Proposal for an Observational Test of the Vainshtein Mechanism

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An observational test of the Vainshtein mechanism

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Modified gravity theories capable of genuine self-acceleration typically invoke a galileon scalar which mediates a long range force, but is screened by the Vainshtein mechanism on small scales. In such theories, non-relativistic stars carry the full scalar charge (proportional to their mass), while black holes carry none. Thus, for a galaxy free-falling in some external gravitational field, its central massive black hole is expected to lag behind the stars. To look for this effect, and to distinguish it from other astrophysical effects, one can correlate the gravitational pull from the surrounding structure with the offset between the stellar center and the black hole. The expected offset depends on the central density of the galaxy, and ranges up to ~ 0.1 kpc for small galaxies. The observed offset in M87 cannot be explained by this effect unless the scalar force is significantly stronger than gravity. We also discuss the systematic offset of compact objects from the galactic plane as another possible signature.

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There has been a lot of interest in theories of modified gravity that might explain the observed accelerated expansion of the universe [1, 2]. Theories capable of genuine self-acceleration – without invoking vacuum energy in the Einstein frame [3] – are especially interesting, and they generally involve introducing a scalar (φ) that respects the galileon symmetry $\varphi \to \varphi + b + c_\mu x^\mu$, where b is a constant and c_μ is a constant vector [4]. This scalar mediates a long-ranged force, the so-called fifth force in addition to the usual gravitational force between objects. Thanks to the Vainshtein mechanism [5], the scalar is screened on small scales, so that solar system constraints are satisfied. To see how it works, let us illustrate with the simplest galileon model, inspired by DGP [6, 7]; the equation for the scalar (in Einstein frame) is [1]:

$$\Box\varphi + \frac{2}{3m^2}\left[(\Box\varphi)^2 - \partial_\mu\partial_\nu\varphi\partial^\mu\partial^\nu\varphi\right] = -8\pi\alpha G\, T_\mu{}^\mu\,, \quad (1)$$

where φ is related to the standard notation π for the galileon by $\pi = \alpha\varphi$, and $T_\mu{}^\mu$ is the trace of the matter energy-momentum, which for our purposes can be equated with (negative of) the matter density ρ, assuming it is non-relativistic. The constant α quantifies the scalar-matter coupling, and is generically of order unity, i.e. of gravitational strength. For instance, it takes the value $1/\sqrt{6}$ in massive gravity models [8]. The mass scale m is generally of the order of the Hubble constant today $m \sim H_0$. We are interested in solutions of this equation in the quasi-static limit, meaning time derivatives can be ignored and $\Box \to \nabla^2$. On large scales, the linear term $\nabla^2\varphi$ dominates; Eq. (1) resembles the Poisson equation, with φ playing the role of the gravitational potential. A localized source ρ yields a profile φ that scales inversely with distance r. However, as one approaches the source, the interaction term (second term on the l.h.s.) dominates, and simple power counting reveals $\varphi \propto \sqrt{r}$. Thus, at small distances, φ is screened relative to the normal $1/r$ gravitational potential. The transition scale is known as the Vainshtein radius, and is roughly given by $(GM/m^2)^{1/3}$ for spherically symmetric configurations, where M is the mass of the source. For instance, the entire solar system fits within the Vainshtein radius of the Sun, about 0.1 kpc, greatly suppressing the scalar force sourced by the Sun. What makes the galileon model attractive from this viewpoint, is that it is the same nonlinear interaction that is responsible both for Vainshtein screening, and for self-acceleration [4].

An important property of Eq. (1) is that it can be rewritten in the form $\partial_\mu J^\mu = -T_\mu{}^\mu$, where J^μ is a nonlinear function of derivatives of φ [4]. One can thus define a scalar "charge" $Q = -\int d^3x\, T_\mu{}^\mu = \int d^3x\, \rho$, which is none other than the mass M, aside from an exception described below [2]. As shown in [9], the scalar charge also quantifies the response of an object to an external field, i.e. an external gravitational $\Phi_{\rm ext}$ + scalar $\varphi_{\rm ext}$ field exerts a net force of $M\ddot{\vec{x}} = -M\vec{\nabla}\Phi_{\rm ext} - \alpha Q\vec{\nabla}\varphi_{\rm ext}$ on an object of mass M and charge Q. It is worth emphasizing that the Vainshtein mechanism does not suppress the scalar charge Q at all – indeed, the fact that $Q = M$ enforces the equivalence principle, i.e. making the motion

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[1] Two additional galileon symmetric interaction terms can be written down in the equation of motion: $(\Box\varphi)^3 - 3\Box\varphi(\partial_\mu\partial_\nu\varphi)^2 + 2(\partial_\mu\partial_\nu\varphi)^3$ and $(\Box\varphi)^4 - 6(\Box\varphi)^2(\partial_\mu\partial_\nu\varphi)^2 + 8\Box\varphi(\partial_\mu\partial_\nu\varphi)^3 + 3[(\partial_\mu\partial_\nu\varphi)^2]^2 - 6(\partial_\mu\partial_\nu\varphi)^4$. All results in this paper apply in the presence of any combination of the galileon terms.

[2] One might wonder if $-\int d^3x T_\mu{}^\mu = \int d^3x\,\rho$ is too strong an assumption, since an apparently non-relativistic object might have relativistic components, e.g. gluons in protons. It turns out that as long as the object in question is stationary, and has a finite extent, $-\int d^3x T_\mu{}^\mu = -\int d^3x T_0{}^0 = \int d^3x\,\rho$ holds, by virtue of a tensorial virial theorem [10]. For further discussions, including the renormalization of Q by quantum effects, see [10, 11].

of an object independent of its mass. What the Vainshtein mechanism suppresses is the external scalar field $\varphi_{\rm ext}$ that an object senses when it is close to the source of that field.

The one exception to $Q = M$ is compact objects, such as black holes. These are objects whose mass receives a significant contribution from the gravitational binding energy, a contribution that is not included in the scalar charge Q. Compact objects such as neutron stars thus have $Q/M < 1$, with black holes as the extreme limit where $Q/M = 0$. This is consistent with the notion of black holes having no hair, more specifically no galileon hair which we prove in a separate paper [12] (see also [13, 14]) [3]. The fact that compact objects have a suppressed scalar charge is not new [15, 16]. What is new with the advent of recent modified gravity models is the screening mechanism, which revives scalar-tensor theories that are otherwise already ruled out by solar system tests. Notice that unlike mass or electric charge, our scalar charge Q does not obey a conservation law. It is thus consistent for an object with $Q/M = 1$ to undergo gravitational collapse and end up with $Q/M = 0$.

Now, recall that Q also quantifies the response of an object to an external scalar field [9]. Thus, under some external $\Phi_{\rm ext}$ and $\varphi_{\rm ext}$, fields a normal non-compact star (as well as dark matter particles) would fall according to $\ddot{\vec{x}} = -\vec{\nabla}\Phi_{\rm ext} - \alpha\vec{\nabla}\varphi_{\rm ext}$, while a black hole would fall according to $\ddot{\vec{x}} = -\vec{\nabla}\Phi_{\rm ext}$, insensitive to the scalar force. The challenge is to find situations where the scalar $\varphi_{\rm ext}$ is not already suppressed by the Vainshtein mechanism. Since both black holes and stars typically reside within galaxies whose Vainshtein radii greatly exceed their sizes, it would seem hopeless to observe the purported difference in the rate of fall between a normal star and a black hole. The galileon symmetry helps save the day. The symmetry tells us that given any solution to the scalar Eq. (1), one can always add a linear gradient, i.e. $\varphi_{\rm ext}$ with a constant $\vec{\nabla}\varphi_{\rm ext}$, and obtain another solution. For any given object, whether such a linear gradient is present and how large it is, depends on the boundary conditions. For a galaxy, the boundary conditions are supplied by the surrounding large scale structure. Interestingly, as is recently demonstrated in a series of numerical simulations [17–20], the galileon scalar indeed obeys linear dynamics on sufficiently large scales ($\gtrsim 10$ Mpc), meaning the large scale structure generates a galileon field that is unsuppressed by the Vainshtein mechanism [4]. The large-scale-structure-generated scalar field has a long wavelength, and can be approximated as a linear gradient on the scale of a galaxy. This linear gradient penetrates the Vainshtein zone of the galaxy, and can act unsuppressed on the galaxy and its constituents. In other words, the galaxy falls according to this unsuppressed scalar $\varphi_{\rm ext}$ induced by large scale structure. So do its constituent dark matter, non-compact stars, *but not its compact objects.* The most readily observable compact object is the central massive black hole if there is nuclear activity. The central black hole, lacking a scalar charge, does not respond to the scalar force, while the stars (and dark matter particles) do. The net effect is that the black hole will lag behind the stars in their overall large-scale-structure-induced motion. In other words, the black hole will be offset from the center of the galaxy, or more precisely, from the minimum of the galactic gravitational potential. The non-zero offset means there's an extra (purely gravitational, not scalar) tug on the black hole from the central region of the galaxy. This suffices to compensate for the lack of a scalar force on the black hole, and keep the black hole and stars in equilibrium, moving in tandem. One can estimate the size of the offset r by equating the extra scalar acceleration sensed by the stars $\alpha|\vec{\nabla}\varphi_{\rm ext}|$ with the extra gravitational tug on the black hole $GM_{\rm gal}(< r)/r^2$, where $M_{\rm gal}(< r)$ is the mass enclosed within radius r of the galaxy. We find a displacement

$$r = 0.1\,{\rm kpc}\left(\frac{2\alpha^2}{1}\right)\left(\frac{|\vec{\nabla}\Phi_{\rm ext}|}{20({\rm km/s})^2/{\rm kpc}}\right)\left(\frac{0.01{\rm M}_\odot{\rm pc}^{-3}}{\rho_0}\right). \quad (2)$$

Here, we estimate $\varphi_{\rm ext}$ by $2\alpha\Phi_{\rm ext}$, since the linear scalar $\varphi_{\rm ext}$ satisfies the same Poisson equation for the gravitational potential $\Phi_{\rm ext}$, but with the source term scaled up by 2α (see Eq. [1]). The typical gravitational acceleration $|\vec{\nabla}\Phi_{\rm ext}|$ can be estimated by the typical peculiar motion multiplied by Hubble: $300\,{\rm km/s} \times 70\,{\rm km/s/Mpc} \sim 20({\rm km/s})^2/{\rm kpc}$. A more careful calculation of the rms $|\vec{\nabla}\Phi_{\rm ext}|$ using the observed matter power spectrum gives a number fairly close to this (e.g. [21]). It should be kept in mind however that $|\vec{\nabla}\Phi_{\rm ext}|$ is a stochastic quantity, and its value depends on environment. The central density of $\rho_0 \sim 0.01{\rm M}_\odot/{\rm pc}^3$ is appropriate for dwarf or low surface brightness galaxies, where the effect is the largest. We provide explicit scaling with the relevant parameters in Eq. (2) so that one can easily extrapolate to other values.

[3] If one thinks of black holes as vacuum solutions, having no scalar charge is certainly *a* solution, but the point of no-hair theorem is to show that it is in fact the only solution, and thus the collapse of an actual star would presumably lead to such a configuration. We should stress that our proof in [12] concerns only spherically symmetric black holes. We take it as suggestive that rotating black holes likely share the same no-galileon-hair property, but it remains to be proven. In any case, the argument for a suppressed Q/M for compact objects is quite robust.

[4] In other words, while the Vainshtein mechanism does operate on small scales, the Vainshtein zones of nonlinear objects do not percolate the universe. Note the Vainshtein radius, estimated from an isolated spherically symmetric object, can be a misleading concept when applied to more complex situations. The tensor structure of Eq. (1) is such that screening works very differently in non-spherically symmetric situations.

In modeling the central region of a galaxy, several issues should be kept in mind. First of all, the above estimate assumes the density profile is approximately flat close to the center. For fixed density at the core radius, a steep profile would imply a small offset. Fortunately, low central density galaxies where the offset is the largest also have fairly flat profiles e.g. [22]. Second, the relevant central density is the one averaged outside the black hole's sphere of influence. Materials within the sphere of influence would simply move with the black hole. It is the materials outside that are important for determining the offset. Thus, Bahcall-Wolf type cusps are not relevant for our considerations [23].

Galaxies for which both the stars and the central massive black hole are readily observable are those with a low level nuclear activity, namely Seyferts. Our estimate in Eq. (2) suggests that the offset would be observable preferentially in small galaxies. Depending on the distance (up to say tens of Mpc), galaxies with a central density in the range $\sim 0.003 - 0.03\,\mathrm{M_\odot/pc^3}$, corresponding roughly to rotational velocities around $\sim 30 - 120$ km/s [22], should have a measurable offset. Until recently, such small Seyferts have not been well explored observationally. A classic case of a dwarf galaxy containing an active nucleus is NGC 4395 [24]. A good size sample (~ 30) of small Seyferts was recently reported by [25]. [5]

An interesting question is how fast the black hole moves relative to the stars, on its way to the equilibrium (offset) position. Using the same set of parameters displayed in Eq. (2), we find a fairly small velocity of ~ 2 km/s. At such a velocity, dynamical friction is unimportant. The time it takes for the black hole to traverse the requisite distance is about 10^8 years. This is fairly close to some estimates of the nuclear activity time-scale [26], though the conditions for nuclear activity are rather uncertain.

M87 is an interesting case, where its massive black hole is known to be offset from the (bulge) stellar center by a projected distance of ~ 7 pc [27]. Its central density is about $\rho_0 \sim 20\,\mathrm{M_\odot/pc^3}$. For our effect to reproduce such an offset thus requires $\alpha \sim 8$, assuming the external gravitational acceleration on M87 is typical. This is a scalar-matter coupling that is quite a bit stronger than gravitational. The more plausible explanations for the observed offset are: acceleration by an asymmetric jet, and gravitational wave recoil from a merger [27]. Two other explanations considered are: the active black hole being a member of a binary, and Brownian motion. The former can be constrained by the lack of a jet precession, and can also be tested by monitoring the system over time. The latter is a small effect, giving an offset < 0.1 pc in the case of M87 [27].

These alternative mechanisms raise a practical question: in the event one observes a black hole offset in a lower central density galaxy, consistent with $\alpha \sim 1$, how does one disentangle the modified gravity effect from other astrophysical effects? One can exploit a distinguishing feature of the modified gravity effect, that is, the offset should be correlated with the gravitational pull of the surrounding large scale structure, in both its direction and strength. For instance, galaxies are expected to stream out of voids, their resident black holes should lag behind the stars in that streaming motion. Voids are especially interesting places to look because the scalar field is expected to be unscreened there. A rough argument goes as follows: Eq. (1) can be rewritten schematically in the form: $H_0^{-2}\partial^2\varphi + (H_0^{-2}\partial^2\varphi)^2 \sim \alpha\rho/\bar{\rho}$. Thus, voids where $\rho/\bar{\rho} < 1/\alpha$ are natural places where one can consistently ignore the nonlinear term compared to the linear term on the left. However, we expect large scale structure to source a linear (i.e. unsuppressed) scalar even away from voids. It would be very useful to map out the precise gravitational and scalar fields in our neighborhood, using fairly detailed knowledge of the mass distribution of the local universe [28]. The two large-scale-structure-generated fields $\Phi_{\rm ext}$ and $\varphi_{\rm ext}$ are expected to roughly align, but an accurate map would aid in isolating the modified gravity effect from other astrophysical effects. Note also that typical astrophysical effects produce a *velocity* offset ($\sim 10 - 1000$ km/s) that is quite a bit larger than what modified gravity predicts.[6]

It is worth emphasizing that this offset effect – namely the lagging of compact objects behind stars in the overall streaming motion of the host galaxy – is not confined to the central massive black hole. Any compact objects, regardless of its mass, will display this effect, though the effect is larger for a black hole than a neutron star i.e. the offset is expected to scale as $2GM/R$, where R is the radius of the object. The advantage of the central massive black hole is that it is readily observable even in a distant galaxy, provided it is active. For other less massive compact objects, the best bet is to look within our own galaxy. One possible signature is to see if compact objects are systematically offset from the galactic plane (defined by the stars), in the opposite direction of our galaxy's streaming motion. Using numbers from Eq. (2), and adopting the solar neighborhood value of $\rho_0 \sim 0.18\,\mathrm{M_\odot/\,pc^3}$, we estimate the offset to be about 2 pc for black holes. It would be useful to compute this offset more carefully by calculating the precise large-scale-structure-generated scalar force on the Milky Way.

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[5] We thank Jules Halpern for pointing out these cases to us.

[6] An exception is Brownian motion which produces small velocity perturbations to the massive black hole: $\sim v_{\rm star}(M_{\rm star}/M_{\rm b.h.})^{1/2}$. Using the relation mass of black hole $M_{\rm b.h.} \sim 10^{8.12}\,\mathrm{M_\odot}(v_{\rm star}/200\,\mathrm{km/s})^{4.24}$ [29], the smallest galaxies ($v_{\rm star} \sim 30$ km/s) have the largest velocity perturbation of ~ 0.2 km/s. For such galaxies, the *spatial* offset is $\sim r_{\rm core}(M_{\rm star}/M_{\rm b.h.})^{1/2} \sim 0.03$ kpc [27], thus comparable to the modified gravity effect. Hence, it is important to use the correlation with large scale structure as a discriminant.

A natural question is: should we expect the same offset effect for other screening mechanisms? Theories that screen by means of scalar self-interactions of the potential type, such as the chameleon [30] or the symmetron [31], operate very differently from those that screen by derivative interactions, i.e. via the Vainshtein effect. Indeed, the scalar force on/from the Sun is screened in non-Vainshtein theories by virtue of the Sun's suppressed Q/M. In chameleon and symmetron theories, objects with a gravitational potential similar to the Sun's ($\sim 10^{-6}$), or deeper, have $Q/M \sim 0$. Thus, main sequence stars and black holes fall at the same rate, and one expects no offset between the two. However, dark matter and stars can fall differently if the host galaxy has a sufficiently shallow gravitational potential [9]. This can lead to an interesting warping of the galactic disk, pointed out by Jain and VanderPlas [32]. For other observational tests of chameleon/symmetron screening, see for instance [9, 30, 31, 33–36].

In summary, we propose a test of Vainshtein screening in galileon theories by comparing the rate of fall for compact objects versus non-relativistic stars. A positive detection of a difference will be a great boost to some of the recent ideas of modifying gravity to explain cosmic acceleration. A negative detection can be used to put an upper limit on the scalar-matter coupling α. A bound reaching 0.1 is conceivable with existing data, and will suffice to rule out most of these recent ideas. Perhaps the most interesting observation is that relatively small scale, local data can shed light on the dark energy problem and the nature of gravity.

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