Ambipolar Transport via Trapped-Electron Whistler Instability Along Open Magnetic Field Lines

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Ambipolar transport via trapped-electron whistler instability along open magnetic field lines

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An open field line plasma is bounded by a chamber wall which intercepts the magnetic field. Steady state requires an upstream plasma source balancing the particle loss to the boundary. In cases where the electrons have long mean-free-path, ambipolarity in parallel transport critically depends on collisionless detrapping of the electrons via wave-particle interaction. The trapped-electron whistler instability, whose nonlinear saturation produces a spectrum of whistler waves that is responsible for the electron detrapping flux, is shown to be an unusually robust kinetic instability, which is essential to the universality of the ambipolar constraint in plasma transport.

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Ambipolarity is a fundamental plasma physics concept in which the electrons and ions have the same particle transport (or loss) rate to a boundary such as the chamber wall in laboratory experiments. It is a robust constraint for plasma transport since even a small amount of net charge can induce an electric field of enormous amplitude. In a magnetized plasma, cross field transport due to collisional drag between electrons and ions are automatically ambipolar. When plasma flow is present, viscosity can break this intrinsic ambipolarity and a cross-field electric field is established to modify the plasma rotation in order to maintain ambipolarity of cross-field transport [1]. In a tokamak this is known as the neoclassical radial electric field and rotation. With an open magnetic field line, which means a field line that intercepts the boundary, for example the tokamak scrape-off layer or Hall-effect thrusters [2], one recovers a situation similar to that of unmagnetized plasmas, where a (pre)sheath electric field must be present to enforce ambipolarity by slowing down the electron flow while accelerating the ion flow to the boundary. The required ambipolar potential drop from the plasma to the boundary is on the order of k_BT_e/e with k_B the Boltzmann constant, T_e the electron temperature, and e the elementary charge.

The crucial role of the parallel (pre)sheath electric field and the transparent physics elucidated by the fluid analysis date back to Bohm and Langmuir [3, 4]. The simplicity of the fluid argument somewhat obscures the rich kinetic physics in even the simplest sheath model as formulated by Langmuir, namely cold ions with collisionless electrons. For example, the presence of near-Maxwellian electrons as required by the fluid model and sometimes observed in low temperature experiments, poses a challenge for theoretical explanation in the name of Langmuir paradox [5]. The potential role of collisionless thermalisation of the distribution function was recently argued [6]. In this Letter, we will show that (1) collisionless detrapping of electrons via wave-particle interaction is an essential mechanism to ensure ambipolarity in a low-collisionality plasma; (2) in an open field line plasma, the trapped-electron population provides a robust drive for whistler wave instability, with a threshold far lower than that of the conventional temperature-anisotropy-driven whistler mode [7, 8]; (3) the prevalence of the trapped electron whistler mode is crucial to supply the detrapping flux via wave-particle interaction to ensure ambipolarity in parallel transport.

The subtle kinetic physics can be demonstrated in the archetypal example of collisionless electrons and cold ions where a uniform magnetic field intercepts a perpendicular, absorbing wall. To establish a steady state, the particle loss to the wall must be replenished by a particle source upstream. Without loss of generality, the upstream source draws from a local Maxwellian of fixed source temperature. The challenge of ambipolarity becomes obvious when one recognizes that the (pre)sheath electric field confines the low energy electrons and sets up a trap-passing boundary in $v_\parallel$ (with respect to magnetic field). The source electrons, which are assumed to be drawn from a local Maxwellian, would populate both the passing and trapped region. In contrast, the ions are accelerated by the (pre)sheath electric field, so all the source ions are passing. To maintain ambipolarity, the source electrons originally in the trapped region (in $v_\parallel$ space) must find a way to cross the trap-passing boundary. In the absence of collisional detrapping, or with inadequate collisional detrapping, a collisionless detrapping mechanism must be present otherwise ambipolarity would be violated. The most obvious candidate for collisionless detrapping is wave-particle interaction. The primary difficulty, in the simple case of an open field line plasma, is to identify a robust plasma instability that would drive plasma waves which efficiently interact with electrons. The "robustness" of the instability can not be overemphasized as by the argument given so far, the requirement of ambipolarity demands an instability at almost arbitrarily small plasma beta, for example.

Kinetic simulations solving the Vlasov-Maxwell equations with VPIC [9] clearly indicate the prevalence of whistler waves propagating along the magnetic field line.

At frequencies near the electron cyclotron frequency, these whistler waves are excellent candidates to interact with the electrons producing the required collisionless detrapping flux. Before evaluating this collisionless detrapping flux, we will first elucidate the nature of the observed whistler wave instability. Interestingly, although parallel transport naturally sets up a plasma temperature anisotropy [10], which would persist in a low collisionality plasma, the well-known temperature-anisotropy-driven whistler wave instability is not the one operating here. The reason is that the instability threshold is usually not satisfied in the low-beta open field line plasma.

To see this, we recall the dispersion relation of a whistler wave propagating along a uniform background magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ with normal mode ansatz $\exp(ikz - i\omega t)$ in a plasma of distribution function $f_{\alpha 0}(v_\parallel, v_\perp)$ for species α [11],

$$\Big(1-\frac{k^2c^2}{\omega^2}\Big)+\sum_\alpha\frac{\omega_{p\alpha}^2}{n_0\omega}\int_{-\infty}^{\infty}\int_0^\infty\Big[\Big(1-\frac{kv_\parallel}{\omega}\Big)\frac{\partial f_{\alpha 0}}{\partial v_\perp^2} + \frac{kv_\parallel}{\omega}\frac{\partial f_{\alpha 0}}{\partial v_\parallel^2}\Big]\frac{v_\perp^3}{\omega-kv_\parallel-\Omega_\alpha}dv_\perp dv_\parallel = 0\,, \quad (1)$$

where $v_\parallel \equiv \mathbf{v}\cdot\hat{\mathbf{z}}$, $\mathbf{v}_\perp = \mathbf{v} - v_\parallel\hat{\mathbf{z}}$ and $\omega_{p\alpha}, \Omega_\alpha$ are the plasma frequency and cyclotron frequency of species α. For a bi-Maxwellian plasma,

$$f_0(v_\parallel, v_\perp) = \frac{2n_0}{\sqrt{\pi}v_{t\perp}^2 v_{t\parallel}} e^{-v_\perp^2/v_{t\perp}^2} e^{-v_\parallel^2/v_{t\parallel}^2}\,, \quad (2)$$

with $v_{t\parallel} = \sqrt{k_B T_\parallel/2m}, v_{t\perp} = \sqrt{k_B T_\perp/2m}$ and $n_0 = \int_{-\infty}^{\infty}\int_0^\infty f_0 v_\perp dv_\perp dv_\parallel$ the plasma density, the dispersion relation takes the simple form,

$$1-\frac{k^2c^2}{\omega^2}+\frac{\omega_{pe}^2}{\omega^2}\left[A+(\frac{\omega}{kv_{t\parallel}}+A\xi)Z(\xi_e)\right]=0\,. \quad (3)$$

Here $A = T_\perp/T_\parallel - 1$ is a temperature anisotropy parameter, $Z(\xi_e)$ is the usual plasma dispersion function, $\xi_e = (\omega - \Omega_e)/kv_{t\parallel}$ and the ion contribution has been dropped since $\omega_{pi}/\omega_{pe} = \sqrt{m_e/m_i} \ll 1$. The ω is generally complex, so $\omega = \omega_r + i\gamma$. The instability criteria (for $\gamma > 0$) can be written as [7]

$$T_\perp/T_\parallel - 1 > (\Omega_e/\omega_r - 1)^{-1}\,. \quad (4)$$

This is a concise but less commonly used form in that the real frequency ω_r explicitly enters Eq. (4). The more popular instability criteria, well-known in the space plasma physics literature, is expressed in terms of the electron $\beta_e = nT_e/B^2$ [8]. Previous results have established that the electromagnetic (whistler) mode becomes stable for low β_e plasma. In our 1-D sheath example, $\beta_e \sim 0.002$, we have observed electromagnetic whistler waves with $\omega_r \simeq 0.9\Omega_e$, while $T_\perp \simeq 6T_\parallel$. Therefore, the temperature anisotropy instability criteria, Eq. (4), is not satisfied.

The actual drive for the observed whistler instability in the simulation is the trapped electron population. As noted earlier, in the presence of a (pre)sheath electric field, the electron distribution has two parts: trapped distribution, $f_t^e(|v_\parallel| < v_c)$, and passing electron distributions, $f_p^e(v_\parallel > v_c)$, where $v_c \equiv \sqrt{2e|\phi_w|/m_e}$ is the trap-passing boundary in $v_\parallel$, ϕ_w is the electrostatic potential difference with the wall. Upon nonlinear saturation, the trapped electron distribution function is dependent on both the source injection (to compensate for the wall loss by passing electrons) and the velocity space diffusion due to wave-particle interaction. For the whistler wave stability analysis, the initial trapped-electron distribution, as seen in our simulations, is well approximated by a Maxwellian distribution with cut-offs in the parallel velocity at $v_\parallel = \pm v_c$,

$$f_t(v_\parallel, v_\perp) = \frac{\alpha n_0}{\sqrt{\pi}v_t^3} e^{-(v_\perp^2+v_\parallel^2)/v_t^2}\Theta(1-v_\parallel^2/v_c^2)\,, \quad (5)$$

where $\Theta(1-v_\parallel^2/v_c^2)$ is the Heaviside step function that vanishes when $|v_\parallel| > v_c$, and $\alpha(v_c) = 2[\mathrm{erf}(v_c/v_t)]^{-1}$ is a normalization factor so $\int f_t v_\perp dv_\perp dv_\parallel = n_0$. Substituting the trapped electron distribution function, Eq. (5), into the dispersion relation, Eq. (1), and ignoring the effect of passing electrons for now since $n_p/n_t \sim m_e/m_i \ll 1$ [12], we obtain the dispersion relation for the trapped-electron whistler mode,

$$D(\omega,k) = 1-\frac{k^2c^2}{\omega^2}+\frac{\alpha\omega_{pe}^2}{2\sqrt{\pi}\omega^2}\frac{\omega}{kv_t}\int_{-\hat{v}_c}^{\hat{v}_c}\frac{e^{-\hat{v}_\parallel^2}}{\hat{v}_\parallel-\xi}d\hat{v}_\parallel + \frac{\alpha\omega_{pe}^2}{2\sqrt{\pi}\omega^2}\frac{\hat{v}_c e^{-\hat{v}_c^2}}{(\hat{v}_c+\xi)(\hat{v}_c-\xi)} \quad (6)$$

where $\hat{v}_{\parallel,\perp,c} = v_{\parallel,\perp,c}/v_t$, $\xi = (\omega - \Omega_e)/kv_t$, and $\Theta'(x) = \delta(x)$ (the dirac-delta function) has been used.

Many physical insights can be obtained from the analytical solutions of Eq. (6) in two limiting cases. In the limit of small cutoff speed, $v_c \ll v_t$, which means that the trapped electrons are cold, we can expand the dispersion relation to the first order of $|\hat{v}_\parallel/\xi|$ by assuming $|\hat{v}_\parallel/\xi| \ll 1$, and further take the limit $k^2c^2/\omega^2 \gg 1$. The solution becomes

$$\omega = \Omega_e + i\omega_{pe}e^{-v_c^2/2v_t^2}\frac{v_b}{c}\,, \quad (7)$$

where $v_b = \sqrt{\alpha v_c v_t/2\sqrt{\pi}}$ is introduced for simplicity. The assumption of $|\hat{v}_\parallel/\xi| \ll 1$ requires $v_c \ll (\omega_{pe}/kc)^2 v_t$ which is usually satisfied for short wave length modes. A finite wave number k will allow particles with almost zero parallel velocity to release their kinetic energy by pitch angle scattering in the wave frame. In the opposite limit of a large cutoff speed, $v_c \gg v_t$, so only the high $v_\parallel$ tail is removed from the Maxwellian. Before going into the calculation, we first note that the third term in Eq.(6) does not contain a singularity for finite positive γ. Intuitively, the kinetic energy of particles with $v_\parallel \lesssim v_c$ decreases if they are scattered into the passing region along the characteristic, $(v_\parallel - \omega/k)^2 + v_\perp^2 = \text{const}$ (constant energy surface in the wave frame), while there are no counter parts with $v_\parallel > v_c$ in the equilibrium distribution. Therefore, the wave amplitude increases and it is reasonable that the most unstable mode satisfies the resonance condition $(\omega - \Omega_e) \simeq kv_c$. In this limit, the integration bound in the dispersion relation, Eq.(6), is extended to infinity. Assuming $k^2c^2/\omega_r^2 \gg 1$, we obtain approximately the real frequency to be

$$\omega_r = \Omega_e[1 - (\frac{v_c}{c}\frac{\omega_{pe}}{\Omega_e})^{\frac{2}{3}}] \tag{8}$$

for $\omega_r/\Omega_e \sim 1$, and

$$\omega_r = \Omega_e[3 + (\frac{v_c}{c}\frac{\omega_{pe}}{\Omega_e})^2]^{-1} \tag{9}$$

for $\omega_r/\Omega_e \ll 1$. In both cases, the growth rate is

$$\gamma = \omega_{pe} e^{-v_c^2/2v_t^2} \frac{1}{\sqrt{2}} \frac{v_b}{c} \sqrt{\frac{\omega_r}{\Omega_e}}\,, \tag{10}$$

where we have assumed $\gamma \ll |\Omega_e - \omega_r|$ and $(\Omega_e - \omega_r)^2\omega_r/\Omega_e \ll \omega_{pe}^2$ for simplicity. For high frequency whistler wave with $\omega \lesssim \Omega_e$, the unstable mode tends to have large phase velocity, $v_p = \omega/k = v_r\omega/(\Omega_e - \omega) \gg v_r$ with v_r being the resonant particle parallel velocity. This is different from the seminal work by Kennel and Petschek [7] who considered bi-Maxwellian electrons and $v_p \sim v_r$. The growth rate given by Eq.(10) for $v_c \gg v_t$ is similar to Eq.(7) for $v_c \ll v_t$, except for a factor of $\sqrt{2}$. In the last term in the dispersion relation, Eq.(6), both $\hat{v}_\parallel + \xi$ and $\hat{v}_\parallel - \xi$ terms can contribute in the $v_c \ll v_t$ case since the resonance is independent of the particular parallel velocity, while only the $\hat{v}_\parallel - \xi$ term contributes in the $v_c \gg v_t$ case since the resonance requires a particular v_r. It is also interesting to note that the imaginary part of $\xi = (\omega - \Omega_e)/kv_t$ dominates in the small v_c limit, while $Re(\xi) \gg Im(\xi)$ in the large v_c limit.

The two limiting results, Eqs. (7,10), show that the trapped-electron distribution, characterized by a sharp gradient in $v_\parallel$ at the trap-passing boundary, is able to drive whistler wave instability below the electron cyclotron frequency for a wide range of v_c. The robust trapped-electron whistler instability is in sharp contrast with the temperature anisotropy whistler mode, Eq.(4), which requires a higher degree of temperature anisotropy to be unstable for the same frequency range. To further illustrate the difference between them, we compare their growth rate and real frequency for $v_c = [0.1v_t, 2v_t]$. The numerical solutions of Eqs. (3) and (6) are shown in Fig. 1, where the most unstable modes are plotted for both a cutoff Maxwellian and a bi-Maxwellian with same perpendicular temperature and equivalent parallel temperature, defined as *i.e.* $T_\parallel = T_\perp \left[1 - \alpha\hat{v}_c\exp(-\hat{v}_c^2)/\sqrt{\pi}\right]$. There are good agreements between numerical results and the analytical results in (1) marginal stability boundary for temperature anisotropy mode, Eq. (4), and (2) the growth rates and real frequency for trapped-electron mode in both the small and large v_c cutoff, Eqs. (7,8,9,10). Over the entire range of $v_c = [0.1v_t, 2v_t]$, it is clearly shown in Fig. 1 that the growth rate of a cutoff-Maxwellian is larger than that of a corresponding bi-Maxwellian for different cutoff speeds. For the low β_e plasma as in our simulation of the 1-D sheath problem, the conventional whistler mode becomes marginally stable when $v_c = 0.5v_t$ or $T_\perp/T_\parallel = 6.42$, while the trapped-electron driven mode still has finite growth rate even for $v_c > 2v_t$. Here we have used the parameters in normalized unit ($\omega_{pe} = 1$) where $\Omega_e = 2\sqrt{2}$, $c = 10v_t$, so that $\beta_e = 0.125\%$. For higher β_e value, the criteria for temperature anisotropy driven instability is shifted to a larger v_c, *i.e.* $v_c \simeq 1.3v_t$ or $T_\perp/T_\parallel \simeq 1.4$, as shown in Fig. 1, and there is substantial increase in the growth rate. [Here, parameters $\omega_{pe} = 1$, $\Omega_e = 0.25$, $c = 10v_t$ have been used, so that $\beta_e = 16\%$.] In contrast, the growth rate of the trapped-electron driven mode is only slightly changed from the low beta case. From these analyses, we conclude that the trapped-electron whistler mode is a robust instability with respect to the electron beta (β_e) and the ambipolar potential (hence v_c/v_t).

In the open field line transport problem, the trapped-electron whistler mode excites whistler waves that produce detrapping electron flux across the trap-passing boundary. The ambipolarity of the parallel transport can only be established in steady state when the statistically averaged detrapping flux matches the source electron injection rate in the trapped phase space. In other words, wave-particle interaction via self-excited whistler instability plays an essential role in maintaining ambipolarity in the parallel transport of open field line plasmas with long mean-free-path electrons. This important physics can be quantified in the 1-D sheath example. In steady state, the ion flux, Γ_i, going into the wall is exactly the same as the injected particle flux. The passing electrons from the source injection contribute to a wall flux

$$\Gamma_e = [1 - \text{erf}(\sqrt{e|\phi_w|/T_s})]\Gamma_i\,, \tag{11}$$

which is obviously less than the ion flux for any finite value of ϕ_w. Here, T_s is the temperature of the Maxwellian source. A mechanism is required to induce detrapping electron flux, in order to satisfy ambipolarity. In short mean-free-path limit , it is Coulomb collision; in the long mean-free-path limit, the candidate is wave-particle interaction. The total electron flux from trapped to passing phase space can be evaluated using the stan-

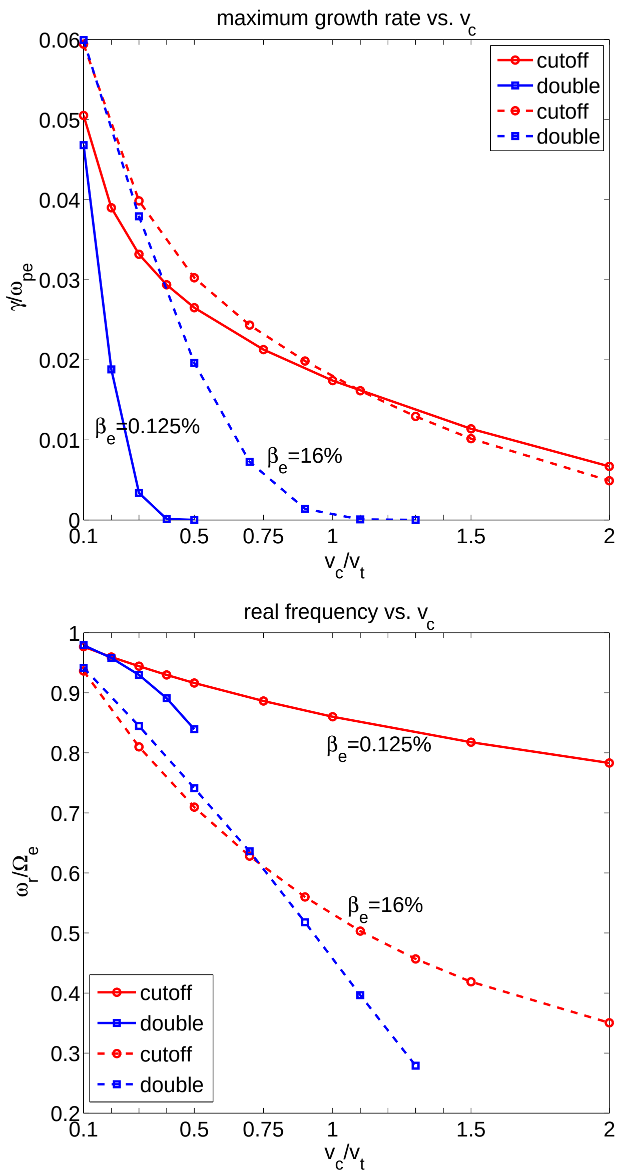

FIG. 1: The growth rate (top) and the real frequency (bottom) of the most unstable mode for both cutoff-Maxwellian and double-Maxwellian distribution with different cutoff parallel velocity v_c or equivalent $T_\parallel$, for $\beta_e = 0.125\%$(solidline) , 16%(dashedline).

dard quasilinear diffusion theory in the literature [13–16],

$$\Gamma_{tp} = \int_0^L \frac{n_0}{v_c v_t}\frac{k_c}{\omega_c}[2(1-\frac{k_c v_c}{\omega_c})g - \frac{k_c v_t}{\omega_c}g']A_{k_c}dl\,, \quad (12)$$

where $g(\hat{v}_\parallel)$ is the saturated local electron distribution function in $v_\parallel$ with a normalization of $\int g(\hat{v}_\parallel)d\hat{v}_\parallel = 1$, L ($k_c L \gg 1$) is the total system size, and $A_{k_c} = (Lq^2/4\pi m^2)\delta\hat{E}_k^2$ denotes the perturbation energy density in k space with $\delta\hat{E}_k$ the kth Fourier component of electric field perturbation. For $\omega \leq \Omega_e$, the resonance condition of electrons with $v_\parallel \geq 0$ requires $k_c < 0$. Therefore, Γ_{tp} is positive when

$$|g'(\hat{v}_c)|/g(\hat{v}_c) > 2(\omega_c/|k_c| + v_c)/v_t\,.$$

Since the instability drive, namely source injection into the trapped region, always presents, the quasilinear diffusion produces a finite detrapping particle flux. We take from the simulation ($\beta_e = 0.125\%$), $v_t = \sqrt{2}$, $L = 200/\sqrt{2}$, $|k_c| \simeq 0.5$, $\omega_c \simeq 2.55$, $|e\phi_w/T_s| \simeq 0.05$ (thus $v_c \simeq 0.32$), and values of $n_0, g(\hat{v}_c), g'(\hat{v}_c), A_{k_c}$ at seven different locations, where we have adopted normalization to the Debye length and electron plasma frequency, and used spatial averaging for simplicity. The electron flux calculated from Eq.(11) is 15% less than the ion flux; while the estimated quasilinear diffusion induced flux from Eq.(12) is approximately 19% of Γ_i, roughly matching the amount required to satisfy ambipolarity.

The nonlinear saturation of the temperature anisotropy whistler instability causes robust temperature isotropization on tens of ω_{pe}^{-1} time scale as shown by Davidson *et al.* [17]. Trapped-electron whistler modes can saturate to a finite level of temperature anisotropy, at which the electron distribution is locally smoothed at the trap-passing boundary such that a sufficiently large gradient of the distribution function is maintained to provide the required ambipolar detrapping flux, Eq. (12). The residual temperature anisotropy in the nonlinearly saturated steady state can be substantial. As an application, our kinetic-Maxwell simulation of a magnetic mirror shows that even with finite collisionality, the trapped electron distribution can retain anisotropy. From the perspective of collisional transport, because the confinement time of plasma in magnetic mirror is much longer than electron-electron collision time, the trapped electrons should be equilibrated into a Maxwellian. However, the collisional isotropization process is subject to a faster wave-particle detrapping process at low collisionality. An electron can be scattered from the trap-zone into the passing-zone before it suffers significant pitch-angle scattering caused by Coulomb collision, yet the detrapping process does not induce energy equipartition between parallel and perpendicular directions. Therefore, the trapped electrons can maintain a significant temperature anisotropy even with finite collsionality, which is confirmed by our kinetic simulations.

In conclusion, we have found that for a broad class of open field line plasma transport problems with particle sink at the wall boundary and upstream plasma source for steady state, the very concept of ambipolarity is intrinsically tied to plasma-wave interaction if the electrons are of long mean-free-path. This comes about because the ambipolar electric field, which slows down electron flow in fluid theory, introduces trap and passing zones in the electron $v_\parallel$ space. As long as the upstream plasma source does not populate the passing zone exclusively, collisionless detrapping via wave-particle interaction plays an essential role in enabling steady-state ambipolarity. The “universal” requirement of ambipolar transport demands an unusually robust instability that interacts strongly with electrons, which we find to be the trapped-electron driven whistler mode.

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[1] P. Helander and D. J. Sigmar, *Collisional Transport in Magnetized Plasmas,* p. 158, Cambridge University Press (2002).
[2] A. Smirnov, Y. Raitses and N. J. Fisch, *Physc. Plasmas*, 14, 057106 (2007).
[3] D. Bohm, *The Characteristics of Electrical Discharges in Magnetic Fields,* Ch. 3, p. 77, New York: McGraw-Hill (1949).
[4] I. Langmuir, Electric discharges in gases at low pressures. *J. Franklin Institute*, 214, 275 (1932).
[5] D. Gabor, E. A. Ash and D. Dracott, *Nature*, 176, 916 (1955).
[6] S. D. Baalrud, J. D. Callen and C. C. Hegna, *Phys. Rev. Lett.*, 102, 245005 (2009).
[7] C. F. Kennel and H. E. Petschek, *J. Geophys. Res.*, 71, 1 (1966).
[8] S. P. Gary and J. Wang, *J. Geophys. Res.*, 101, 10749 (1996).
[9] K. Bowers, *et al. Phys. Plasmas* **15**, 055703 (2008).
[10] X. Z. Tang, *Plasma Phys. Control. Fusion* **53**, 082002 (2011).
[11] N. A. Krall and A. W. Trivelpiece. *Principles of Plasma Physics.* p. 414, San Francisco Press, Inc. (1986).
[12] Zehua Guo and Xian-Zhu Tang, *Phys. Plasmas*, 19, 062501 (2012).
[13] A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, *Nucl. Fusion*, 1, 82 (1961); *Nucl. Fusion Suppl.*, Pt. 2, 465 (1962).
[14] W. E. Drummond and D. Pines, *Nucl. Fusion Suppl.*, Pt. 3, 1049 (1962).
[15] C. F. Kennel and F. Engelmann, *Phys. Fluids*, 9, 2377 (1966).
[16] A. N. Kaufman, *J. Plasma Phys.*, 8, 1 (1972).
[17] R. C. Davidson, D. A. Hammer, I. Haber, and C. E. Wagner, *Phys. Fluids*, 15, 317 (1972).