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Nonlinear excitations of zonal structures by Toroidal Alfvén Eigenmodes

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Zonal flows and, more generally, zonal structures are known to play important self-regulatory roles in the dynamics of microscopic drift-wave type turbulences. Since Toroidal Alfvén Eigenmode (TAE) plays crucial roles in the Alfvén wave instabilities in burning fusion plasmas, it is, thus, important to understand and assess the possible roles of zonal flow/structures on the nonlinear dynamics of TAE. It is shown that zonal flow/structure spontaneous excitation is more easily induced by finite amplitude TAEs including the proper trapped ion responses, causing the zonal structure to be dominated by the zonal current instead of the usual zonal flow. This work shows that proper accounting for plasma equilibrium geometry as well as including kinetic thermal ion treatment in the nonlinear simulations of Alfvénic modes are important ingredients for realistic comparisons with experimental measurements, where the existence of zonal fields has been clearly observed.

Zonal flows and, more generally, zonal structures are known to play important self-regulatory roles in the dynamics of microscopic drift-wave type turbulences. In fact, zonal structures have a unique role in the cross-scale coupling of disparate spatiotemporal scales in burning plasmas as complex systems [1], for they are predominantly only radially varying on *mesoscales*, intermediate between those of turbulence and macroscopic plasma equilibrium. In the absence of velocity-space free energy, zonal structures are linearly stable, due to their intrinsic symmetry [2], but may be forced driven via nonlinear mode coupling. In such case, their damping effect on the driving modes is proportional to the zonal structures intensity and dissipation rate; i.e., it vanishes in the dissipationless limit, which is the relevant case for burning plasmas of fusion interest. Thus, the self-regulation is essentially achieved via *spontaneous excitations of modulational instabilities* above a critical threshold in the driving fluctuation intensity, by which zonal structures act as non-local spectral transfer of energy. Meanwhile, the driving instabilities are themselves scattered by the zonal structures into shorter radial wavelength sidebands, and, consequently, into the short-radial wavelength stable domain.

Zonal electric fields and corresponding zonal flows have been widely measured in experiments and their observed properties are consistent with the existing general theoretical framework [3]. Meanwhile, zonal magnetic fields (zonal currents), predicted by theoretical analyses [3–6], have been only recently observed in experiments in the compact helical system (CHS) [7]. Such observation is important for the implications that zonal magnetic fields may have in understanding fundamental processes, such as magnetic field dynamo, as well as for the understanding of nonlinear dynamics of Toroidal Alfvén Eigenmodes (TAEs), which play crucial roles among Alfvén wave instabilities in magnetized plasmas of fusion interest.

The effects of zonal flows [8] and nonlinear mode couplings, due to zero-frequency axisymmetric nonlinear distortions of equilibrium magnetic field [9] or density [10], were proposed as possible saturation mechanism in the nonlinear dynamics of TAEs. More recently, numerical simulation results have shown that low frequency *forced driven* zonal flows may have a role in the nonlinear TAE saturation [11], but have not observed *spontaneous* excitation of zonal structures.

In this work, we address the *spontaneous* excitation of zonal structures by TAEs, i.e. of both zonal flows and currents, and show that spontaneous excitation is more easily induced by finite amplitude TAEs including the proper trapped ion responses, causing the zonal structure to be dominated by the zonal current instead of the usual zonal flow. Our analysis is carried out assuming a simple tokamak equilibrium with shifted circular magnetic flux surfaces; however, results summarized by Eq. (22) are very general and show that the branching ratio (relative strength) of zonal flows and currents and the onset condition for modulational instability crucially depend on plasma equilibrium and kinetic response. Thus, for realistic comparisons with experimental observations in toroidal plasmas, theoretical analyses and numerical simulations must rely on kinetic descriptions in realistic equilibrium geometries.

Here, we adopt the theoretical approach of [4] as well as [12]. Thus, the field variables $\delta\phi$ and δA_{\parallel} are used to investigate the nonlinear couplings among the pump TAE, (ω_0, \mathbf{k}_0) , the upper and lower TAE sidebands, $(\omega_{\pm}, \mathbf{k}_{\pm})$, and the zonal mode (ω_z, \mathbf{k}_z) . Indicating TAE and zonal mode with the subscripts A and z , respectively, one then has, for example, $\delta\phi = \delta\phi_A + \delta\phi_z$ and $\delta\phi_A = \delta\phi_0 + \delta\phi_+ + \delta\phi_-$. Assuming, for simplicity, that we deal with high toroidal mode numbers TAE, as those expected in ITER [13, 14], we adopt the well known ballooning-mode decomposition in (r, θ, ϕ) field-aligned toroidal flux coor-

ordinates [15]

$$\begin{aligned}\delta\phi_0 &= A_0 e^{i(n\phi - m_0\theta - \omega_0 t)} \sum_j e^{-ij\theta} \Phi_0(x-j) + \text{c.c.}, \\ \delta\phi_{\pm} &= A_{\pm} e^{\pm i(n\phi - m_0\theta - \omega_0 t)} e^{i(\int^r k_z dr - \omega_{\pm} t)} \\ &\quad \times \sum_j e^{\mp ij\theta} \begin{bmatrix} \Phi_0(x-j) \\ \Phi_0^*(x-j) \end{bmatrix} + \text{c.c.},\end{aligned}\quad (1)$$

and

$$\delta\phi_z = A_z \exp \left[i \left(\int^r k_z dr - \omega_z t \right) \right] + \text{c.c.} \quad (2)$$

Here, $(m = m_0 + j, n)$ are poloidal and toroidal mode numbers, m_0 is the reference poloidal mode number, $nq(r_0) = m_0$, $q(r)$ is the safety factor, $x = nq - m_0 = nq'(r - r_0)$ and A_0 , A_{\pm} and A_z are the envelope amplitudes of TAE pump, sideband and zonal mode, respectively, having used $\int |\Phi_0|^2 dx = 1$ as normalization condition. The same decomposition of Eqs. (1) and (2) is assumed for the parallel vector potential.

Considering $|k_{\perp} \rho_i|^2 \sim |k_z \rho_i|^2 < \epsilon = r_0/R_0 < 1$, with ρ_i the thermal ion Larmor radius and R_0 the torus major radius, we obtain the vorticity equation of the zonal mode from [4] (where (r, ϕ, θ) flux coordinates were adopted)

$$\begin{aligned}i\omega_z \chi_{iz} \delta\phi_z &= \frac{c}{B_0} k_z k_{\theta} k_z^2 \rho_i^2 \\ &\quad \times \left\langle \left(1 - \frac{k_{0\parallel}^2 v_A^2}{\omega_0^2} \right) \right\rangle_x (A_0^* A_+ - A_0 A_-).\end{aligned}\quad (3)$$

Here, $\chi_{iz} \simeq 1.6q^2 \epsilon^{-1/2} k_z^2 \rho_i^2$ [2], $k_{\theta} = nq/r$, $k_{\parallel} = x/qR_0$ and $\langle \dots \rangle_x \equiv \int dx |\Phi_0|^2(\dots)$. Noting that $|\Phi_0|^2(x)$ is localized at and even with respect to $|x| = 1/2$, with a characteristic width $\Delta_x \sim \mathcal{O}(\epsilon)$, we then have

$$\begin{aligned}\langle (\omega_0^2 - k_{0\parallel}^2(x) v_A^2) \rangle_x &\simeq \left\langle \left(\omega_0^2 - \frac{\omega_A^2}{4} \right) \right\rangle \\ &+ \left[\frac{\omega_A^2}{4} - k_{0\parallel}^2(x) v_A^2 \right]_x \simeq \left(\omega_0^2 - \frac{\omega_A^2}{4} \right) + \mathcal{O}(\epsilon^2),\end{aligned}\quad (4)$$

where $\omega_A = v_A/(qR_0)$; thus, Eq. (3) becomes

$$i\omega_z \chi_{iz} \delta\phi_z = \frac{c}{B_0} k_z k_{\theta} k_z^2 \rho_i^2 \left(1 - \frac{\omega_A^2}{4\omega_0^2} \right) (A_0^* A_+ - A_0 A_-).\quad (5)$$

Meanwhile, considering the strong electron current screening effect on scale lengths that are longer than the collisionless skin depth $\delta_e = c/\omega_{pe}$, with ω_{pe} the electron plasma frequency, and noting that $\delta_e \ll \rho_i$ for $m_e/m_i \ll \beta \ll 1$, β denoting the ratio between kinetic and magnetic energy densities, the evolution equation for $\delta A_{\parallel z}$ or $\delta\psi_z \equiv \omega_0 \delta A_{\parallel z} / ck_{0\parallel}$ is $\delta j_{z\parallel e} \simeq 0$, i.e.

$$\delta\psi_z = i \frac{c}{B_0} \frac{k_z k_{\theta}}{\omega_0} (A_0^* A_+ + A_0 A_-).\quad (6)$$

Including the nonlinear correction to the ideal Ohm's law, for the $(\omega_{\pm}, \mathbf{k}_{\pm})$ TAE sidebands, the vorticity equations can be rendered into the following forms [4];

$$\begin{aligned}A_{\pm} \mathcal{L}_{\pm} \begin{bmatrix} \Phi_0(x) \\ \Phi_0^*(x) \end{bmatrix} &= 2i \frac{c}{B_0} k_{\theta} k_z \omega_0 \begin{bmatrix} A_0 \\ A_0^* \end{bmatrix} \\ &\quad \times (\delta\phi_z - \delta\psi_z) \nabla_0^2 \begin{bmatrix} \Phi_0(x) \\ \Phi_0^*(x) \end{bmatrix},\end{aligned}\quad (7)$$

where

$$\mathcal{L}_{\pm} = \omega_A^2 x \nabla_{\pm}^2 x - \omega_{\pm}^2 (1 + \epsilon_0 T) \nabla_{\pm}^2, \quad (8)$$

$$T \nabla_{\pm}^2 \begin{bmatrix} \Phi_0(x) \\ \Phi_0^*(x) \end{bmatrix} = \nabla_{\pm}^2 \begin{bmatrix} \Phi_0(x+1) + \Phi_0(x-1) \\ \Phi_0^*(x+1) + \Phi_0^*(x-1) \end{bmatrix}, \quad (9)$$

$$\nabla_+ \Phi_0 = [-\hat{\theta} i k_{\theta} + \hat{r} (i k_z + nq' \partial_x \ln \Phi_0)] \Phi_0, \quad (10)$$

$$\nabla_- \Phi_0^* = [\hat{\theta} i k_{\theta} + \hat{r} (i k_z + nq' \partial_x \ln \Phi_0^*)] \Phi_0^*, \quad (11)$$

$\epsilon_0 = 2\epsilon + \Delta'$ [16], Δ' is the Shafranov shift and

$$\nabla_0 \Phi_0 = (-\hat{\theta} i k_{\theta} + \hat{r} nq' \partial_x \ln \Phi_0) \Phi_0. \quad (12)$$

Performing $\int_{-\infty}^{\infty} dx (\Phi_0, \Phi_0^*)$ on Eq. (7), we obtain

$$A_{\pm} \epsilon_{A\pm} b_{\pm} = -2i \frac{c}{B_0} k_{\theta} k_z \omega_0 b_0 \begin{pmatrix} A_0 \\ A_0^* \end{pmatrix} (\delta\phi - \delta\psi)_z, \quad (13)$$

where $b_0 = \rho_i^2 \langle |\nabla_0 \Phi_0|^2 \rangle_x$, $b_+ = \rho_i^2 \langle |\nabla_+ \Phi_0|^2 \rangle_x = b_0 + b_z$, $b_z = k_z^2 \rho_i^2$ and $b_- = b_+$. Meanwhile [19],

$$\epsilon_{A\pm} = \left(\frac{\omega_A^4}{\epsilon_0 \omega^2} \Lambda_T(\omega) D(\omega, k_z) \right)_{\omega=\omega_{\pm}}, \quad (14)$$

$$D(\omega, k_z) = \left(\Lambda_T(\omega) - \delta \hat{W}(\omega, k_z) \right), \quad (15)$$

with $\Lambda_T = \sqrt{-\Gamma_- \Gamma_+}$, $\Gamma_{\pm} = (\omega^2/\omega_A^2 - 1/4) \pm \epsilon_0 \omega^2/\omega_A^2$ [16] and $\delta \hat{W}(k_z, \omega)$ playing the role of a normalized potential energy, which, besides its obvious dependence on k_z and other parameters characterizing the local plasma equilibrium, can also depend on the mode frequency, since the TAE mode structure in the ideal MHD region depends on the TAE mode frequency inside the toroidicity induced gap in the shear Alfvén continuous spectrum. Solutions of $D(\omega, k_z) = 0$ are $\omega = \pm \omega_T(k_z)$, with the pump TAE frequency given by $\omega_0 = \omega_T(k_z = 0)$.

Combing Eq. (13) with Eqs. (5) and (6) and letting $-i\omega_z = \gamma_z$ yield

$$\begin{aligned}\delta\phi_z &= 2i \left(\frac{c}{B_0} k_{\theta} k_z \right)^2 \left(1 - \frac{\omega_A^2}{4\omega_0^2} \right) \frac{b_z}{\chi_{iz}} \frac{b_0}{b_+} \\ &\quad \times \frac{\omega_0}{\gamma_z} |A_0|^2 \left(\frac{1}{\epsilon_{A+}} - \frac{1}{\epsilon_{A-}} \right) (\delta\phi - \delta\psi)_z,\end{aligned}\quad (16)$$

and

$$\begin{aligned}\delta\psi_z &= 2 \left(\frac{c}{B_0} k_{\theta} k_z \right)^2 \frac{b_0}{b_+} |A_0|^2 \\ &\quad \times \left(\frac{1}{\epsilon_{A+}} + \frac{1}{\epsilon_{A-}} \right) (\delta\phi - \delta\psi)_z.\end{aligned}\quad (17)$$

Noting that

$$D(\omega_{\pm}, k_z) = \pm \frac{\partial D}{\partial \omega_0} (i\gamma_z \mp \Delta_T) \quad , \quad (18)$$

with $\Delta_T \equiv \omega_T(k_z) - \omega_0$, Eqs. (16) and (17) further reduce to

$$\begin{aligned} \delta\phi_z &= 2 \left(\frac{c}{B_0} k_{\theta} k_z |A_0| \right)^2 \left(\frac{\omega_0^2}{\omega_A^2} - \frac{1}{4} \right) \left(\frac{b_z}{\chi_{iz}} \right) \\ &\times \left(\frac{b_0}{b_+} \right) \frac{\epsilon_0}{\Lambda_T(\omega_0)} \frac{2\omega_0/\omega_A^2}{\partial D/\partial \omega_0} \frac{(\delta\phi - \delta\psi)_z}{\gamma_z^2 + \Delta_T^2} \\ &\equiv -\alpha_{\phi T} \frac{(\delta\phi - \delta\psi)_z}{\gamma_z^2 + \Delta_T^2} \quad , \end{aligned} \quad (19)$$

and

$$\begin{aligned} \delta\psi_z &= -2 \left(\frac{c}{B_0} k_{\theta} k_z |A_0| \right)^2 \left(\frac{b_0}{b_+} \right) \\ &\times \frac{\epsilon_0 \omega_0^2/\omega_A^2}{\Lambda_T(\omega_0)} \frac{2\omega_0/\omega_A^2}{\partial D/\partial \omega_0} \frac{\Delta_T/\omega_0}{\gamma_z^2 + \Delta_T^2} (\delta\phi - \delta\psi)_z \\ &\equiv -\alpha_{\psi T} \frac{(\delta\phi - \delta\psi)_z}{\gamma_z^2 + \Delta_T^2} \quad . \end{aligned} \quad (20)$$

Equations (19) and (20) then yield the following desired dispersion relation

$$\gamma_z^2 = \alpha_{\psi T} - \alpha_{\phi T} - \Delta_T^2 \quad ; \quad (21)$$

i.e., modulational instability will set in when

$$\begin{aligned} &\left(\frac{c}{B_0 \omega_0} k_{\theta} k_z |A_0| \right)^2 \left(\frac{b_0}{b_+} \right) \frac{\epsilon_0}{\Lambda_T(\omega_0)} \frac{4\omega_0/\omega_A^2}{\partial D/\partial \omega_0} \\ &\left[\frac{\Delta_T}{\omega_0} \frac{\omega_0^2}{\omega_A^2} + \frac{b_z}{\chi_{iz}} \left(\frac{\omega_0^2}{\omega_A^2} - \frac{1}{4} \right) \right] > \left(\frac{\Delta_T}{\omega_0} \right)^2 \quad . \end{aligned} \quad (22)$$

Note that, typically, $|\Delta_T/\omega_0| \sim O(\epsilon_0)$ and $|b_z(1 - \omega_A^2/4\omega_0^2)/\chi_{iz}| \sim O(\epsilon_0^{3/2}/q^2)$. Furthermore, we generally have $\omega_0 \partial D/\partial \omega_0 > 0$ in the ideal MHD first stability region for ideal ballooning modes [19]. Thus, Eq. (22) becomes approximately

$$\Delta_T/\omega_0 > 0 \quad , \quad (23)$$

and

$$\begin{aligned} &\left(\frac{c}{B_0 \omega_0} k_{\theta} k_z |A_0| \right)^2 \left(\frac{b_0}{b_+} \right) \\ &\times \frac{\epsilon_0 \omega_0^2/\omega_A^2}{\Lambda_T(\omega_0)} \frac{4\omega_0/\omega_A^2}{\partial D/\partial \omega_0} > \left(\frac{\Delta_T}{\omega_0} \right) \quad . \end{aligned} \quad (24)$$

This condition is essentially determined by the *spontaneous excitation* of the zonal field $\delta\psi_z$, given by Eqs. (6) and (20), which dominates over the usual zonal flow $\delta\phi_z$, defined in Eqs. (5) and (19). As to the sign of Δ_T/ω_0 , Eq. (23), it depends on specific equilibria and parameters, and must be calculated for individual cases. Note that, for $\Delta_T/\omega_0 < 0$, Eq. (22) may still be satisfied for $\omega_0^2 > \omega_A^2/4$

and small $|\Delta_T/\omega_0|$; however, with $\delta\phi_z$ dominating over $\delta\psi_z$.

In order to give quantitative estimates for the onset condition of the modulational instability, we recall that linear TAE analysis gives [17–19]

$$\frac{\epsilon_0 \omega_0^2/\omega_A^2}{\Lambda_T(\omega_0)} \frac{2\omega_0/\omega_A^2}{\partial D/\partial \omega_0} \sim 1 \quad . \quad (25)$$

Thus, assuming $b_z \lesssim k_{\theta}^2 \rho_i^2 \sim \epsilon_0 b_0$ and noting $k_{\parallel} \simeq 1/2qR_0$, the threshold condition for spontaneous excitation becomes

$$\left(\frac{c}{B_0 \omega_0} k_{\theta} k_z |A_0| \right)^2 \sim \left| \frac{\Delta_T}{\omega_0} \right| \sim \epsilon_0 \frac{b_z}{k_{\theta}^2 \rho_i^2} \sim \frac{b_z}{\epsilon_0} \quad , \quad (26)$$

having considered the maximum $b_0 \sim \epsilon_0$; or, in terms of $\delta B_r/B_0$,

$$\left| \frac{\delta B_r}{B_0} \right|_{th}^2 \sim \frac{\rho_i^2}{4\epsilon_0 (qR_0)^2} \quad . \quad (27)$$

This estimate yields $|\delta B_r/B_0|_{th}^2 \sim O(10^{-8})$ for some typical tokamak parameters. This suggests that spontaneous excitation of zonal structures may be a process effectively competing with other nonlinear dynamics in determining the saturation level of TAE modes. Above threshold, one can estimate

$$\gamma_z \simeq \epsilon_0^{-1/2} b_z^{1/2} k_z v_A \left| \frac{\delta B_r}{B_0} \right| \quad , \quad (28)$$

and $b_z \sim \epsilon_0^2$ for the most unstable growing zonal structures with $\gamma_z \simeq \epsilon_0^{1/2} k_z v_A |\delta B_r/B_0|$, consistently with Eq. (26).

It is important to note that Eqs. (23) and (24) have been derived under the condition $|k_{\perp} \rho_i|^2 \sim |k_z \rho_i|^2 < \epsilon = r_0/R_0 < 1$, which is reasonable and usually applies for TAEs excited by energetic ions in burning plasmas of fusion interest [14]. For shorter wavelengths, or equivalently $\epsilon \rightarrow 0$, both $\delta\phi_z$ and $\delta\psi_z$ become increasingly smaller, as we can readily recognize from Eqs. (19) and (20), since $\omega_0^2/\omega_A^2 - 1/4 \rightarrow 0$ and $\Delta_T/\omega_0 \rightarrow 0$. This is due to the cancellation of the Reynolds and Maxwell stresses, yielding the well known properties of the *Alfvénic state* [20–24], which is broken in the present case by the toroidal geometry of the considered plasma equilibrium, showing the importance of equilibrium geometry in determining both linear and nonlinear plasma dynamic behaviors. Thus, at sufficiently short wavelengths or in simpler plasma equilibria, Eqs. (19) and (20) must be suitably modified (see e.g. [25] and references therein for a recent discussion of this issue); such analysis, however, is beyond the scope of the present work and will be discussed elsewhere. It is also important to note that the zonal structure is dominated by the zonal current instead of the usual zonal flow because of magnetically trapped-ion enhanced polarizability, $\chi_{iz} \simeq 1.6q^2 \epsilon^{-1/2} k_z^2 \rho_i^2$ [2],

$$|\delta\phi_z|/|\delta\psi_z| \approx |k_z \rho_i|^2/|\chi_{iz}| \approx O(\epsilon^{1/2}/q^2) < 1 \quad (29)$$

Indeed, if one adopts the MHD model without trapped ions, so that, $\chi_{iz} \simeq k_z^2 \rho_i^2$, and, correspondingly, $\delta\phi_z \approx \delta\psi_z$, spontaneous excitation of zonal structures is still possible, given $\Delta_T/\omega_0 > 0$; i.e., Eq. (23). However, in the lower half of the TAE frequency gap, $\omega_0^2/\omega_A^2 < 1/4$, where TAE are preferentially located when strongly driven by supra-thermal ions [17, 18], the threshold condition is larger than that of Eq. (24). Meanwhile, for $\Delta_T/\omega_0 < 0$, spontaneous excitation of zonal structures is found only in the upper half of the TAE frequency gap, $\omega_0^2/\omega_A^2 > 1/4$, as noted above; contrary to the case including the proper trapped ion responses. All these considerations may provide a plausible explanation for the simulation results of Todo et al. [11], where the zonal mode response is found to be forced driven by TAE rather than spontaneously excited. The importance of including proper trapped-ion dynamics for the correct prediction of the spontaneous excitation of zonal flows by electrostatic drift-type turbulence in toroidal plasmas was pointed out in [26], where a comparative analysis of slab and toroidal plasma equilibria is discussed. In that case, the result of adopting a simplified geometry description was quantitatively different but qualitatively the same as in the more realistic plasma equilibrium. In the present case, however, the physics picture changes both quantitatively and qualitatively. Including kinetic thermal ion treatment in the nonlinear simulations of Alfvénic modes [27–29] is, thus, an important ingredient for realistic comparisons with experimental measurements, where the existence of zonal currents/magnetic fields has been clearly observed [7]. Furthermore, these results demonstrate the crucial roles played by equilibrium geometry in determining the nonlinear dynamics of Alfvén modes, with obvious impact on the fluctuation induced radial transport of energetic particles and, ultimately, on the fusion performance.

As a final remark, it is worthwhile mentioning some further reflection based on the structure of Eq. (22). In the ideal MHD second stability region for ideal ballooning modes, which may be of interest for high performance burning plasma operations, TAE modes generally have $\omega_0 \partial D / \partial \omega_0 < 0$. This means that zonal structures would be still dominated by zonal currents, but for equilibria such that $\Delta_T/\omega_0 < 0$. We also note that the structure of Eq. (22) is specific to tokamaks only through the quantity entering Eq. (25), Δ_T/ω_0 and b_z/χ_{iz} . These quantities regulate the branching ratio (relative strength) of zonal flows and currents and the onset condition for the modulational instability; in other words, the self-regulatory effect of zonal structures on TAE modes. It, therefore, will be interesting, by a suitable extension of these terms, to generalize the present theoretical framework to other toroidal configurations; each maintaining its specificities via the TAE local dispersiveness and the branching ratio of zonal currents and zonal flows being set by size of the ratio b_z/χ_{iz} .

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