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Interplay of disorder and interaction in Majorana quantum wires

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We study the interplay between disorder and interaction in one-dimensional topological superconductors which carry localized Majorana zero-energy states. Using Abelian bosonization and the perturbative renormalization group (RG) approach, we obtain the RG-flow and the associated scaling dimensions of the parameters and identify the critical points of the low-energy theory. We predict a quantum phase transition from a topological superconducting phase to a non-topological localized phase, and obtain the phase boundary between these two phases as a function of the electron-electron interaction and the disorder strength in the nanowire. Based on an instanton analysis which incorporates the effect of disorder, we also identify a large regime of stability of the Majorana-carrying topological phase in the parameter space of the model.

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Introduction. The search for topological phases of matter has become an active and exciting pursuit in condensed matter physics [1]. Among the many important examples of such phases are topological superconductors (SC) supporting zero-energy Majorana bound states (MBS) [2–11]. A particularly promising realization of topological superconductivity is one-dimensional (1D) semiconductor/SC heterostructures [10, 11]. In addition to being one of the simplest examples of fractionalization, zero-energy MBS quasiparticles have Ising-like non-Abelian braiding properties [12–15] and can be used for topological quantum computation [16].

The distinct feature of topological SCs is the ground-state degeneracy due to the fermion parity encoded in the exponentially localized zero-energy MBS [3, 17]. In a finite-length 1D wire, this degeneracy is approximate and there is an exponentially small energy splitting $e^{-L/\xi}$ due to a finite overlap of MBS. Here L and ξ are the length of the wire and superconducting coherence length, respectively. The presence of impurities in 1D p-wave SCs with broken time-reversal and spin $SU(2)$ symmetry (class D) [18] adversely affects the stability of the topological phase and drives a transition to a non-topological insulator phase [19–27]. The aforementioned QPT transition between topological and non-topological (localized) thermal insulator phases is accompanied by the change of the ground-state degeneracy splitting from exponential to algebraic in L [22]. In other words, increasing the disorder strength leads to a topological quantum phase transition (QPT) from the Majorana-carrying topological SC phase with quantum degeneracy to a trivial phase with no end-MBS in the wire. The effect of electron-electron interactions in the disordered SC wires have *not* been taken account before. The latter may have important implications for the topological phase, and there may be QPTs associated with the tuning of the interaction strength. Indeed, it is well known that the low-energy properties of 1D conductors are strongly affected by both electron-electron interactions and disorder [28]. Clarification of their combined effect is crucial for our complete understanding of the topological phase diagram of the system and ultimately for the experimental realization of Majorana quantum wires in the laboratory [29], where both disorder and interactions would be inevitably present.

In this Letter, we investigate an important question concerning the effect of both disorder and interaction on the stability of the topological phase and provide a theoretical framework that generalizes previous important results on disordered non-interacting systems [19–27] and on interacting clean Majorana wires [30–33]. We consider a generic 1D p-wave SC and include the effects of both quenched disorder and interaction using the bosonization and replica method [34]. We derive a set of coupled renormalization-group (RG) equations for the parameters of the model, obtaining in the process the quantum phase diagram of the system. Using these results in combination with an instanton analysis allows us to analyze the topological stability of MBS under the influence of both interaction and disorder. In general, disorder and repulsive interactions reinforce their detrimental effects on the topological SC phase and tend to eliminate the exponentially-split ground state MBS degeneracy associated with different fermion parity [35]. However, for a sufficiently strong initial induced pairing Δ, we predict a stable topological phase at low temperatures, even in the presence of disorder and interaction. Our results are relevant to recent experiments on semiconductor nanowires with strong spin-orbit and Zeeman interactions, proximity-coupled to a s-wave bulk SC [29], whose low-energy Hamiltonian was shown to reduce to an effective 1D spinless p-wave SC [10, 11], and shed light on the question of the stability of MBS in realistic situations.

Theoretical model. We start with a model for p-wave spinless fermions in a clean, single channel conductor of length L with open boundary conditions. In that case, the Hamiltonian for the 1D SC wire in the continuum is

$$H_0^{(1)}=\int_0^L dx\ \psi^\dagger\left(-\frac{\partial_x^2}{2m}-\mu\right)\psi-\Delta\psi\left(\frac{i\partial_x}{k_F}\right)\psi+\text{H.c.},$$

where $\hbar=1$, $\psi\left(x\right)$ is the fermionic annihilation field operator, m is the effective mass, μ is the chemical potential and Δ is the p-wave pairing interaction. In absence of interactions and disorder, the Hamiltonian $H_0^{(1)}$ can be straightforwardly diagonalized by the means of a standard Bogoliubov transformation. However, introducing interactions considerably complicates the theoretical description and a different approach is needed. We therefore start from the limit $\Delta=0$, and linearize the spectrum $\xi_k=k^2/2m-\mu$ around the Fermi points $\pm k_F$. This allows to express the fermion field $\psi\left(x\right)$ as a sum of right- and left-movers $\psi\left(x\right)=e^{ik_Fx}\psi_R\left(x\right)+e^{-ik_Fx}\psi_L\left(x\right)$, and to introduce the standard Abelian bosonization procedure of Fermi fields $\psi_r=\frac{1}{\sqrt{2\pi a}}U_re^{-i(r\phi-\theta)}$, where $r=\{R\left(+\right),L\left(-\right)\}$, and $a\sim k_F^{-1}$ is the short-distance cutoff of the continuum theory, The bosonic fields $\phi\left(x\right),\theta\left(x\right)$ are conjugate canonical variables obeying the commutation relation $\left[\phi\left(x\right),\theta\left(y\right)\right]=i\pi\text{sign}\left(y-x\right)/2$, and U_r are the standard Klein factors [28]. Physically, $\phi\left(x\right)$ represents slowly-varying fluctuations in the electronic density $\rho\left(x\right)\simeq\rho_0-\partial_x\phi\left(x\right)/\pi$, and $\theta\left(x\right)$ is related to the SC order parameter through the relation $-i\psi\left(x\right)\partial_x\psi\left(x\right)\propto\psi_R\left(x\right)\psi_L\left(x\right)\propto e^{i2\theta(x)}$, where we have neglected less relevant higher-order terms in $\partial_x\theta\left(x\right)$. A short-range interaction $H_0^{(2)}=g\int dx\ \ \psi_R^\dagger\left(x\right)\psi_R\left(x\right)\psi_L^\dagger\left(x\right)\psi_L\left(x\right)$ acquires a simple form in terms of the bosonic fields, and the Hamiltonian $H_0=H_0^{(1)}+H_0^{(2)}$ is therefore given by

$$H_0{=}\int dx\left[\frac{vK}{2\pi}\left(\partial_x\theta\right)^2{+}\frac{v}{2\pi K}\left(\partial_x\phi\right)^2{+}\frac{2\Delta}{\pi a}\sin\left(2\theta\right)\right].\quad(1)$$

For $\Delta=0$, Eq. (1) reduces to the Luttinger liquid (LL) model [28], which describes gapless plasmon excitations in the wire propagating with velocity $v\simeq v_F$, and is parametrized by the the dimensionless Luttinger parameter $K=\sqrt{\frac{1-g/\pi v_F}{1+g/\pi v_F}}$ representing repulsive (attractive) interactions for $K<1(K>1)$. The hypothesis of a short-ranged interaction in $H_0^{(2)}$ requires the presence of strong screening in the nanowire. In a realistic situation, such as the case of Ref. [29], we assume that this screening is provided by electrons in the semiconductor and surrounding SC. We also assume henceforth that the filling is incommensurate with the lattice and the length of the wire $L\gg L_c\equiv\left[4k_F-2\pi/a\right]^{-1}$, in which case the umklapp scattering term $\cos\left(2\phi-4k_Fx\right)$ becomes strongly oscillating at lengthscales larger than L_c and averages out to zero [30].

As follows from the analysis of Eq. (1) made below, the SC pairing Δ around $K\approx1$ is relevant [see Eq. (7)], and flows to strong coupling. Thus, at large enough Δ, the field $\theta\left(x\right)$ is pinned to the minima of $\sin2\theta$ and the SC state breaks $\mathbb{U}(1)$ symmetry down to $\mathbb{Z}_2$. In the infinite system $L\to\infty$, the latter corresponds to two degenerate minima at $\theta\left(x\right)=-\pi/4,\ 3\pi/4$ which are related to each other by the global $\mathbb{Z}_2$ transformation $\theta\to\theta+\pi$ [35]. Such a transformation is implemented by the fermion parity operator $P=\left(-1\right)^{N_F}=\exp\left[-i\int_0^L\partial_x\phi\left(x\right)dx\right]$ with N_F the total fermion number operator. The degenerate ground states characterized by different fermion parity read $\left|\text{even/odd}\right\rangle=\left(\left|-\pi/4\right\rangle\pm\left|3\pi/4\right\rangle\right)/\sqrt{2}$. In the case of a large but finite L, the two degenerate ground-states are split in energy due to quantum tunneling between the two minima $\theta\left(x\right)=-\pi/4,\ 3\pi/4$. The splitting energy can be calculated using instanton analysis $\delta E=A_fe^{-S_{\text{inst}}}$, where S_{inst} is the action of the Euclidean instanton $\theta_0\left(x,\tau\right)$ (where τ is the imaginary-time), obeying the boundary conditions $\theta_0\left(x,-\infty\right)=-\pi/4$ and $\theta_0\left(x,\infty\right)=3\pi/4$, and A_f is a prefactor due to quantum fluctuations around those minima [36]. The instanton configuration minimizing S_{inst} is spatially uniform rendering effectively a $0+1$ dimensional problem, whose corresponding action is [35]

$$S_{\text{inst}}=\frac{4\sqrt{K}}{\pi}\frac{L}{\xi},\quad(2)$$

with $\xi=v/\Delta$ the SC coherence length. The instanton-analysis therefore predicts an energy splitting scaling as $\delta E\propto\exp(-\frac{4\sqrt{K}}{\pi}\frac{L}{\xi})$, in agreement with the non-interacting Majorana chain [3].

We now introduce quenched disorder into model (1). We consider the case of a short-range Gaussian disorder potential $V\left(x\right)$ that couples to the fermionic density, $H_{\text{dis}}=-\int dx\ \ V\left(x\right)\rho\left(x\right)$ and characterized by $\left\langle V\left(x\right)V\left(y\right)\right\rangle=D_b\delta\left(x-y\right)$. In bosonized language, the disordered Hamiltonian is [28]

$$H_{\text{dis}}\ =\ \int dx\left[-\eta\left(x\right)\frac{\partial_x\phi\left(x\right)}{2\pi}+\xi\left(x\right)\frac{e^{-i2\phi}}{2\pi a}+\text{H.c.}\right](3)$$

where we have defined the disordered potentials $\eta\left(x\right)\equiv\frac{1}{N}\sum_{q\sim0}e^{iqx}V\left(q\right)$ and $\xi\left(x\right)\equiv\frac{1}{N}\sum_{q\sim0}e^{iqx}V\left(q-2k_F\right)$. The forward scattering term $-\eta\left(x\right)\partial_x\phi\left(x\right)/2\pi$ can be eliminated by the means of a gauge transformation $\phi\left(x\right)\to\phi\left(x\right)-\frac{K}{v}\int^x dy\ \ \eta\left(y\right)$, reflecting the fact that forward scattering does not affect the thermodynamic properties of the system [37]. We next implement the replica method, that consists in introducing the set of independent "replicas" of the system $\{\phi\left(x\right),\theta\left(x\right)\}\to\{\phi_i\left(x\right),\theta_i\left(x\right)\}$, with $i=1,2,\ldots,n$ and commutation relations $\left[\phi_l\left(x\right),\theta_m\left(y\right)\right]=i\frac{\pi}{2}\text{sign}\left(y-x\right)\delta_{lm}$, allowing a simpler integration over different disorder configurations [28, 34]. At the end of the calculation, we take a limit

$n \to 0$. The integration of the Gaussian field $V(x)$ results in the replicated action of the 1D system

$$S = \sum_{j=1}^{n} \int d\tau \left[\int dx \, \frac{\partial_x \phi_j}{i\pi} \dot{\theta}_j + H_{0,j}(\tau) \right] - \sum_{i,j=1}^{n} \frac{D_b}{(2\pi a)^2} \times \int dx d\tau d\tau' \cos 2 \left[\phi_i(x,\tau) - \phi_j(x,\tau') \right], \quad (4)$$

where the Hamiltonian $H_{0,j}$ is defined in Eq. (1). In the absence of SC pairing, this model was studied by Giamarchi and Schulz in the context of the localization transition, predicted to occur at the critical value $K_c = 3/2$ for spinless fermions, in the limit of weak disorder [37]. For $K < K_c$, disorder flows to strong coupling and the groundstate corresponds to a pinned charge-density-wave (PCDW), characterized by a localization length $\xi_{\text{loc}} \propto D_b^{1/(3-2K)}$. Above K_c, the LL phase remains stable, describing a "delocalized" electronic fluid. In the presence of SC pairing, the LL fixed-point is never stable, as we show below.

RG analysis. The critical properties of model (4) can be studied in the framework of perturbative RG around the LL fixed-point. Following standard derivations [28, 38], we expand the partition function corresponding to action S at first-order in the small parameter D_b, and up to second order in Δ. We implement the RG procedure in real-space, which leaves invariant the LL fixed-point Hamiltonian, and obtain the following system of RG-flow equations

$$\frac{dK(\ell)}{d\ell} = y_\Delta^2(\ell) - K^2(\ell)\, y_b(\ell), \quad (5)$$

$$\frac{dv(\ell)}{d\ell} = -v(\ell)\, K(\ell)\, y_b(\ell), \quad (6)$$

$$\frac{dy_\Delta(\ell)}{d\ell} = \left[2 - K^{-1}(\ell) \right] y_\Delta(\ell), \quad (7)$$

$$\frac{dy_b(\ell)}{d\ell} = \left[3 - 2K(\ell) \right] y_b(\ell), \quad (8)$$

where we have introduced the dimensionless variables $y_\Delta = 2\Delta a/v$ and $y_b = D_b a/4\pi v^2$. Physically Eq. (5) describes the renormalization of interactions in the wire [parametrized by $K(\ell)$] induced by superconductivity and disorder. While $y_\Delta(\ell)$ couples to field $\theta(x)$, favoring a SC ground state with broken $\mathbb{Z}_2$-symmetry, the parameter $y_b(\ell)$ couples to the dual field $\phi(x)$ and tries to pin the density to the disorder potential, thus opposing a SC ground state. These competing effects are reflected in the different signs of the prefactors in Eq. (5): $y_\Delta(\ell)$ renormalizes $K(\ell)$ to larger values, inducing attractive interactions in the wire, while $y_b(\ell)$ drives $K(\ell) \to 0$ enhancing the effect of repulsive interactions. In the limit $\{y_\Delta(\ell), y_b(\ell)\} \to 0$ the properties of the system are determined by the value of $K(\ell)$, i.e., the coupling $y_\Delta(\ell)$ becomes relevant (in the RG sense) for $K(\ell) > 1/2$, whereas $y_b(\ell)$ is relevant for $K(\ell) < 3/2$ [37, 38]. From

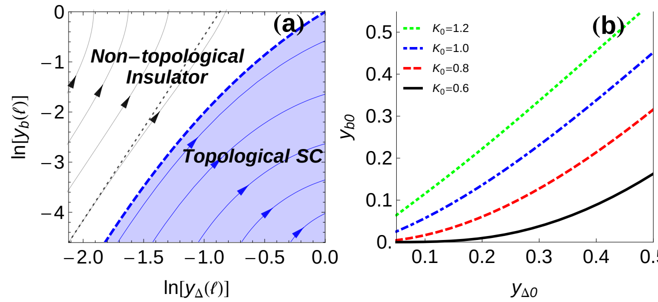

FIG. 1: (a) Parametric dependence of $y_b(\ell)$ vs $y_\Delta(\ell)$, as obtained from the numerical solution of the RG-flow Eqs. (5)-(8), for fixed initial parameter $K_0 = 0.65$ (log-log scale). The thick dashed curve is the critical line, separating the topological SC phase (shaded area) from the non-topological disordered phase, and the thin dotted line is our analytical estimate $y_b \sim y_\Delta^\nu$, valid in the limit $\{y_b(\ell), y_\Delta(\ell)\} \to 0$. (b) Phase diagram in $y_{\Delta 0}, y_{b0}$ space obtained for $y_{s0} = 0$ and different values of K_0. The curves correspond to the critical lines y_{b0} vs $y_{\Delta 0}$, satisfying the condition $y_\Delta(\ell^*) = y_b(\ell^*) = 1$. The area below each curve represents the regime for which topological SC is expected to dominate over disorder.

this RG-analysis we extract two important conclusions: 1) the non-interacting limit $K = 1$ is an unstable point in parameter-space, and 2) repulsive interaction and disorder reinforce each other's detrimental effects on the topological SC. Note that within the experimentally interesting regime $1/2 < K(\ell) < 3/2$ both $y_\Delta(\ell)$ and $y_b(\ell)$ are competing perturbations flowing simultaneously to strong coupling. Moreover, in the non-interacting case $K = 1$, $y_\Delta(\ell)$ and $y_b(\ell)$ have the same scaling dimension. In order to maintain the internal consistency of our perturbative approach, the RG flow has to be stopped at a value ℓ^* for which one of the couplings reaches the strong-coupling regime, i.e., $\max\left[y_\Delta(\ell^*), y_b(\ell^*) \right] = 1$. Although strictly speaking our approach is not applicable in the strong-coupling regime, the fact that $\theta(x)$ and $\phi(x)$ are dual fields that cannot order simultaneously allows us to reasonably conjecture that there are no intermediate fixed-points in the RG flow, and therefore to classify the nature of the ground state according to the coupling that first reaches the above condition [38]. When the two competing couplings reach the strong coupling regime simultaneously [i.e., $y_b(\ell^*) = y_\Delta(\ell^*) = 1$], the system does not order and this condition defines a critical line of QPTs that separates the topological SC phase with broken $\mathbb{Z}_2$ symmetry from the PCDW insulating phase (cf. thick dashed line in Fig. 1(a)).

From the lowest-order RG equations one obtains the approximate solutions $y_b(\ell) = y_{b0} e^{(3-2K)\ell}$, $y_\Delta(\ell) = y_{\Delta 0} e^{\left(2-K^{-1}\right)\ell}$, which together produce the relative scaling $y_b \sim y_\Delta^\nu$ with $\nu = (3-2K)/\left(2-K^{-1}\right)$. Physically, this means that interactions (encoded in ν) determine the scaling of disorder strength relative to the SC order parameter: for $K > 1$ (attractive interactions) disorder grows slower than SC, while the inverse occurs for $K < 1$ (repulsive interactions). In Fig. 1(a) we show the parametric dependence of $y_b(\ell)$ as a function of $y_\Delta(\ell)$, for the initial condition $K_0 = 0.65$. The continuous lines correspond to the numerical solution of Eqs. (5)-(8), and the dotted line is our analytical result $y_b \sim y_\Delta^\nu$, valid in the limit $\{y_b(\ell), y_\Delta(\ell)\} \to 0$. At the phase boundary (thick dashed line), this result implies the approximate relation $y_{b0} \sim y_{\Delta 0}^\nu$ for the initial values, which together with the relation: $D_b = 2\pi v_F/\tau_e$ (where τ_e is elastic scattering time), produces $1/\tau_e E_F \sim (\Delta/E_F)^\nu$. Interestingly, for $K = 1$ we find that the critical condition for the topological-non-topological transition is $1/\tau_e \sim \Delta$, which exactly coincides with the results obtained in the non-interacting case [19–23, 27]. Note, however, that in the interacting case the equation for the phase boundary involves an additional energy scale E_F and has a non-trivial dependence on the electron-electron interactions.

The above procedure leads to a qualitative "phase-diagram" in terms of the initial parameters of the model. In Fig. 1(b) we plot the critical curves in $y_{\Delta 0}$-y_{b0} space, for different initial values of interaction K_0. The area below each curve represents the regime for a stable topological SC supporting MBS. Starting from the inital value $K_0 = 0.6$ (i.e., strongly interacting wire), note that the topological region expands as the interaction becomes increasingly attractive.

Topological stability of MBS. To study the effect of interaction and disorder on the stability of MBS, we evaluate the energy-splitting δE in the regime where Δ flows first to strong coupling. As mentioned before, in order for the topological SC phase to be stable, δE should scale exponentially with L. Our approach therefore consists in integrating the RG-flow up to the scale $\ell^* = \ln(y_{\Delta 0}) / \left(K_0^{-1} - 2\right)$ [i.e., such that $y_\Delta(\ell^*) = 1$], and calculating there the instanton action S_{inst} in presence of the backscattering term in (3). Since in that regime $y_b(\ell^*) \ll 1$, the effect of backscattering can be accounted for perturbatively, and we can make use of the instanton solution $\theta_0(\tau)$ found in the clean case. The contributions of backscattering to S_{inst} can be divided into: a) an *explicit* contribution, arising from the presence of the term $\sim D_b(\ell^*) \langle \cos[2\phi(x,\tau_1) - 2\phi(x,\tau_2)] \rangle$ in the action, and b) an *implicit* contribution, originated in the indirect effect of $y_b(\ell)$ on the other couplings through the RG-flow equations. Since in the regime of interest Δ "locks" the phase θ to the minima of the $\sin 2\theta$ potential, ϕ becomes a strongly fluctuating field and therefore the contribution a) is strongly suppressed, i.e. it scales as $\langle \cos[2\phi(x,\tau) - 2\phi(x,0)] \rangle \sim \exp(-|\tau E_F| L/\xi)$ [39]. This constitutes a subleading correction to S_{inst} which is neglected in the following analysis. We therefore focus on the more important contribution b). The expression of the instanton action $S_{\text{inst}}(\ell^*)$ is formally identical to Eq. (2) with the change $K \to K(\ell^*)$. Integrating RG-flow Eq. (5) up to the scale ℓ^* yields (at lowest order in the parameters y_Δ and y_b) $K(\ell^*) = K_{\text{cl}} - \delta K_{\text{dis}}$, where $K_{\text{cl}} = K_0 + K_0 (4K_0 - 2)^{-1}$ is the renormalized Luttinger parameter in the clean limit $l_e = v\tau_e \to \infty$, and where $\delta K_{\text{dis}} = K_0^2 (3 - 2K_0)^{-1} (k_F l_e)^{-1} (k_F \xi/2)^\nu$ is the effect of disorder [39]. Replacing $K(\ell^*)$ into (2) yields

$$S_{\text{inst}} = \frac{4\sqrt{K_{\text{cl}}}}{\pi} \left[\frac{L}{\xi} - \frac{L}{2l_e} \frac{K_0^2}{K_{\text{cl}}(3 - 2K_0)} \left(\frac{k_F \xi}{2} \right)^{\nu - 1} \right] \quad (9)$$

This result encodes the interplay of interaction and (weak) disorder on the topological degeneracy of MBS through the relation $\delta E \propto e^{-S_{\text{inst}}(\ell^*)}$, and constitutes an important generalization of the non-interacting results in Ref. [23] to the interacting case. Physically, it expresses the fact that MBS are stable as long as disorder is weak, such that $\xi (k_F \xi)^{\nu - 1} \ll l_e$. Note that the internal consistency of the bosonization approach requires the energy cutoff $\Lambda_0 = v_F k_F$ to be much larger than Δ. This implies that $k_F \xi \gg 1$ and we therefore conclude that effect of disorder on MBS energy splitting is enhanced (lessened) for repulsive (attractive) interactions, which is one of the main results of this paper. Interestingly, one can notice that the non-interacting results of Ref. [23] are recovered for $K_0 = 1$ and $\nu = 1$. While Eq. (9) is only valid in the regime $1/2 < K_0 < 3/2$ due to the lowest-order approximation in the integration of the RG-flow, a numerical integration of Eqs. (5)-(8) allows to generalize it to any K_0.

Conclusions. We have carried out a RG analysis of the topological superconductivity in a 1D p-wave SC wire in the presence of both electron-electron interaction and disorder, treating them on equal footing. Our results provide useful insights into their interplay and are relevant to understand more realistic situations (e.g., Ref. [29]). The solution of the RG-flow Eqs. (5)-(8) combined with the calculation of the instanton action in Eq. (9) demonstrate that a topological SC state that supports stable non-Abelian MBS could be in principle realized on a large regime of parameter space.

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