Phase Transition of Light in Cavity QED Lattices

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Phys. Rev. Lett. **109**, 053601 — Published 3 August 2012

DOI: 10.1103/PhysRevLett.109.053601

On the phase transition of light in cavity QED lattices

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(Dated: May 4, 2012)

Systems of strongly interacting atoms and photons, that can be realized wiring up individual cavity QED systems into lattices, are perceived as a new platform for quantum simulation. While sharing important properties with other systems of interacting quantum particles here we argue that the nature of light-matter interaction gives rise to unique features with no analogs in condensed matter or atomic physics setups. By discussing the physics of a lattice model of delocalized photons coupled locally with two-level systems through the elementary light-matter interaction described by the Rabi model, we argue that the inclusion of counter rotating terms, so far neglected, is crucial to stabilize finite-density quantum phases of correlated photons out of the vacuum, with no need for an artificially engineered chemical potential. We show that the competition between photon delocalization and Rabi non-linearity drives the system across a novel Z_2 parity symmetry-breaking quantum criticality between two gapped phases which shares similarities with the Dicke transition of quantum optics and the Ising critical point of quantum magnetism. We discuss the phase diagram as well as the low-energy excitation spectrum and present analytic estimates for critical quantities.

PACS numbers: 42.50 -o, 42.50 Pq, 73.43.Nq, 05.30.Rt

Introduction - Interaction between light and matter is one of the most basic processes in nature and represents a cornerstone in our understanding of a broad range of physical phenomena. In the study of strongly correlated systems and collective phenomena, light has traditionally assumed the role of a spectroscopic probe. The increasing level of control over light-matter interactions with atomic and solid-state systems [1–3] has brought forth a new class of quantum many body systems where light and matter play equally important roles in emergent phenomena: photon lattices [4–15]. The basic building block of such systems is the elementary Cavity QED (CQED) system formed by a two-level system (TLS) interacting with a single mode of an electromagnetic resonator. When CQED systems are coupled to form a lattice, the interplay between photon blockade [17–19] and inter-cavity photon tunnelling leads to phenomenology akin to those of Hubbard models of massive bosons as realized e.g. by ultracold atoms in optical lattices [20]. The possibility of quantum phase transitions of light between Mott-like insulating and superfluid phases has stimulated a great deal of discussion recently [4–9, 11–14]. The excitement about these systems stems from their potential as dissipative quantum simulators that provide full access to individual sites through continuous weak measurements [16].

While sharing important features with conventional condensed matter or atomic physics setups, systems of strongly correlated photons have their own unique properties that ultimately derive from the nature of the fundamental light-matter interaction. As photons can disappear by interacting with the matter field, their number is not conserved but rather fixed by the condition of thermal equilibrium. To describe this situation for a photon gas in equilibrium with either photonic or dipolar bath – such as in a blackbody – one says that photons have zero chemical potential [21]. From the point of view of bosonic Hubbard models this has rather dramatic consequences, as one would then require an external non-equilibrium drive in order to engineer non-trivial quantum many body states other than the vacuum [22]. For a Lattice CQED system however, as we will show in this Letter, this is remarkably not so. If the vacuum Rabi frequency becomes comparable to the TLS transition frequency, a regime referred to as the "ultra-strong coupling regime", the lattice can spontaneously generate photons out of vacuum. A non-trivial ground state is then achieved in the ultra-strong coupling limit. By considering the original light-matter interaction Hamiltonian described by the Rabi model, we show that a lattice of CQED systems displays a novel Z_2 parity-breaking quantum phase transition where the two-level systems polarize to generate a ferroelectrically ordered state and the photons acquire a non vanishing expectation value due to coherent hopping. This novel quantum criticality, described by a delocalized super-radiant quantum critical point, shares some similarity with the Dicke phase transition of quantum optics and turns out to be in the universality class of the Ising model.

Single Resonator - The elementary light-matter interaction between a photonic mode of a resonator and a TLS is described by the Rabi model [1]

$$\mathcal{H}_R = \omega_r\, a^\dagger\, a + \omega_q \sigma^+\, \sigma^- + g\, x\, \sigma_x \tag{1}$$

where ω_r is the frequency of the resonator, ω_q the qubit transition frequency, g the light-matter coupling strength and $x = a + a^\dagger$. In addition, depending on the specific context, an extra term should be added to Eq. (1) where the field appears quadratically, $H_{A^2} = D\,(a+a^\dagger)^2$. We will discuss its implications at the end of the paper and first address the physics of the Rabi model (1) and its lattice extension. When the coupling g is sufficiently smaller than the frequencies ω_r, ω_q one can safely neglect processes where simultaneously atomic and photonic excitations are created, described by the counter-rotating terms $a^\dagger\,\sigma^+ + \sigma^-\,a$. In this so called rotating-wave approximation the Hamiltonian (1) reduces to the Jaynes-Cumming (JC) model, $H_{JC} = \omega_r\,a^\dagger\,a + \omega_q\sigma^+\,\sigma^- + g\left(a^\dagger\,\sigma^- + \sigma^+\,a\right)$ used widely in discussions of CQED physics. While appropriate in many relevant cases, recent implementations of circuit QED [23, 24] achieved coupling strengths g where the counter-rotating terms begin to show significant deviations from the expectations of the JC model [1, 25–31]. This is the so called "ultra-strong coupling" regime of parameters $g \sim \omega_r$. Although the physics of the Rabi model has been widely studied and well-understood [32–34] it is attracting renewed attention recently [35, 36]. Here we will examine the Rabi-Hubbard model as realized e.g. by a lattice of circuit QED cavities where each node is described by $\mathcal{H}_R$. As we discuss below, this system forms a viable platform for studying non-trivial strongly correlated phases of light. Experimental efforts to fabricate on-chip photonic lattices of circuit QED systems are currently underway [37]. Before we introduce the lattice, we consider the generalized Rabi model

$$\mathcal{H}_{gR}[a,a^\dagger] = \omega_r\,a^\dagger\,a + \omega_q\sigma^+\,\sigma^- + g\,\left(a^\dagger\,\sigma^- + \sigma^+\,a\right) + \\ +g'\,\left(a^\dagger\,\sigma^+ + \sigma^-\,a\right) \quad (2)$$

We would like to stress that we introduce this model to explore the role of counter-rotating terms in a controlled fashion. This model interpolates between the JC Hamiltonian for $g' = 0$ and the standard Rabi Hamiltonian for $g' = g$. In the following we will restrict ourselves to the resonant case $\omega_r = \omega_q = \omega_0$. For $g' = 0$ i.e. in the JC limit, the above model conserves the total number of excitations, $\mathcal{N} = a^\dagger\,a + \sigma^+\sigma^-$. The resulting continuous $U(1)$ symmetry allows an exact analytic solution of $\mathcal{H}_{gR}$ in terms of dressed states of photons and TLS excitations, the polaritons. The ground-state shows an interesting evolution upon increasing the coupling g/ω_0, with an infinite series of level crossings for $g_c(n) = \omega_0\,\left(\sqrt{n+1} + \sqrt{n}\right)$ where the number of excitations increases from n to $n+1$, resulting in a characteristic staircase structure (see Fig. 1).

For an arbitrarily small g', counter-rotating terms break the continuous $U(1)$ symmetry down to a discrete Z_2 group associated with parity $\mathcal{P} = e^{i\pi\mathcal{N}}$. Under this unitary operator the photon field a and the TLS operator σ_x transform respectively as $\mathcal{P}^\dagger a\mathcal{P} = -a$ and $\mathcal{P}^\dagger\sigma_x\mathcal{P} = -\sigma_x$, from which the invariance immediately follows, namely $[\mathcal{P}, \mathcal{H}_{gR}] = 0$. A direct consequence of the parity symmetry is that, while $\langle\,a\rangle = \langle\,\sigma_x\rangle = 0$ in the ground state of $\mathcal{H}_{gR}$ much as in the JC limit, the photon field in the Rabi ground state is squeezed, i.e. $\langle\,a^{2n}\rangle \neq 0$. While the discrete Z_2 symmetry prevent a full closed-form solution, the model (2) in the Rabi limit $g' = g$ has recently been shown to be nevertheless integrable [35]. Important features of this exact solution that will be relevant for our discussion below are that (i) no level crossing between states of different parities can occur as a function of g/ω_0 (note that $g' = g$), which in turn implies that the ground state of the Rabi model remains an even parity state for any g/ω_0, (ii) the ground state and the first excited state are quasi-degenerate in the deep ultra-strong coupling limit $g/\omega_0 \gg 1$.

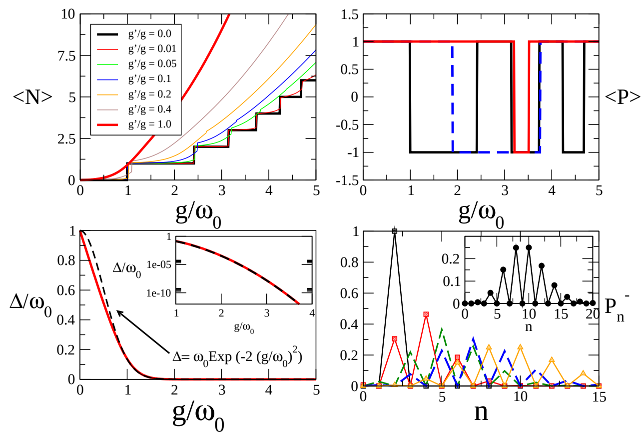

FIG. 1: Evolution of the single site generalized Rabi Model ground state properties upon increasing the strength of counter-rotating terms g'/g. Top panels: number of excitations and parity of the ground state as a function of g/ω_0 and for different g'/g. Bottom panels: (left) decay of level splitting Δ in the ultra-strong coupling regime and (right) probability of having n polaritons in the Rabi ground state (inset) and its evolution with $g'/g = 0.0, 0.25, 0.5, 0.75, 1.0$.

To get further insight into the structure of the Rabi ground state we numerically diagonalize the Hamiltonian (2). In Fig. 1 (top panels) we plot the number of excitations and the parity of the ground state as a function of g/ω_0 for different values of g'/g. Upon increasing the strength of counter-rotating terms the JC plateaux are gradually smeared out. Though the parity remains well-defined, the evolution with g' reveals multiple crossings between eigenstates switching the parity of the ground state, ultimately resulting in an even parity ground state when $g' = g$. We also plot (bottom panel, Fig. 1) the scaled level splitting Δ/ω_0 between the ground state and the first excited state, which vanishes as $\Delta \sim e^{-2(g/\omega_0)^2}$

for large g/ω in agreement with degenerate perturbation theory. Instead, the gap to the next energy level stays of order one (not shown) at large g/ω_0. Polaritonic dressed states are not anymore exact eigenstates for $g' \neq 0$ and turning on g' results in a smooth broadening and shift of the eigenstates of the generalized Rabi model when projected on the polaritonic eigenstates of the corresponding JC model, as shown in the right panel of Fig. 1.

Lattice model of interacting atoms and photons – We now come to the main subject of this Letter, which is the physics of the Rabi-Hubbard model, a model of itinerant photons hopping between neighboring resonators and interacting on-site with a TLS according to the local Hamiltonian (2). The full many-body Hamiltonian for this system reads

$$\mathcal{H} = -J \sum_{\langle \mathbf{R}\mathbf{R}' \rangle} a^\dagger_{\mathbf{R}}\, a_{\mathbf{R}'} + \sum_{\mathbf{R}} \mathcal{H}_{gR}[a_{\mathbf{R}}, \sigma^+_{\mathbf{R}}]. \qquad (3)$$

We stress here that, with an eye on possible future experiments on a circuit QED architecture, we do not include any chemical potential to tune the density of excitations in the ground state. The goal here is to exploit the spontaneous polarization of the Rabi vacuum that emerges when the system enters the ultra-strong coupling regime.

We start considering first the $g' = 0$ JC limit and study the phase diagram in the $g - J$ plane [42]. In the absence of hopping, the ground state is an exact dressed state of polaritons. A gapped and incompressible Mott Insulating (MI) phase of polaritons survives at finite hopping until a critical value of J is reached. The phase boundary $J_c(g)$ features characteristic Mott lobes (see Fig. 3), a legacy of the level crossings of the single site JC model discussed above. For hopping strengths larger than J_c the system is in a superfluid (SF) compressible phase with gapless excitations associated to phase fluctuations of the $U(1)$ order parameter. It is now well-established that the JC lattice model is in the same universality class as the Bose-Hubbard model [8, 11–13, 38, 39]. Crucially for our purpose here, the experimental realization of this MI-SF quantum phase transition requires an external driving or a suitably engineered chemical potential in order to counter-balance photon losses into the vacuum.

We now argue that the inclusion of counter-rotating terms in Eq. (3) has a dramatic effect on the above physics. The roots of this can be traced back to the single resonator limit. As discussed above, the counter-rotating terms leave the system with a discrete Z_2 symmetry associated to parity. Photon hopping in (3) can trigger a spontaneous breaking of this parity symmetry above some critical coupling $J_c(g)$, toward a phase where both $\langle a_{\mathbf{R}} \rangle \neq 0$ and $\langle \sigma^x_{\mathbf{R}} \rangle \neq 0$. As the broken symmetry is discrete, this quantum phase transition is fundamentally different from the JC one. Indeed it can be seen as a delocalized super-radiant quantum critical point reminiscent of the multi-mode Dicke transition of quantum optics. In order to see that a non-zero J favors ordering, we start from the full Hamiltonian (3) and notice that photons can be integrated out exactly in an imaginary-time action formalism to obtain an effective model for the TLSs only. The result of this calculation [43] reveal that photon mediates an effective Ising-like coupling between TLS which is retarded and long-range, $J^{eff}_{\mathbf{R}-\mathbf{R}'}(\tau) = -g^2/2\, \langle T_\tau x_{\mathbf{R}}(\tau)\, x_{\mathbf{R}'}(0) \rangle$. The scaling with g implies that at sufficiently large g/ω_0 and for finite J, a ferromagnetically ordered Ising phase emerges with $\langle \sigma^x_{\mathbf{R}} \rangle \neq 0$. Further insight into this emerging Z_2 degree of freedom are obtained from the single site limit. As we discussed, at large g/ω_0 the ground state and the first excited state are almost degenerate, with an exponentially small splitting and a gap to the next level which stays of order one. These two states $|\pm\rangle$ have opposite parity and can be thought as eigenstates of an effective pseudospin 1/2 degree of freedom, $\Sigma^z_{\mathbf{R}}$. In addition we notice that the photon operator $a_{\mathbf{R}}$ does not couple states with same parity. Its expression in the restricted $|\pm\rangle$ subspace reads $a_{\mathbf{R}} \to \beta\, \Sigma^+_{\mathbf{R}} + \gamma\, \Sigma^-_{\mathbf{R}}$, where the dependence of the coefficients β, γ on the coupling g can be obtained numerically. In the limit $g/\omega_0 \gg 1$ one can analytically show that a linear scaling holds $\beta = \gamma \sim g/\omega_0$. Armed with these results we can rewrite the Rabi-Hubbard Hamiltonian as

$$\mathcal{H}_{eff} = -\sum_{\langle \mathbf{R}\mathbf{R}' \rangle} J^x\, \Sigma^x_{\mathbf{R}}\, \Sigma^x_{\mathbf{R}'} + J^y\, \Sigma^y_{\mathbf{R}}\, \Sigma^y_{\mathbf{R}'} + \frac{\Delta}{2} \sum_{\mathbf{R}} \Sigma^z_{\mathbf{R}} \, (4)$$

where the couplings $J^{x,y} = J(\gamma \pm \beta)^2/2$ depend on g as shown in Fig. 2. This effective model describes a pseudospin anisotropic XY model in a longitudinal magnetic field $\Delta/2$, which is known to display a quantum phase transition toward a Z_2 broken symmetry phase which is in the Ising universality class for any finite anisotropy, $J^x \neq J^y$ [41]. The effective psuedospin description highlights once more the differences between the Rabi and the JC case. Indeed here both the disordered and the ordered phases are gapped except right at the critical point where the gap is expected to vanish as a power-law.

Mean Field Phase Diagram and Fluctuations - We now use Gutzwiller mean field theory to confirm the general picture we have drawn for the transition. By decoupling the hopping term in (3) we reduce the original lattice problem to an effective single site problem, $H_{eff}[\psi] = H_{loc} - \psi \left(a^\dagger + a\right)$ in a self-consistent field $\psi = Z\, J \langle a \rangle_\psi$. In addition, by expanding the energy to second order in ψ we can get the mean field phase boundary $J_c(g)$, above which a parity symmetry broken phase emerges with both $\langle a_{\mathbf{R}} \rangle \neq 0$ and $\langle \sigma^x_{\mathbf{R}} \rangle \neq 0$. In Fig. 3 we plot the mean field phase boundary in the J, g plane for different values of g'/g from the JC to the Rabi limit. The Mott lobes for $g' = 0$ are gradually suppressed as the ratio g'/g is increased. For intermediate values a residual lobe structure remains, which reflects the level crossings already discussed in the single-site problem. However we stress that no Mott insulator

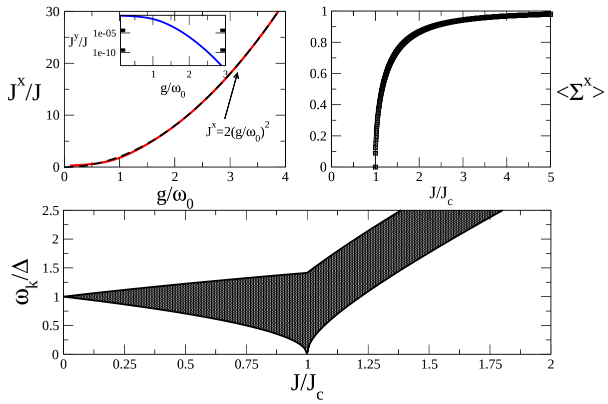

FIG. 2: Top panels: (Left) Scaled ferromagnetic coupling $J^{x,y}/J$ of the effective spin model as a function of g/ω_0. While $J^x \sim (g/\omega_0)^2$ at ultra-strong coupling, the coupling along y is vanishingly small. (Right) Mean-field order parameter for the effective spin model. Bottom panel: Band of spin-wave excitations above the ground-state.

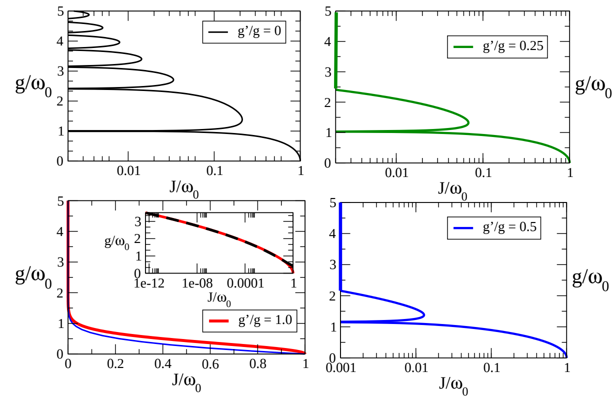

FIG. 3: Mean field phase diagram for the generalize Rabi lattice model for different values of g'/g. For the Rabi case, $g' = g$, we compare with the mean field phase diagram for the effective spin model (left bottom panel). In the inset, we show the decay of the critical coupling J_c at ultra-strong coupling $J_c = \omega_0^3\, e^{-2(g/\omega_0)^2}/4g^2$ [44]

exists for any finite g'. Further insight on the transition can be gained from the effective spin Hamiltonian (4). A linearized fluctuation analysis gives a critical coupling $J_c = \Delta/\left(\beta+\gamma\right)^2 \sim \omega_0^3\, e^{-2(g/\omega_0)^2}/4g^2$ which correctly matches the numerical results in the large g/ω_0 regime (see figure 3). In addition the effective Hamiltonian also gives access to the spectrum of low-lying excitations $\omega_{\mathbf{k}}$, plotted in Fig. 2, that as expected is gapped on both sides and vanishes in a power-law fashion at the transition, $E_g \sim |J - J_c|^{1/2}$. The low-energy spectrum is gapless at the critical point, with a linear dispersion $\omega_{\mathbf{k}} = c\,|\mathbf{k}|$.

Discussion - The physical picture we have drawn from our analysis of the lattice Rabi model reveals a striking feature of hybrid systems made of atoms and photons. Due to the nature of the fundamental light-matter interaction, which allows non-trivial vacuum fluctuations, no external driving forces or artificially engineered chemical potentials are in principle required to stabilize finite density quantum phases of correlated atoms and photons. Rather it is the coupling between matter and light that will trigger this non-trivial vacuum polarization. An important question for a possible experimental realization e.g. on a circuit QED platform concerns the stability of the above picture against photon leakage that is an inherent feature of any quantum optical system. Physical intuition would suggest that at least for a small coupling to a low-temperature photonic bath the ordered phase would be protected by the discrete nature of the Z_2 symmetry. However an in-depth study of the phase diagram and a full understanding of quantum criticality in the open system limit is an important fundamental problem that we leave to future investigation.

We now briefly discuss the role of the A^2 term $H_{A^2} = D\,(a+a^\dagger)^2$ in the picture that emerges from the above discussion. Recently it has been argued that such a term, generally assumed to be small, can become relevant in certain cavity QED realizations of the *single-mode* Dicke Model, where an ensemble of many TLSs is coupled to a single mode of a cavity. Indeed when the coupling D scales sufficiently fast with light matter interaction g, $D > g^2/\omega_q$, the super-radiant critical point disappears. While this condition is realized in cavity QED setups with real atoms coupled via electric dipole and results in so called no-go theorems, the situation with some circuit QED implementations, where TLSs couple capacitively to the resonator, is currently subject of scientific debate [45, 46]. We note that in contrast to the single-mode Dicke model, in our system the Z_2 parity symmetry breaking emerges from a non-trivial competition between hopping delocalization of photons and local light-matter interactions. As a result the critical boundary $J_c(g)$ can be accessed by increasing the hopping strength J, at fixed (and even moderate) light-matter coupling. While the inclusion of H_{A^2} in our lattice Hamiltonian may change quantitatively the shape of the phase boundary [43], especially in the ultrastrong coupling regime, it is rather an issue of the specific implementation that will determine the ideal architecture to realize the Rabi phase transition in an experimental system. Finally we note that circuit QED implementations can be engineered where the A^2 term is irrelevant. This is the case, for example, of flux qubits inductively-coupled to resonators [47], a setup that in principle [48] is ideally suited to access the ultra-strong coupling regime and that has been recently explored experimentally in single circuit QED units [23].

Conclusions - In this work we have explored the physics of itinerant photons hopping between neighboring resonators of a lattice of CQED systems. We have studied its equilibrium phase diagram as a function of the atom-photon coupling g and shown that this system displays a novel parity symmetry breaking quantum phase transition, belonging to the Z_2 Ising universality class, between two gapped phases. Simultaneously, the photonic degrees of freedom acquire a non-vanishing expectation value, displaying a delocalized superradiant phase above a critical hopping.

Acknowledgements - We thank David Huse for valuable and stimulating discussions. This work was supported by the National Science Foundation through the Princeton Center for Complex Materials under grant no. DMR-0819860 and by the Swiss NSF through grant no. PP00P2-123519/1.

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[43] See Supplementary Material.
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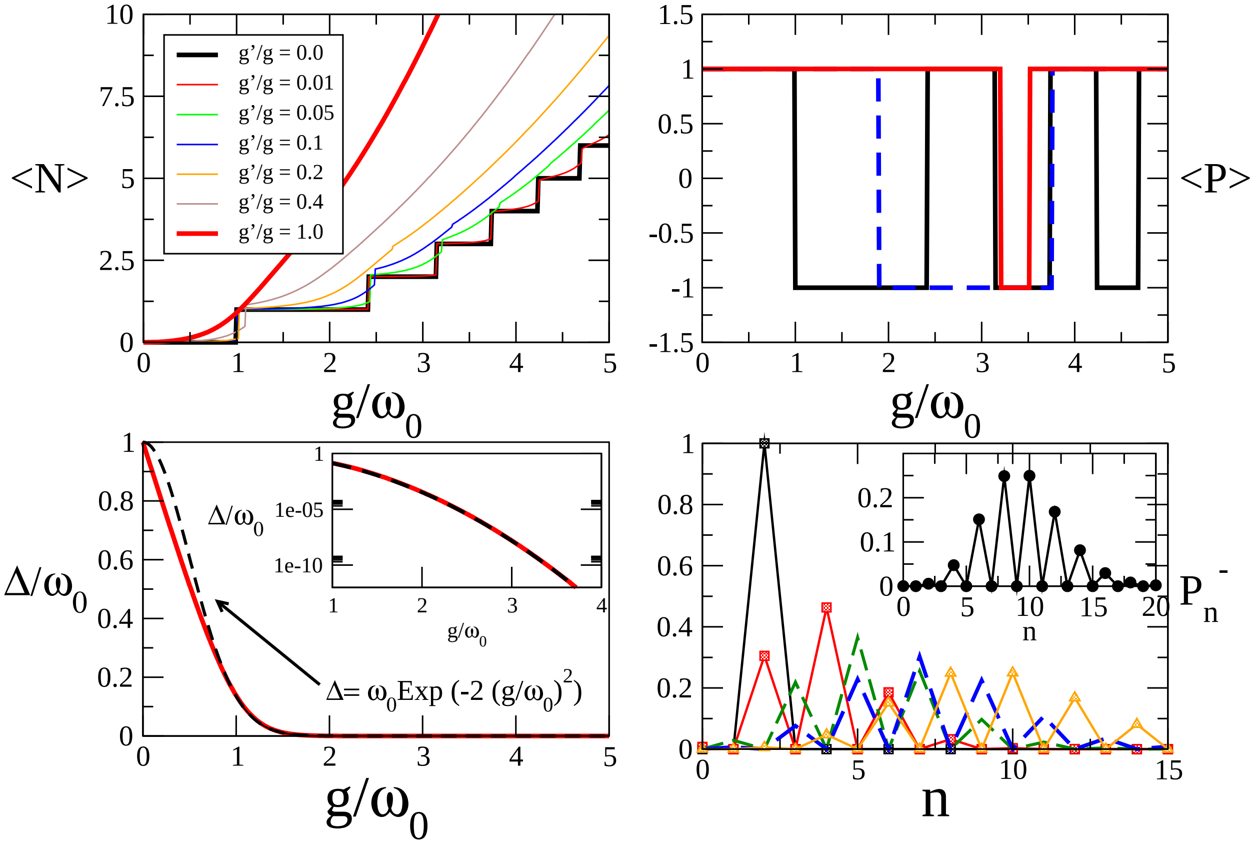

Figure 1 LP12649 03MAY12

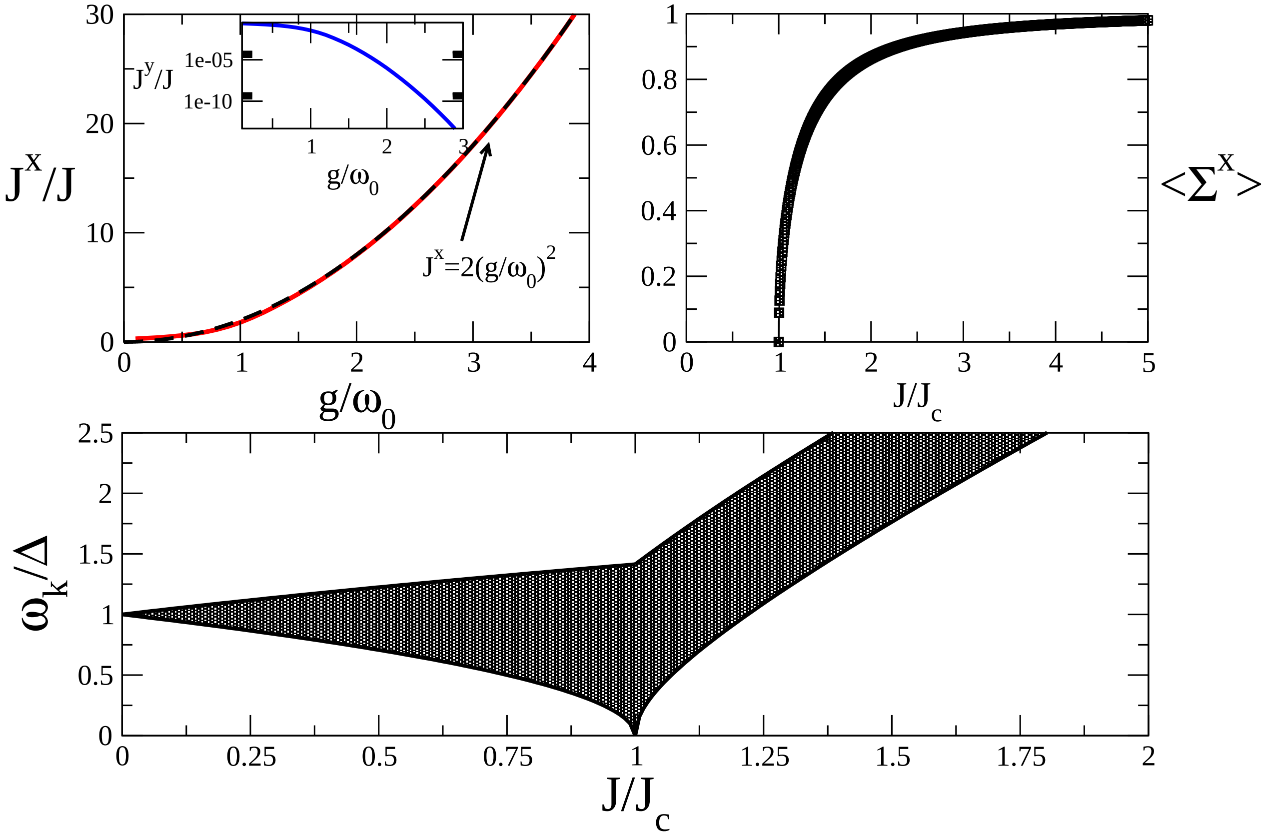

Figure 2 LP12649 03MAY12

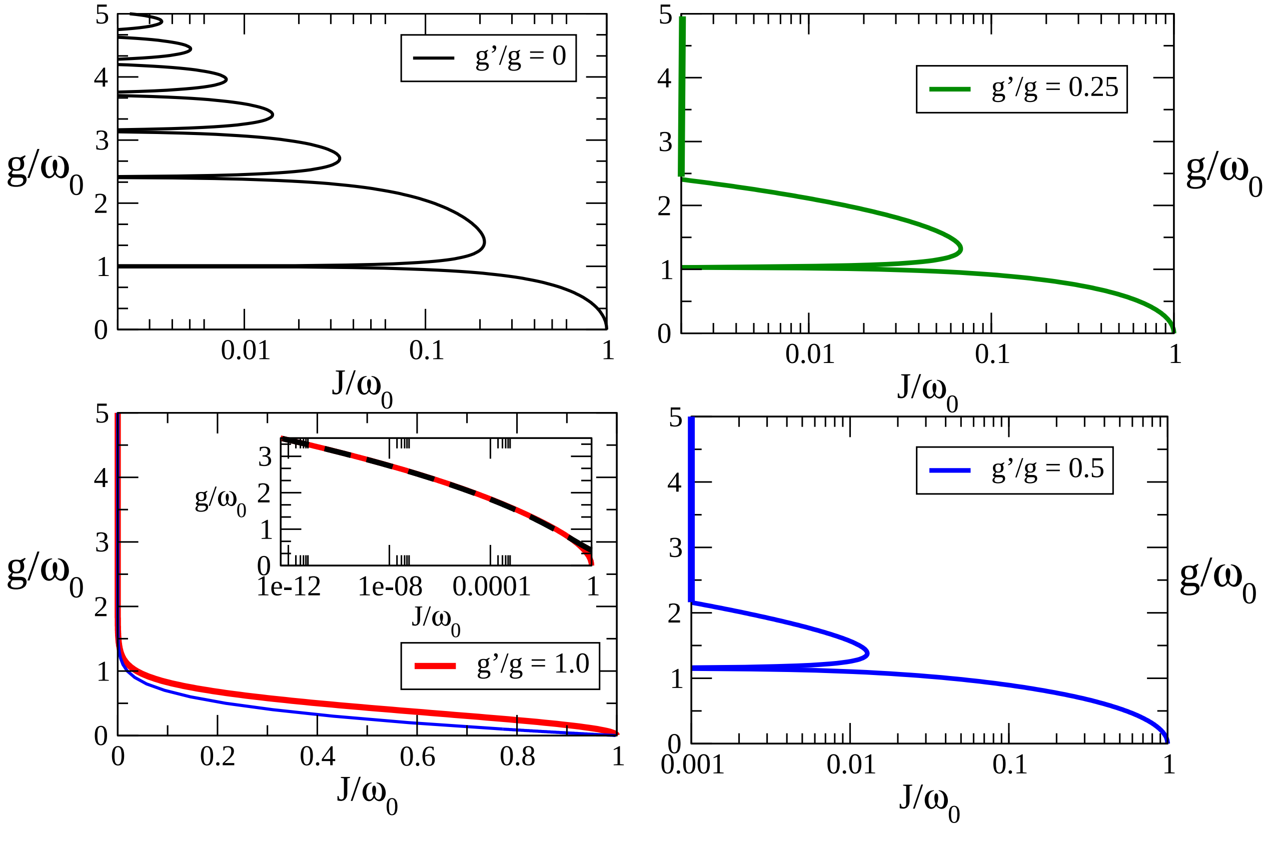

Figure 3 LP12649 03MAY12