# Spontaneous Generation of a Sheared Plasma Rotation in a Field-Reversed θ-Pinch Discharge

Y. A. Omelchenko and H. Karimabadi

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# Spontaneous Generation of a Sheared Plasma Rotation in a Field-Reversed θ-Pinch Discharge

Y.A. Omelchenko and H. Karimabadi

*SciberQuest, Del Mar, CA 92014*

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## Abstract

By conducting two-dimensional hybrid simulations of an infinitely long field-reversed θ-pinch discharge we discovered a new type of plasma rotation, which rapidly develops at the plasma edge in the ion diamagnetic direction due to the self-consistent generation of a Hall-driven radial electric field. This effect is different from the previously identified end shorting and particle loss mechanisms. We also demonstrate flute-like perturbations frequently inferred in experiments and show that in the absence of axial contraction effects they may quickly alter the toroidal symmetry of the plasma.

## 1. Introduction

We report results from hybrid (kinetic ions, massless fluid electrons) simulations of a field-reversed θ-pinch discharge (FRTD) in an infinite cylindrical plasma chamber. FRTD is a well-developed method for creating the field-reversed configuration (FRC), which is considered to be one of the most promising candidates for an economical fusion reactor (for a recent review of FRC research see Ref. [1]).

FRTD is achieved by creating a plasma frozen in a bias axial magnetic field and then reversing and ramping the magnetic field at the cylindrical wall, allowing it to compress and penetrate into the plasma. The plasma extends to the geometric axis and the combination of internal plasma and external coil toroidal currents leads to the formation of open and closed magnetic field lines predominantly in the poloidal (z-r) plane (with some equilibrium toroidal field). θ-pinch is usually a rather violent process, leading to radial shocks and associated plasma heating. In addition, flute modes can lead to large-scale toroidal instabilities, which may greatly reduce the trapped poloidal flux [1].

The FRC notably stands apart from a number of other fusion concepts because of its relatively small magnetic field in the core, which provides a natural high-β plasma configuration with large ion orbits, singular (O- and X-) points and significant sheared poloidal flows. These effects give rise to the physics not fully captured by MHD and extended-MHD models. As a result, fully kinetic or hybrid models are required to study FRC formation and stability.

One of the most intriguing issues in the FRC physics is the spontaneous toroidal plasma spin-up observed in all FRCs formed by θ-pinch [1]. Experimental observations suggest that there is strong shear in plasma rotation velocity, with rotation being surprisingly fast in the FRC's edge layer at the early stage of formation [1-3]. The edge rotation is transferred to the core plasma through viscous friction, causing it to rotate in the ion diamagnetic direction [2]. Ion rotation is routinely considered to be part of FRC equilibrium models in both MHD and hybrid simulations but the exact theoretical origin of this spin-up remains unclear because radial profiles of the rotation data are not available for most experiments [1,4]. Understanding the cause for this phenomenon is important because rotation is known to drive various instabilities that may significantly degrade the FRC confinement.

A number of physical effects have been sought to explain the cause of spontaneous FRC spin-up [1]. Among these effects, which are not based on frictional or particle-loss mechanisms, the most notable one is end-shorting of open magnetic field lines (through the modification of cross-field electric field) in the edge layer of an FRC [2,4]. The end-shorting effect results in launching a torsional Alfvén wave, which imparts a torque on the plasma. The essential point in this theory is that the torque, which changes the ion diamagnetic drift velocity, is assumed to be imparted to the plasma *by the wall boundary*, with the energy of the resulting spin-up coming from the release of electrostatic potential energy [2]. The end-shorting mechanism was shown to generate large rotation just outside the FRC separatrix in the same direction as the rotation in the core plasma, whereas the particle loss results in an oppositely directed rotation [2,4].

## 2. Hall-driven rotation

In this Letter we present computational results and simple supporting theory that suggest an alternative mechanism for the spontaneous spin-up of the edge plasma at early time in a

FRTD. This theory does not concern the nature of the contact of open field lines with wall boundaries or particle losses. We show that the Hall-driven rotation (HDR) can be initiated *internally* by a radial electric field, which arises in the plasma outer edge layer as a result of the Hall effect and magnetic field frozen-in condition. This fast rotation develops rapidly outside the separatrix in the ion diamagnetic direction. Deeper into the plasma it reverses its direction twice in a narrow middle layer near the magnetic null (O-point) due to the change of sign in the magnetic field and its radial derivative. The energy going into the sheared plasma rotation comes from the imploding magnetic field. In a quasi-steady regime the Hall-induced rotation vanishes in the core plasma.

The HDR effect can be explained in simple terms for the case of an infinite cylindrical plasma. Let z be the axial axis and x,y coincide with the local radial and toroidal (θ) directions, respectively, with the x-axis being directed positively along the radius towards the wall.

The radial electric field can be found by combining Ampere's law and the generalized Ohm's law (in the following theoretical analysis we neglect the plasma resistivity):

$$E_x = -\frac{V_y}{c} B_z - \frac{B_z}{4\pi e n_e}\frac{\partial B_z}{\partial x} - \frac{1}{e n_e}\frac{\partial P_e}{\partial x}\,. \qquad (1)$$

In Eq. (1) $V_y$ is the ion rotation velocity ($V_y \approx 0$ initially), $n_e$ is the electron number density. $B_z$ is the total axial magnetic field, i.e., a superposition of an internally generated wave field, an externally applied forward magnetic field ($B_z^{coil} > 0$), and the reverse bias field ($B_z^{bias} < 0$) initially frozen in the plasma. The second term in Eq. (1) represents the nonlinear Hall term. For parameters in question, the electron pressure term in Eq. (1) is small compared to the Hall term when both the magnetic field $B_z$ and its gradient $\partial B_z / \partial x$ deviate significantly from zero. Since this is true almost everywhere in the edge layer (except the narrow vicinity of the magnetic null), this term is omitted from further analysis.

The magnetic field penetration into a plasma was studied analytically in the EMHD (electron magnetohydrodynamics) limit [5]. It was shown to be governed by the Burgers equation, which describes the convective transport of magnetic field frozen in the massless electron fluid and conditioned by magnetic diffusion. In our case the dynamics of the imploding magnetic field is similar. The inertial plasma implosion makes the plasma edge undergo rapid (compared to the Alfven timescale) compression so that $\partial n_e / \partial x < 0$ for $x > 0$. When this compression proceeds faster than the resistive penetration, then the magnetic field frozen-in condition holds and the penetrating magnetic flux $B_z > 0$ increases radially inward:

$$\partial B_z / \partial x \approx (B_z / n_e) \partial n_e / \partial x < 0 \, , \; x > 0 \, . \qquad (2)$$

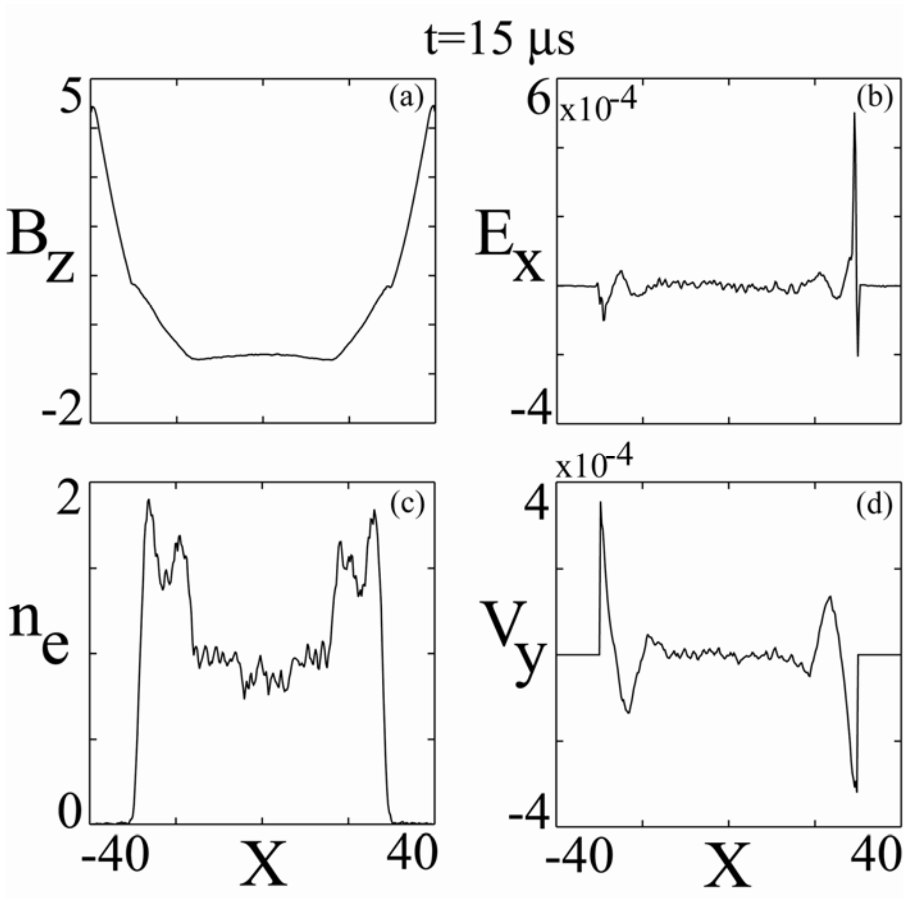


**FIG. 1.** Simulation profiles along the mid-plane X-axis (cm) at t=15μsec: (a) axial magnetic field, $B_z$ (kG), (b) radial electric field, $E_x$ (kG), (c) plasma density, $n_e$ normalized to the initial value, and (d) ion rotation velocity, $V_y$ normalized to the speed of light.

The occurrence of condition (2) is shown in Fig. 1(a). As a consequence, the Hall term in Eq. (1) produces a large *dynamic* radially outward field $E_x > 0$ (Fig. 1(b), $x > 0$), which makes edge ions *spin up* (via the $\mathbf{E} \times \mathbf{B}$ drift) in the diamagnetic direction with $V_y < 0$ (Fig. 1(d)). When the diffusion at the plasma edge prevails over the compression then $\partial B_z / \partial x \approx 0$ (e.g., see Fig. 1(a), $x < 0$), the Hall term in Eq. (1) dynamically vanishes and *the already existing* ion toroidal rotation accounts (via the first term on the RHS of Eq. (1)) for a smaller radially outward electric field outside the separatrix (Fig. 1(b), $x<0$).

Further away from the edge, the θ-pinch driven magnetic field ($B_z>0$) proceeds to smoothly penetrate into the plasma ($\partial B_z/\partial x>0$, $x>0$). This results in reversing the sign of the Hall term in Eq. (1), which induces ion spin-up in the opposite direction. After passing the O-point ($B_z=0$) the magnetic field becomes negative ($B_z<0$), which makes the Hall-driven term in Eq. (1) positive again. At this point, however, the plasma dynamics become greatly affected by the electron pressure term and ion FLR effects. The velocity shear width is determined by the characteristic size of ion orbits around the magnetic O-point. In the core plasma, after a series of initial bouncing shock waves, the Hall-driven effect vanishes and the concomitant rotation is negligible.

As already seen above, simple theoretical considerations for the Hall-driven spin-up are confirmed by results from our hybrid simulations conducted with a novel asynchronous hybrid code, HYPERS [6] for initial ($t=0$) parameters typical of the modern field-reversed θ-pinches [4]: $n_e \approx 2\times10^{14}cm^{-3}$, $T_e=T_i\approx 5eV$, $B_z^{bias}=-0.7kG$ (the bias field), $B_{z,\max}^{coil}=4.3kG$ (the crow-bar forward field at the wall), $t_{1/4}\approx 12\mu s$ (the discharge quarter-period), $t_R\approx 2\mu s$ (the field reversal time). In simulations discussed in this paper we used a constant resistivity, $\eta/\mu_0=250m^2/s$. The resistive model was also used to account for the propagation of magnetic field in low-density plasma regions (where $n_e$ falls below a small cutoff).

A cylindrical wall with the radius, $R=40cm$ was represented with a staircase boundary in a rectangular computational domain covered with a uniform mesh of $N_x\times N_y=300\times300$ cells with equal mesh spacings in both dimensions: $\Delta_x=\Delta_y=\lambda_i/6$ ($\lambda_i=c/\omega_{pi}(t=0)\approx 1.6cm$ is the characteristic ion inertial length). This physical resolution is sufficient for hybrid simulations and results in a converged numerical solution.

In order to initiate the externally driven plasma implosion, a time-dependent external (θ-pinch) electric field, $E_\theta^{pinch}$ was assumed to penetrate into the plasma in a narrow skin layer with the radial width of order $\delta\approx\lambda_i$ according to the Faraday law:

$$\frac{1}{r}\frac{\partial}{\partial r} r E_\theta^{pinch} = -\frac{1}{c}\frac{\partial}{\partial t} B_z^{coil},$$

where $r$ is the radius and $B_z^{coil}$ is the external magnetic field known as a function of time.

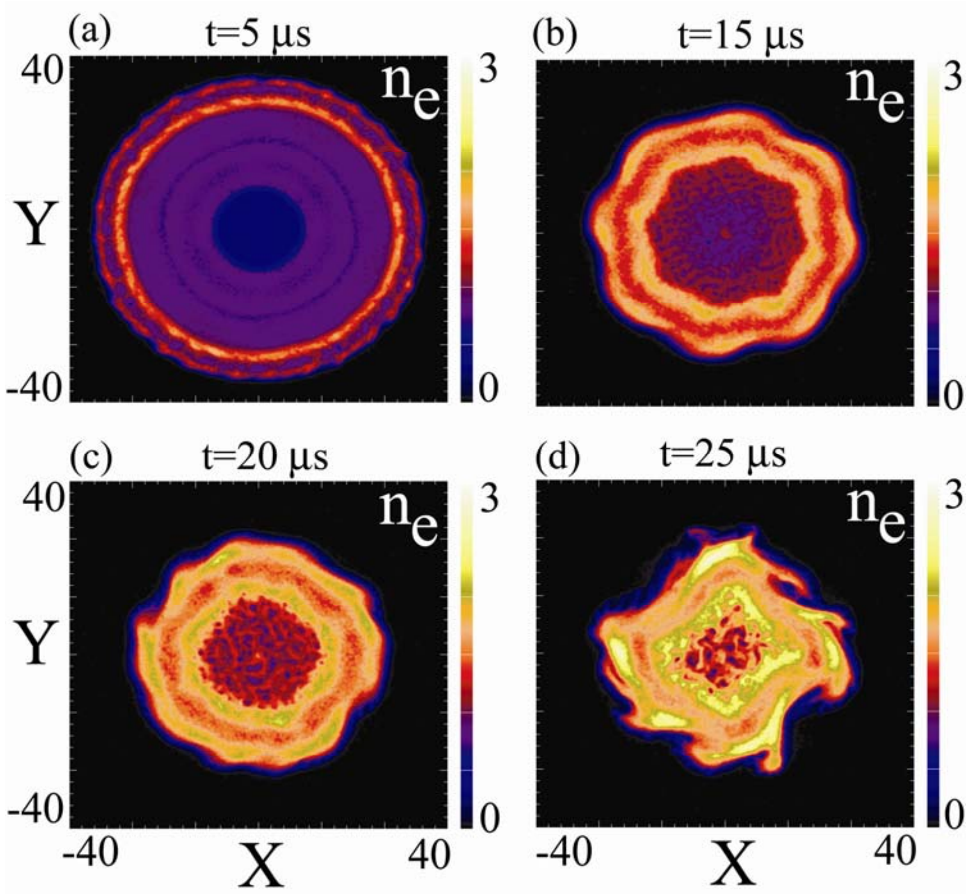


**FIG.2**. Evolution of plasma density in a hybrid simulation showing radial shocks and large-amplitude toroidal pertubations.

The temporal evolution of the plasma density in FRTD is shown in Fig. 2. The plasma, imploding under the influence of the coil-driven electric field, generates a series of compressional magnetosonic shock waves, which bounce between the outer edge of the plasma and the geometric axis (Fig. 2(a)).

As a result of the fast magnetic implosion the plasma edge undergoes the Raleigh-Taylor instability cascading into low-n toroidal (flute) modes, which persist along with the fast edge rotation (Fig.2(b-d)). These perturbations are frequently inferred in θ-pinch experiments [1]. Interestingly, similar plasma density structures were earlier observed in the hybrid simulation of a cylindrical plasma column expanding into a magnetized background plasma [7]. Realistic modeling of the dynamics of these modes, including their effect on global FRC plasma properties, requires further three-dimensional studies.

Note that in this Letter our primary goal is to demonstrate a new effect – the Hall-driven plasma rotation. Exploring concrete FRC physics is our next target, which will require following the long-time behavior of an FRTD plasma under realistic experimental conditions in three dimensions.

The future studies should also clarify the importance of the Hall-driven rotation compared to the short-ending mechanism studied in [4]. We note, however, that an appreciable plasma rotation outside the separatrix in MHD simulations [4] develops on the characteristic Alfvén time, $\tau_A = L_{1/2}/v_A \approx 22\mu s$ (where $L_{1/2} = 3m$ is the initial axial half-length). The effect demonstrated in our study is generated much faster on the implosion time scale, $\tau_H \leq 10\mu s$, with the magnitude of ion rotation velocity exceeding that in [3] by a significant factor of 3-5. This may provide a reasonable explanation why observations consistently indicate that the plasma edge layer rotates much faster than the core plasma [1].

### 3. Discrete-event simulation

The HYPERS code [6] used in this study notably differs from all conventional hybrid codes. HYPERS does not step variables synchronously in time but instead performs time integration by executing discrete events – asynchronous updates of individual particles and local fields, which are carried out as frequently as dictated by their physical time scales. The multi-scale nature of discrete-event simulation (DES) is illustrated in Figs. 3-4.

Fig. 3 shows distributions of particle timesteps (in units of $\omega_{pi}^{-1}(t=0)$) and particle numbers as functions of energy (in logarithmic scale). Ions get energized by local electric fields which arise at the fronts of compressional shocks. Accurate numerical pushing in space of these particles automatically requires much smaller timesteps. As a result, even though the number of energetic particles remains relatively small during simulation compared to the number of thermal bulk particles, conventional (synchronous time-stepping) hybrid codes either have to discard them altogether or decrease the global timestep for all particles *simultaneously*. In contrast, HYPERS treats all particles (with timesteps varying by more than an order of magnitude in these simulations) asynchronously without suffering noticeable performance penalty.

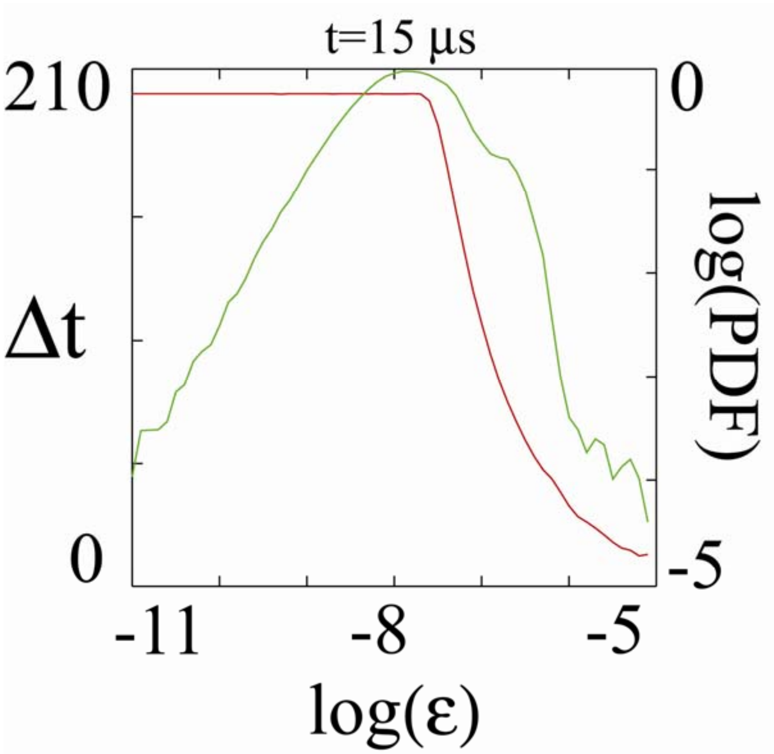


**Fig. 3.** Distributions of ion particle timesteps (red) and numbers (green) as functions of energy, ε normalized to the proton rest energy.

Fig. 4 demonstrates the similar temporal adaptivity of the HYPERS field solver, which adjusts local field update rates in accordance with local physical timescales. In our simulations these scales differ by more than two orders of magnitude, with the smallest timesteps originating in the narrow edge layer, where they are necessitated by the need to account for the proper electron (Hall) physics. To achieve a similar accuracy level, a conventional explicit hybrid code would have to reduce the global field timestep to the smallest value everywhere, which would increase the CPU runtime by one-two orders of magnitude compared to event-driven simulations. Moreover, automatic reduction of the global timestep in time-stepped codes may trigger numerical instabilities which do not arise in DES codes because event-driven changes to all variables are always bounded by physically limiting thresholds [6].

Self-adaptivity of DES was previously shown to result in stable, accurate and fast simulations of strongly inhomogeneous space plasmas [6]. End-to-end hybrid simulations of formation and sustainment schemes for modern FRCs [1] need to time-accurately treat multi-scale Hall and kinetic plasma effects in the presence of strong applied magnetic fields and vacuum regions over long axial distances (meters) and times (up to a fraction of millisecond). Therefore, temporal adaptivity and numerical robustness (stability) of HYPERS are essential for enabling simulations of realistic FRCs in the future.

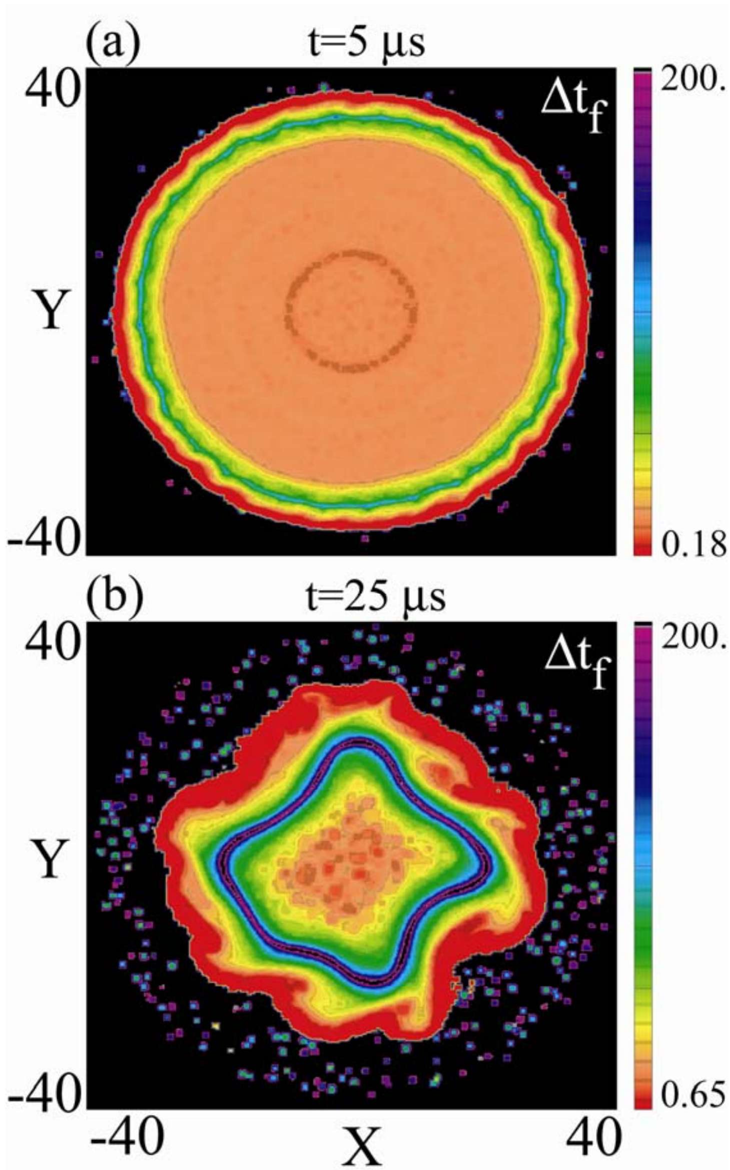


**Fig. 4.** Snapshots of local field timestep distributions at the beginning (a) and end (b) of a hybrid DES run. The noisy "halo" is indicative of a low-density plasma left over after compression.

**4. Conclusion**

Results from event-driven hybrid simulations and simple supporting theory presented in this Letter indicate that a fast plasma rotation may spontaneously occur at the plasma edge in early stages of FRTD in the absence of conventional end-shorting or particle loss mechanisms.

The sheared rotation rapidly develops in the ion diamagnetic direction, with its direction changing sign twice in a narrow radial layer located close to the magnetic null position. We show that this new type of plasma spin-up is caused by a radial electric field generated due to the Hall physics. In addition, the plasma is shown to develop flute modes that can significantly degrade the radial confinement – especially for fast magnetic compression rates. The impact of end shorting effects on θ-pinch dynamics will be studied in out next three-dimensional work.

Recently large-size hot FRCs have been produced by merging θ-pinch-formed, high-β, compact toroids [1]. A successful attempt was made to control radial electric field in the plasma sheath outside of the FRC separatrix with a plasma gun [8]. It was reported that a radially *inward* electric field, produced by the gun, effectively countered the usual FRC spin-up and resulted in better plasma confinement. Although these experimental findings may be related to other confinement issues, our theory does predict that the rapid plasma spin-up would be eliminated if a large outward electric field at the plasma edge were somehow neutralized.

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