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Microwave characterization of Josephson junction arrays: implementing a low loss superinductance

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We have measured the plasma resonances of an array of Josephson junctions in the regime $E_J \gg E_C$, up to the ninth harmonic by incorporating it as part of a resonator capacitively coupled to a coplanar waveguide. From the characteristics of the resonances, we infer the successful implementation of a superinductance, an electrical element with a non-dissipative impedance greater than the resistance quantum ($R_Q = h/(2e)^2 \simeq 6.5 \text{ k}\Omega$) at microwave frequencies. Such an element is crucial for preserving the quantum coherence in circuits exploiting large fluctuations of the superconducting phase. Our results show internal losses less than 20 ppm, self-resonant frequencies greater than 10 GHz, and phase slip rates less than 1 mHz, enabling direct application of such arrays for quantum information and metrology. Arrays with a loop geometry also demonstrate a new manifestation of flux quantization in a dispersive analog of the Little-Parks effect.

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The emerging field of quantum electronics exploiting large fluctuations of the superconducting phase is challenged by the engineering of an electromagnetic environment which suppresses simultaneously the quantum fluctuations of charge and the random low-frequency fluctuations of offset charges. The small value of the fine structure constant $\alpha = 1/137$ entails a fundamental asymmetry between flux and charge quantum fluctuations, strongly favoring the latter. To illustrate this, let us consider the simplest case of a dissipationless LC oscillator, where the charge Q on the capacitor plates and the generalized flux Φ across the inductor are conjugate variables. The ratio between quantum fluctuations of charge, $\delta q = \delta Q/(2e)$, and flux, $\delta \varphi = \delta \Phi/\Phi_0$, in the ground state of the oscillator is given by $\delta \varphi/\delta q = Z_0/R_Q$. Here $Z_0 = \sqrt{L/C}$ is the characteristic impedance of the oscillator and $R_Q = h/(2e)^2 = 6.5 \text{ k}\Omega$ is the superconducting resistance quantum. Using only geometrical inductors and capacitors, the characteristic impedance of the oscillator Z_0 cannot exceed the vacuum impedance $Z_{vac} = \sqrt{\mu_0/\epsilon_0}$, thus imposing quantum fluctuations of charge at least an order of magnitude larger than flux fluctuations: $\delta q/\delta \varphi > R_Q/Z_{vac} = 1/(8\alpha)$.

On-chip resistors and long chains of Josephson junctions (JJs) in the dissipative regime have already been used to provide high impedance environments for the phase across a Josephson element [1–3]. However, these Ohmic components cannot screen charge offsets efficiently and, being dissipative, tend to destroy the quantum coherence of the devices. We thus need a circuit element which possesses three key attributes: high impedance at frequencies of interest, perfect conduction at DC and extremely low dissipation. These attributes define the so-called “superinductance” [4, 5].

There are currently two leading candidates for implementing superinductances. The first is superconducting nanowires [6, 7]. Unfortunately, they appear to show sig-

nificant internal dissipation, which is not yet well understood, and they are challenging to fabricate. The second implementation exploits the large kinetic inductance of arrays of JJs. Though amenable to design parameters, arrays may also suffer from dissipation due to coupling to internal degrees of freedom [8, 9] or to coupling to a dissipative external bath [10]. Indeed, transport measurements on large arrays of JJs show the appearance of a superconducting to insulating transition (SIT) with decreasing Josephson energy E_J [11–13]. Previous measurements on resonators where the inductive energy is dominated by JJ arrays have yielded internal quality factors in the range of a few thousands [14, 15]. The long lifetime of the fluxonium qubit was the first indirect evidence of a low-loss superinductance based on JJ arrays [5, 16]. However, this realization suffered from coherent quantum phase-slips (CQPS) [17, 18], which constitute additional degrees of freedom, difficult to control experimentally.

In this Letter, we report microwave characterization of arrays of large JJs (see Fig 1(a)) with Josephson energy $E_J \simeq 180 E_C$, where $E_C = e^2/(2C_J)$ is the charging energy of one junction. The parasitic capacitance of the array to ground is such that the CQPS rate is exponentially suppressed with E_J/E_C [19].

Our arrays consist of closely spaced Josephson junctions on a C-plane sapphire substrate, as shown in Fig. 1. We fabricated the junctions by e-beam lithography and double angle evaporation of aluminum using the bridge-free technique of [20, 21]. Prior to aluminum deposition, the substrate is cleaned of resist residues using an oxygen plasma [22]. We minimize the width of the connecting wires between junctions in order to reduce parasitic capacitances to ground, which ultimately lower the self-resonant frequency of the superinductance [23].

We characterize our superinductances at low temperatures and microwave frequencies by incorporating them

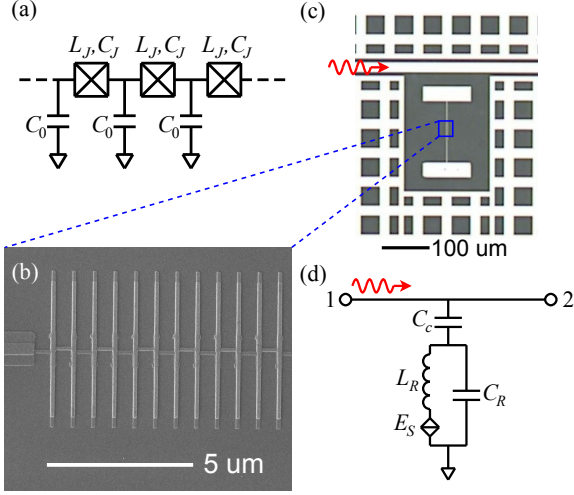


FIG. 1. (a) Schematic representation of an array of Josephson junctions. Capacitances of islands to ground and across tunnel junctions are denoted with C_0 and C_J , respectively. The junction inductance is given by L_J . (b) SEM image of the Josephson junction array fabricated using the bridge-free technique [20, 21]. (c) Optical image of an LC resonator. The large pads implement the resonator capacitance as well as coupling capacitances to the CPW feedline. An array of Josephson junctions between the pads implements the superinductance. The ground plane is patterned with flux-trapping holes. (d) Low-frequency model for the device shown in (c). The phase slip element (split diamond) in series with the superinductance represents the collective contribution of phase slips through all junctions in the array. The characteristic energy of the phase-slip element is denoted by E_S .

in lumped element LC resonators capacitively coupled to a co-planar waveguide in the hanger geometry. The resonator response is measured in transmission. An optical image of a typical device and its low-frequency circuit model are shown in Figs. 1(c,d).

The samples were mounted on the mixing chamber stage (15 mK) of a dilution refrigerator inside a copper box, enclosed in an aluminum-Cryoperm-aluminum shield with a Cryoperm cap. We used two 4-12 GHz Pamtech isolators and a 12 GHz K&L multi-section low-pass filter before the HEMT amplifier, and installed copper powder filters on the input and output lines.

The internal quality factor of a resonator is extracted by fitting the transmission data about the resonance with the response function [24]

$$S_{21}(f) = 1 - \frac{Q_{ext}^{-1} - 2i\frac{\delta f}{f_R}}{Q_{tot}^{-1} + 2i\frac{f - f_R}{f_R}}, \quad (1)$$

where Q_{tot} and Q_{ext} are the total and external quality factors, f_R is the resonant frequency, and δf characterizes asymmetry in the transmission response profile. The internal quality factor is given by $Q_{int} = \frac{Q_{ext}Q_{tot}}{Q_{ext} - Q_{tot}}$. We show a typical measured response for a resonator with an

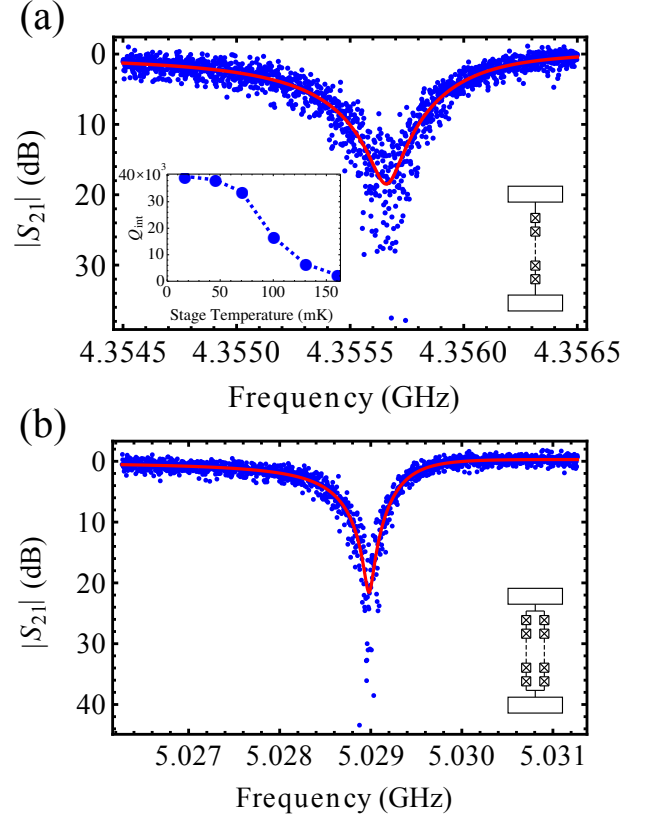


FIG. 2. (a) Typical microwave transmission data for an 80-junction resonator measured with order one photon circulating power; solid line is theoretical prediction corresponding to $Q_{int} > 37,000$. Inset: Temperature dependence of the internal quality factor of the resonator. Dashed line is a guide for the eye. (b) Same measurement as in (a) for a 160-junction array loop; solid line corresponds to $Q_{int} > 56,000$. Parameters for the low-frequency model in Fig. 1(d) are $C_c = 1.6$ fF, $C_R = 7.2$ fF and $L_R = 150$ nH for (a) and $C_c = 1.8$ fF, $C_R = 11$ fF and $L_R = 76$ nH for (b).

80-junction superinductance in Fig. 2(a). This yields an internal quality factor of 37,000 for the resonator at 15 mK (stage temperature), corresponding to a loss in the superinductance of better than 27 ppm. We note that an unknown portion of the internal loss comes from the capacitors. The inset of Fig. 2(a) shows the dependence of the internal quality factor on the stage temperature. The quality factor appears to saturate at low temperatures; this can either indicate the presence of non-equilibrium quasiparticles, or more likely the approach of the noise floor of our measurement setup, limiting the resolvable values for the Q . A similar resonator with two 80-junction arrays in parallel was measured to have a quality factor of 56,000 (Fig. 2(b)), corresponding to a superinductance loss of smaller than 18 ppm, an order of magnitude below the previously reported values [14]. The external quality factors, Q_{ext} , of both resonators were 5,000.

We next evaluate the self-resonant modes of an N -junction array. The Lagrangian of the array is

$$\mathcal{L} = \sum_{n=1}^N \frac{1}{2} C_0 \dot{\Phi}_n^2 + \frac{1}{2} C_J (\dot{\Phi}_n - \dot{\Phi}_{n+1})^2 - \frac{1}{2} \frac{(\Phi_n - \Phi_{n+1})^2}{L_{J0}}, \quad (2)$$

where Φ_n are the node fluxes associated with each superconducting island. We express each node flux as a superposition of discrete Fourier mode amplitudes,

$$\Phi_n = \frac{1}{\sqrt{N}} \sum_{k=1}^N e^{i \frac{\pi k}{N} n} \Phi_k, \quad (3)$$

which leads to a diagonal Hamiltonian of the following form:

$$\mathcal{H} = \sum_{k=-N/2}^{N/2} \mathbb{C}_{kk'}^{-1} Q_k Q_{-k'} + \mathbb{L}_{kk'}^{-1} \Phi_k \Phi_{-k'}. \quad (4)$$

Here Q_k are the canonical conjugate “charge” variables to the fluxes Φ_k , and

$$\mathbb{C}_{kk'} = \delta_{kk'} \left[\frac{C_0}{2} + C_J \left(1 - \cos \frac{\pi k}{N} \right) \right]; \quad k \in \left[-\frac{N}{2}, \frac{N}{2} \right] \quad (5)$$

$$\mathbb{L}_{kk'} = \delta_{kk'} \frac{L_{J0}}{\left(1 - \cos \frac{\pi k}{N} \right)}, \quad (6)$$

are respective capacitance and inductance matrices in the Fourier basis. This immediately leads to a dispersion relation of the form,

$$\omega_k = (L_{kk} C_{kk})^{-1/2} = \omega_0 \sqrt{\frac{1 - \cos \frac{\pi k}{N}}{\frac{C_0}{2C_J} + \left(1 - \cos \frac{\pi k}{N} \right)}}, \quad (7)$$

where $\omega_0 = 1/\sqrt{L_J C_J}$ is the plasma frequency of a single junction. The coupling capacitance pads at each end of the array (see Fig. 1(c)) load down the eigenfrequencies (see supplementary information for calculation details).

The data presented in Fig. 2 corresponds to measurements of the $k = 1$ mode. The frequencies of higher modes, $k \geq 2$, of the array lie outside the band of our measurement setup. In order to observe these modes we exploit their cross-Kerr interaction, which is induced by the junction nonlinearity. This interaction leads to a frequency shift of the lowest mode ($k = 1$) when higher modes are excited. In Fig. 3(a) we show the results of a two-tone measurement of an 80-junction resonator (same device as presented in Fig. 2(a)), where we continuously monitor the $k = 1$ mode while sweeping the frequency of a probe tone. When the probe tone is resonant with a higher mode of the array, we observe a drop in the $k = 1$ frequency.

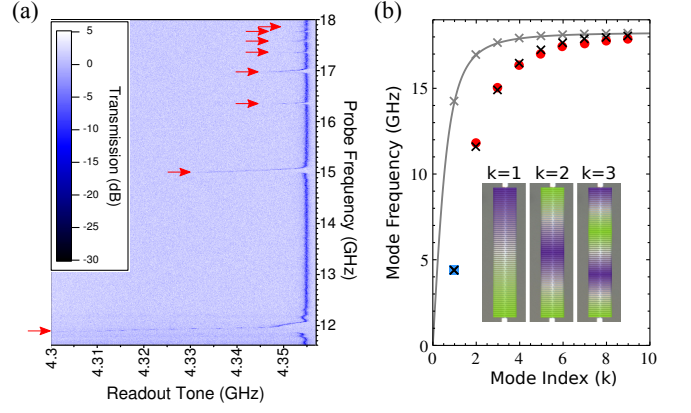


FIG. 3. (a) Shift of the lowest frequency mode of the 80-junction resonator upon application of a probe tone. Due to the weak non-linearity of the array (theory predicts of order 10 MHz/photon [25]), shifts occur when the probe tone matches a plasmonic resonance of the array (marked with red arrows). (b) Measured plasma mode frequencies of the 80-junction array: The blue square represents the fundamental resonant frequency of the resonator (4.355 GHz), and red circles indicate the plasma modes pointed out by red arrows in (a). The black crosses denote the calculated plasma frequencies. The gray curve represents the dispersion relation without including corrections due to the coupling capacitors (open boundary conditions), and markers show corresponding plasma frequencies. The inset in (b) shows the voltage profile along the array for the first three plasma modes.

Fig. 3(b) shows the comparison between the measured frequencies of the array modes and the theoretically predicted values. The black crosses show the renormalized mode frequencies calculated after incorporation of the corrections due to coupling capacitance pads (see supplementary information), which are in good agreement with the measured frequencies represented by colored markers. The parameters extracted from the fit were $C_0 = 0.04$ fF, $C_J = 40$ fF and $L_J = 1.9$ nH, with a confidence range of 20%. The simulated value of the capacitance to ground $C_0 = 0.09$ fF agrees within a factor of 2 with the inferred value from the fit. Room temperature resistance measurements of the junction arrays yield a value of $L_J = 2.1$ nH, which agrees with the fit within 10%. Using the fit parameters we calculate the dispersion relation for the bare superinductance, shown as a gray curve in Fig. 3(b). We note that the lowest resonant frequency of the bare superinductance is 14.2 GHz, which meets the design criterion of having self-resonances well above the frequency range of interest (1 - 10 GHz).

In order to characterize the phase slip rate of the superinductances, we monitor the frequency of the $k = 1$ mode of a resonator formed by two superinductances in parallel, while sweeping an external magnetic field. As flux bias is increased, the persistent current induced in the loop increases, and results in a drop in frequency of

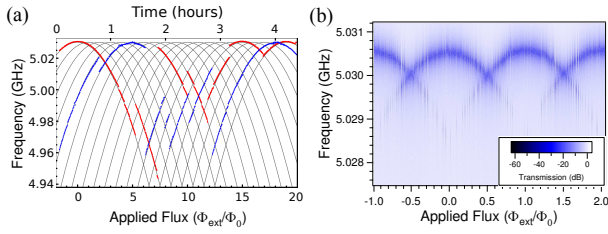


FIG. 4. (a) Lowest mode frequency versus applied flux bias for resonator with 160-junction array loop. Flux bias is swept up (red) and down (blue) over the course of several hours. Gray curves represent quasi-classical predictions for resonator frequencies at different integer values of flux quanta in the array loop. The only adjustable parameter is the number of junctions, found to be 150 ± 10 . (b) Same measurement as in (a) over a smaller flux range, but before each measurement a high power pulse at one of the plasma modes is applied in order to reset the resonator to the lowest flux state.

the mode (Fig. 4(a)). This behavior can be described to lowest order in junction nonlinearity by the quasi-classical expression

$$f(\Phi_{ext}) = \frac{f_R}{\sqrt{1 + \frac{1}{2} \left(\frac{2\pi}{N} \left(\frac{\Phi_{ext}}{\Phi_0} - m \right) \right)^2}}, \quad (8)$$

where Φ_{ext} is the applied flux bias, and m is the integer number of flux quanta inside the loop. A phase slip event is associated with an integer change in m , which we observe as a jump in resonator frequency. As we increase the flux bias, the phase slip rate is enhanced and frequency jumps become more probable [19]. We swept the flux bias applied to the loop over several flux quanta before a phase slip event occurred. The typical duration between phase slips recorded in this experiment was over an hour, Fig. 4(a). This is a remarkably low phase slip rate of well under 1 mHz for a loop of 160 junctions, significantly lower than previously reported values on shorter arrays [17, 26].

Due to the extremely low phase slip rate, in order to measure the resonator in the lowest flux state, we employ an active resetting scheme. We apply a high power pulse at one of the $k \geq 2$ modes before measuring the location of the lowest resonant frequency. This high power pulse activates phase slips, relaxing the loop to the lowest flux state. Remarkably this allows a change in persistent current at constant flux bias. Using this protocol, we tracked the resonant frequency in the lowest flux state, as shown in Fig. 4(b), and observed discrete inverted parabolas. This periodic modulation of the resonator frequency with flux is akin to the Little Parks effect [27] measured dispersively in a non-dissipative regime. We exploit this effect to unambiguously calibrate the number of flux quanta in the loop. The fitted value for the total number of junctions differs by 6% from the actual number, which can be explained by the classical nature of the theory which

does not take quantum fluctuations into account.

In conclusion, microwave measurements of superconducting Josephson junction arrays demonstrate that superinductances in the range of 100–300 nH, with self-resonant frequencies above 10 GHz, internal losses less than 20 ppm, and phase slip rates below 1 mHz can be successfully implemented. With parameters such as measured in our experiment, Josephson junction array superinductances significantly enrich the quantum electronics toolbox. Applications would include further suppression of offset charges in superconducting qubits [16], high Q and tunable lumped-element resonators, on-chip bias tees, and kinetic inductance particle detectors [28]. In addition, these arrays exhibit rich dynamics owing to many internal degrees of freedom, thus making them perfect candidates for the study of quantum many-body phenomena such as quantum impurity models [29, 30], microwave photonics [31], and measurements of Bloch oscillations [32].

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