistance displays a pronounced voltage dependence in the range of parameters where either $\hbar/\tau\sim\varepsilon_0(\mathcal{E})$ or $\mathcal{E}\sim\mathcal{E}_c$. So, $\delta R(V)$ behaviour can be controlled via changing τ by cooling or heating the system.

The regime $|\mathcal{E}|<\mathcal{E}_c$ when the system is $\mathcal{PT}$-symmetric favors fluctuational Cooper pairs. When $|\mathcal{E}|>\mathcal{E}_c$ the spectrum of fluctuation propagator becomes complex that implies breaking of the Cooper pairs by the electric field with the rate $\tilde\Gamma\neq 0$ that increases with the increase of $\mathcal{E}$. It follows from Eqs. (13) and Eq. (14) that then the correction to the resistance from the fluctuating Cooper pairs quickly switches off. The same conclusion follows from the investigation of the superconducting wire where the $\mathcal{P}$-symmetry is broken due to geometrical imperfection, see Fig. 2.

What we have investigated above was the behavior of the superconducting fluctuations within the framework of the quadratic Keldysh action describing the fluctuations of the order parameter, see Ref. [11]. The natural question that arises is whether the revealed bifurcation picture retains in case of large fluctuations where one has to go beyond the Gaussian approximation. We expect the affirmative answer since the predicted instability follows from the symmetry considerations analogous to those in the general theory of the second phase transitions which, as one can prove [11], do not change upon appearance of the higher order terms. To cast the above reasoning into a mathematical form we note that on the heuristic level the large fluctuations would result in modifying (3) into the nonlinear, but having the same symmetry, Schrödinger equation. The corresponding generalization of Eq.(3) has the form:

$$\theta^{''}+2i(E+x\mathcal{E})\sin\theta=0\,, \quad (15)$$

The boundary conditions at $x=\pm L/2$ we take in a more general form: $\theta(0)=\theta_{s1}$ and $\theta(L)=\theta_{s2}$, where $\theta_{s1,2}$ are parameterized as follows: $\theta_{s1,2}(E)=\frac{1}{2}(\pi+i\ln\frac{\Delta+E\mp V/2}{\Delta-E\pm V/2})$, where Δ is constant. [For $\mathcal{E}=0$ Eq. (15) formally coincides with the Usadel equation [19] for the θ angle parameterizing the quasiclassical retarded Greens function in the superconducting weak link with the order parameter Δ in the reservoirs.]. Expanding $\sinh\theta$ and identifying $2iE$ with $\epsilon-\tau^{-1}$ and θ with ψ one recovers Eq.(3). We solved Eq. (15) numerically and found that the first fold-bifurcation appears at $\mathcal{E}_c\approx 5E_{\rm Th}/L$ rather than $\approx 49E_{\rm Th}/L$ found in Eq.(3). We thus have demonstrated that even in case of large fluctuations, where the extension beyond the linear approximation is required, the bifurcation of the fluctuation spectrum preserves, while the value of the critical field $\mathcal{E}_c$ where it

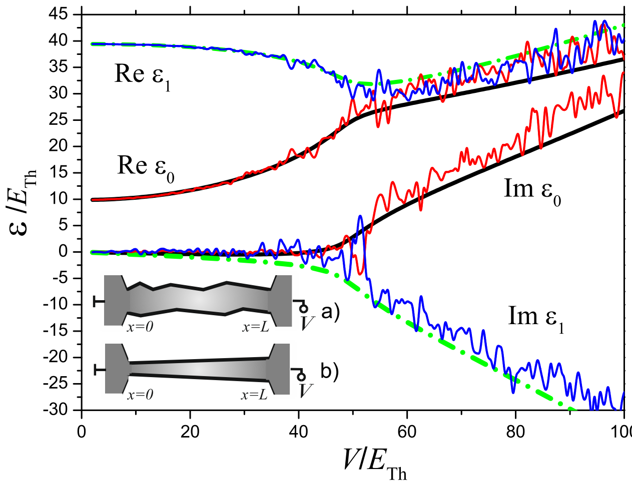

FIG. 2. Evolution of the two lowest energy levels, ε_0 and ε_1 upon applying bias $V=\mathcal{E}L$ when the wire is not $\mathcal{P}$-symmetric [so $\mathcal{PT}$-symmetry is also broken] due to the spatial variation of the wire thickness d with 5% relative amplitude: a) d fluctuates according to the gaussian law, ε_0 is the red curve and ε_1 is the blue one and b) the wire width monotonically changes, ε_0 is the black curve and ε_1 is the green one. In both cases the bifurcation is smoothed and the energies become complex below $\mathcal{E}_c$. So $\Gamma>0$ and as the result we get the suppression of the fluctuational Cooper pair contributions to the conductivity compared to the case of the $\mathcal{P}$-symmetric wire, see Fig. 1.

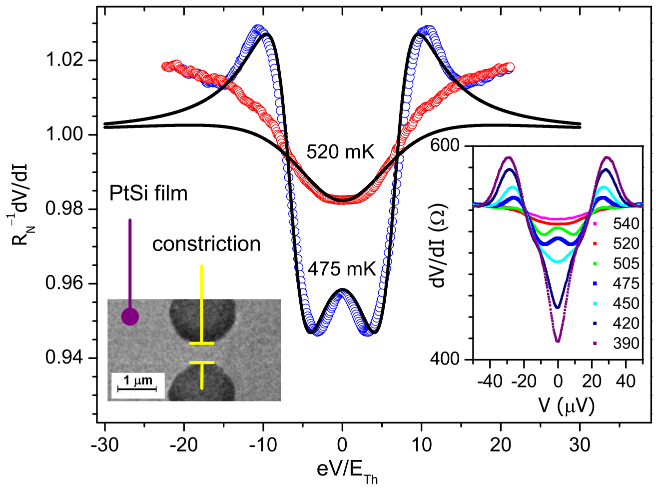

FIG. 3. The differential resistance of the superconducting weak link. The left inset presents scanning electron microscope image of the experimentally studied system, the constriction made by the electron beam lithography out of the PtSi film of the thickness $d = 6\,\mathrm{nm}$ and having $T_c = 560\,\mathrm{mK}$. The details of the system preparation are given in [15]. The right inset displays the full set of the differential resistance vs. V dependences at different temperatures given in mK in the legend. The central panel shows comparison of two exemplary experimental curves (symbols), normalized to $R_N = 536\,\Omega$, with the dV/dI calculated from Eqs. (8) and (14) (solid lines). We took $L = 0.4\,\mu\mathrm{m}$ and $D = 6\,\mathrm{cm}^2/\mathrm{s}$ [15], which gave $E_{\mathrm{Th}} = 2.5\,\mu\mathrm{eV}$, and chose $\hbar\Gamma/E_{\mathrm{Th}} = 0.176$.

occurs may change.

We have focused here on the superconducting wire of relatively small length that generated us the characteristic energy scale E_{Th}. Our solution for the $\mathcal{PT}$-symmetry breaking bifurcation and the fluctuations heavily relied on the discrete nature of H_{eff} spectrum. In the infinite geometry, $L \to \infty$, $E_T \to 0$, and the spectrum of H_{eff} is continuous. Then there is no $\mathcal{PT}$-symmetry breaking bifurcation. Taking the integral in Eq.(13) over the continues spectrum of H_{eff} we would get fluctuation corrections to the resistance with the form different from Eq. (14). Then the effective pair breaking electric field $\mathcal{E}_c \sim 1/\xi\tau$ [20] as follows from the uncertainty relation between τ and $\Gamma \sim \xi\mathcal{E}_c$, where $\xi = \sqrt{D\tau}$.

In order to test our theory we designed the experiments on mesoscale-patterned ultrathin PtSi wires having small constriction, as shown in Fig. 3. The details of the system preparation and parameters of the films are given in [15] and the Supplementary. The constriction plays the role of a weak link where fluctuation effects are expected to be very strong. The dimensions and the material characteristics were chosen to create the most favourable conditions for manifestation of the $\mathcal{PT}$-symmetry breaking effect in the system response to applied voltage bias. Namely, since the characteristiv energy scale, E_{Th}, is inversely proportional to L^2, the length of the constriction should not be too large in order to diminish the disguising effect of the thermal broadening. Another restriction on L is dictated by the condition that the characteristic drive $\mathcal{E}_c L$ remained less than superconducting gap. At the same time, in order to suppress Josephson coupling which could prevail over the fluctuation contribution, one has to take $L \gg \xi_N$, where $\xi_N = \sqrt{\hbar D/2\pi k_B T}$ is the decay length for the pair amplitude in diffusive normal conductor. Taking into account that, according to our calculations, the characteristic energy scale where fluctuations are important is about $10E_{Th}$, the above conditions imply that L should not be much larger than 10 ξ_N.

Figure 3 shows the differential resistance, dV/dI, of the superconducting weak link as function of the applied voltage bias, V. Upon cooling the system down from the critical temperature, the shape of the measured dV/dI-V dependencies near $V = 0$, transforms from the convex one, with the shallow minimum, into the W-shaped curve with a peak at $V = 0$. With further decreasing temperature, the central knob inverts, and $dV/dI(V)$ acquires a pronounced progressively deepening V-shape developing on top of shallow minimum. Importantly, the width of the deep remains equal to that of the maximum (see the curves corresponding to $T \leq 450\,\mathrm{mK}$ in the right inset to Fig. 3). The solid lines in the main panel present the dV/dI vs. V dependences calculated according to Eqs. (8) and (14), with Γ being the only fitting parameter. The fit traces perfectly traces the temperature evolution of $dV/dI(V)$, and, most strikingly, the W-shape at $T = 475\,\mathrm{K}$ in all its details, including maxima in dV/dI at $|eV| \approx 10E_{\mathrm{Th}}$ and the central knob $5E_{\mathrm{Th}}$ wide.

The similar behaviour of differential resistivity, the evolution from the shallow minimum to maximum and then to the dip again with the decreasing temperature, was observed in Ref. [17], where the quest for the theoretical explanation of this effect was formulated. Using the parameters given in Ref. [17] (see Fig. 2 there) we estimate $\xi_N = 0.14\,\mu\mathrm{m}$ at $T = 1\,\mathrm{K}$, the bridge length being $2.8\,\mu\mathrm{m}$. Furthermore, the corresponding $E_{\mathrm{Th}} \simeq 1.4\mu\mathrm{V}$, and one sees that the characteristic voltage of "saddled" shaped structure around zero bias in [17] is about $40E_{\mathrm{Th}}$ in accord with our notion that the dV/dI features develop on the voltage scale well exceeding Thouless energy.

As a final remark, we stress that the temperature evolution of dV/dI shape results from the confluence of the voltage-dependent fluctuation conductivity, stemming from the Maki-Thompson and Aslamazov-Larkin mechanisms, and the low-voltage quadratic dispersion [$\varepsilon_0(V) = \varepsilon_0(0) + aV^2$, see Fig.1] of the ground state energy. Importantly, the width of the central knob/peak is $\approx 5E_{Th}$, in a contrast to the more narrow dip in the tunnelling conductivity [16] (the knob in dV/dI corresponds to the groove in dI/dV), having the width of $|eV| \approx E_{\mathrm{Th}}$ reflecting the suppression of the electronic density of states by the proximity effect. The observed effect also differs from the zero-bias conductance peak in NS and SNS junctions at low temperatures [18] originating from the phase-coherent Andreev reflection.

In conclusion, we have demonstrated that the $\mathcal{PT}$-symmetry favors fluctuating Cooper pairs in the superconducting weak link. We have found that the applied electric field exceeding the critical value, $\mathcal{E}_c$, breaks down the $\mathcal{PT}$-symmetry and destroys the superconducting fluctuations in the weak link and derived the expression for $\mathcal{E}_c$. Combining effects of superconducting fluctuations and the low-voltage dispersion of the ground state energy of the effective non-Hermitian Hamiltonian of the fluctuating Cooper pairs we have quantitatively described the experimentally observed differential resistance of the weak link in the vicinity of the critical temperature.

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