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Kaon B-parameter from improved staggered fermions in $N_f = 2 + 1$ QCD

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We present a calculation of the kaon B-parameter, B_K, using lattice QCD. We use improved staggered valence and sea fermions, the latter generated by the MILC collaboration with $N_f = 2+1$ light flavors. To control discretization errors, we use four different lattice spacings ranging down to $a \approx 0.045$ fm. The chiral and continuum extrapolations are done using SU(2) staggered chiral perturbation theory. Our final result is $\hat{B}_K = 0.727 \pm 0.004(\text{stat}) \pm 0.038(\text{sys})$, where the dominant systematic error is from our use of truncated (one-loop) matching factors.

CP violation was first observed in the kaon system in 1964 [1], but only in the last few years has it been possible to use this classic result to learn about the parameters of the Standard Model (SM). CP violation in the SM is due to the phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and leads to CP violation in kaon mixing—so-called indirect CP violation—through Feynman diagrams involving virtual charm and top quarks. Integrating out heavy quarks (c, b and t) and the W and Z-bosons, one finds that a prediction for indirect CP violation requires the calculation of the matrix element of a local four-fermion operator between a K_0 and a $\overline{K}_0$. To calculate this matrix element (which is parametrized by the kaon B-parameter, B_K) requires control over the non-perturbative physics of the strong interactions. The only known quantitative method to calculate such matrix elements from first principles is lattice Quantum Chromodynamics (LQCD), and it is only very recently that lattice calculations have begun to control all sources of error.

In this note we present a calculation of B_K using improved staggered fermions. Fully controlled results using other types of lattice fermion have been obtained previously, using valence domain-wall quarks with staggered sea quarks [2], using domain-wall valence and sea fermions [3, 4]. and, very recently, using Wilson valence and sea fermions [5]. We also presented a result using a partial data set in Ref. [6], which we refer to as SW-1 in the following. What sets our work apart from that using valence domain-wall fermions is that we are able to use smaller values of the lattice spacing, a, thus potentially providing better control over the continuum limit. We also have very small statistical errors, such that the final statistical error in B_K is $\sim 0.5\%$. In addition, our use of a different fermion discretization provides a highly non-trivial cross-check of the other results, analogous to the use of a different experimental technique.

The required matrix element is parametrized by

$$B_K(\mu, \mathrm{R}) = \frac{\langle K^0 | O_{\Delta S=2}(\mu, \mathrm{R}) | \bar{K}^0 \rangle}{8 f_K^2 M_K^2 / 3} \tag{1}$$

where R is a specific regularization scheme chosen to define the operator $O_{\Delta S=2} = \sum_\nu [\bar{s}\gamma_\nu(1-\gamma_5)d][\bar{s}\gamma_\nu(1-\gamma_5)d]$, and μ is the corresponding renormalization scale. We use lattice regularization, and then convert to the continuum $\overline{\mathrm{MS}}$ scheme (using naive dimensional regularization for the γ_5) using one-loop matching factors from Ref. [7]. At the end we convert to the renormalization-scale invariant quantity $\widehat{B}_K$.

We use improved staggered fermions for both valence and sea quarks. The advantages of staggered fermions are that they are computationally inexpensive and that they retain a remnant of chiral symmetry. The latter property implies that the matrix element in the numerator of Eq. (1) vanishes when $M_K \to 0$ because of the "left-left" chiral structure of $O_{\Delta S=2}$. Without chiral symmetry, $O_{\Delta S=2}$ mixes with operators with "left-right" structure, whose matrix elements do not vanish when $M_K \to 0$, and are thus enhanced. For both staggered and domain-wall fermions, such mixing is not allowed. Mixing with chirally enhanced operators is allowed for Wilson fermions, but appears now to be controllable [5].

For the sea quarks, we use the MILC collaboration's publicly available ensembles generated using $2+1$ flavors of asqtad staggered fermions [8]. Here "$2+1$" indicates degenerate up and down quarks and a heavier, nearly physical, strange quark. Each staggered lattice flavor describes four continuum fermions, usually called "tastes". This unwanted degeneracy is removed by the fourth-root prescription, and we assume this leads to no problems with the continuum limit.

For the valence sector we choose HYP-smeared staggered fermions. We prefer these to asqtad fermions because they are more continuum-like, e.g. the breaking of

TABLE I. MILC ensembles used to calculate B_K [8]. a is the nominal lattice spacing, m_ℓ (m_s) the light (strange) sea-quark mass, "ens" the number of gauge configurations and "meas" the number of measurements per configuration. "Status" indicates changes since SW-1: "old" is unchanged, "`new`" is a new ensemble, and `update` indicates more measurements.

a (fm)	am_ℓ/am_s	size	ID	ens × meas	status
0.12	0.03/0.05	$20^3 \times 64$	C1	564×9	`update`
0.12	0.02/0.05	$20^3 \times 64$	C2	486×9	`update`
0.12	0.01/0.05	$20^3 \times 64$	C3	671×9	old
0.12	0.01/0.05	$28^3 \times 64$	C3-2	275×8	old
0.12	0.007/0.05	$20^3 \times 64$	C4	651×10	old
0.12	0.005/0.05	$24^3 \times 64$	C5	509×9	`update`
0.09	0.0062/0.031	$28^3 \times 96$	F1	995×9	`update`
0.06	0.0036/0.018	$48^3 \times 144$	S1	744×2	old
0.045	0.0028/0.014	$64^3 \times 192$	U1	705×1	`new`

taste symmetry is smaller by a factor of ~ 3 [9]. Nevertheless, taste-breaking induced by discretization errors leads to significant complications in the analysis. This enters both through mass splittings between kaons of different tastes and by inducing additional operator mixing. Both these effects, as well as those due to the use of different types of valence and sea quarks, and to the use of the fourth-root prescription, can be incorporated into the chiral effective theory describing staggered fermions—staggered chiral perturbation theory (SChPT) [6, 10–12]. Specifically, we use SU(2) SChPT—in which only the up and down quarks are treated as light. After fitting to the forms predicted by SChPT, one can then remove "by hand" the taste-breaking discretization errors.

We use the MILC asqtad lattices listed in Table I for the present work. The most important changes since our previous work, SW-1, are the addition of a fourth, finer, lattice spacing (the "ultrafine" ensemble U1) and the 9 or 10-fold increase in the number of measurements on several of the "coarse" ensembles ($a \approx 0.12$ fm) and also on the "fine" ($a \approx 0.09$ fm) ensemble F1.

The sea-quark masses in these ensembles are not physical. The strange quark is somewhat too heavy, requiring a small correction. The light quarks are too heavy, requiring an extrapolation to the physical mass. The lightest sea-quark pion on the coarse ensembles has $m_\pi^{\rm min}(\text{sea}) \approx 280$ MeV. This will turn out to be light enough for a controlled extrapolation, because the dependence on sea-quark masses turns out to be mild.

On each lattice, we create and destroy kaons using two wall-sources, which are separated by Euclidean time Δt. The sources have the property of creating only kaons with the desired Goldstone taste, ξ_5, and with vanishing spatial momenta. The discretized version of the operator $O_{\Delta S=2}$ is placed between the two sources, and summed over space. Δt is chosen large enough that there is a plateau region with minimal contamination from excited states, and small enough that effects from states propagating "around the world" in the time direction can be ignored. We obtain B_K by fitting to a constant over the plateau region.

We use 10 different valence quark masses, $am_V = am_s^{\rm nom}(n/10)$ for $n = 1, 2, \ldots, 10$, with the nominal strange quark masses being 0.05, 0.03, 0.018 and 0.014 for coarse, fine, superfine and ultrafine lattices, respectively. (For equal bare masses, HYP-smeared quarks have smaller physical masses than asqtad quarks, because of differing renormalization factors.) We use the lightest four valence masses for the valence d quark (m_x), and the heaviest three valence masses for the valence s quark (m_y). This maintains the relations $m_x \ll m_y \sim m_s^{\rm phys}$, as required for the applicability of SU(2) ChPT.

The four values of am_x allow us to extrapolate to the physical down-quark mass. This extrapolation is much shorter than that for the sea-quarks, both since our valence quarks are lighter (the lightest $\bar{x}x$ pion has a mass ≈ 200 MeV), and because we are extrapolating to $m_d^{\rm phys}$ rather than $(m_u^{\rm phys}+m_d^{\rm phys})/2$ (so that $M_{\bar{x}x}$ must be extrapolated to 158 MeV, the mass of an unphysical $\bar{d}d$ meson). We fit the dependence on $X = M_{\bar{x}x}^2$ (for pseudoscalar taste) to the next-to-leading order (NLO) form predicted by SU(2) SChPT. In SW-1 we used uncorrelated fits and did not include finite-volume corrections. Here we correct both shortcomings. To obtain satisfactory correlated fits (with $\chi^2/\text{d.o.f.} \lesssim 1$) we need to include higher-order terms with coefficients constrained by Bayesian priors. Specifically, we fit B_K for fixed m_y to

$$c_1 F_0(X) + c_2 X + c_3 X^2 + c_4 X^2 \ln^2 X + c_5 X^2 \ln X + c_6 X^3 \,, \quad (2)$$

where $F_0(X)$ contains the leading order constant term as well as the chiral logarithms. The latter include taste-breaking effects and finite-volume dependence (see SW-1 and Ref. [13] for the explicit form). The terms multiplied by c_{3-5} are the generic NNLO forms in the continuum. Since these are not known analytically, we include them with coefficients whose magnitude is constrained by Bayesian priors to be of the size expected by naive dimensional analysis. We also include a single analytic NNNLO term (with coefficient c_6).

Examples of the resulting fits are shown in Fig. 1. With the fit parameters in hand, we can extrapolate m_x and $m_d^{\rm sea}$ to $m_d^{\rm phys}$, $m_u^{\rm sea}$ to $m_u^{\rm phys}$, the volume to infinity, and remove taste-breaking discretization errors. We note that finite volume shifts are small for our pion masses, even though our lightest pions have $m_\pi L \approx 3$ (with L the box size). We have checked this directly by comparing results on ensembles C3 and C3-2 [13].

We estimate the systematic error of our X-fits by doubling the allowed widths of the Bayesian priors, and by dropping the NNNLO term. The former gives the larger effect, and we take the largest size of this shift (0.33%, on the S1 ensemble) as the error estimate.

The three values of am_y allow us to extrapolate to the physical strange quark mass. We find that a linear fit

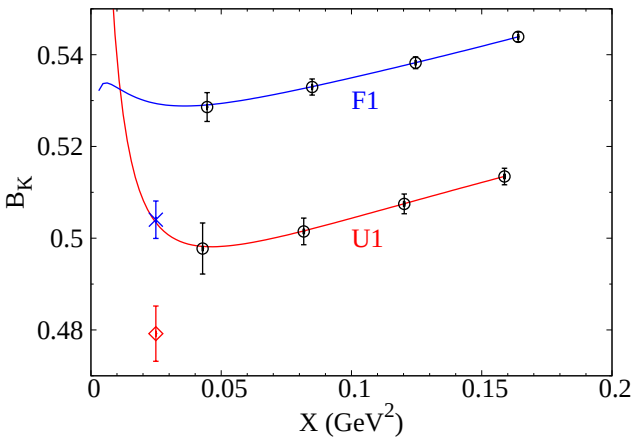

FIG. 1. $B_K(\mathrm{NDR}, 1/a\ \mathrm{GeV})$ vs. X on the U1 and F1 ensembles. The [red] diamond ([blue] cross) shows the result on the U1 (F1) ensemble, after extrapolation and removal of taste-breaking artifacts as described in the text.

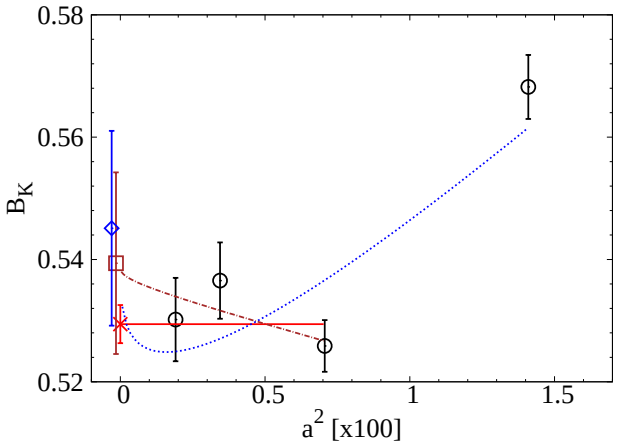

FIG. 2. Continuum extrapolation of $B_K(\mathrm{NDR}, 2\ \mathrm{GeV})$. The fits are described in the text.

to m_y represents the data very well, and use this for our central values. We use a quadratic "Y-fit" to estimate a fitting systematic.

At this stage the values of B_K on different ensembles differ primarily because of taste-conserving discretization and matching errors. To remove the main part of these errors we use ensembles C3, F1, S1 and U1, which have very similar sea-quark masses. In Fig. 2, we show the dependence on a^2, and present the results of several methods of extrapolation to $a = 0$. We note that the simple linear dependence observed in SW-1 (for the largest three lattice spacings) has been resolved by improved errors, and by the addition of the U1 point, into a less smooth dependence. On theoretical grounds [12], the expected dependence is

$$d_1 + d_2(a\Lambda)^2 + d_3(a\Lambda)^2\alpha_s + d_4\alpha_s^2 + d_5(a\Lambda)^4 + \dots\,, \quad (3)$$

where $\alpha_s = \alpha_s^{\overline{\mathrm{MS}}}(1/a)$. We fit to this 5-parameter form applying Bayesian constraints on d_{2-5}—the expected values are taken to be 0, while the standard deviations are set to 2, having chosen $\Lambda = 300$ MeV. The fit is shown by the [blue] dotted curve, and gives the extrapolated value shown by the diamond. The fit, however, is very poor, with $\chi^2/\mathrm{d.o.f.} = 4.5$. This problem is not resolved by adding terms of one higher order. Thus we drop the coarse lattice from the fits, and then find good fits to a constant (solid [red] line and cross), a linear dependence on a^2 (not shown) and to the constrained form (3) (dashed [brown] line and square). We take the constant fit for the central value, and the difference between it and the constrained fit as the systematic error in the continuum extrapolation. For more discussion of fits see Ref. [14].

After the preceding analysis, B_K can still have a residual dependence on the sea quark masses. In SU(2) SChPT, the dependence on m_ℓ is linear at NLO. We have investigated the m_ℓ dependence in detail on the coarse lattices, with results shown in Fig. 3. We plot versus L_P, the squared mass of the sea-quark pion, and find a linear behavior with a small slope $\approx -1/(2.9\ \mathrm{GeV})^2$. In SW-1, with errors 3 times larger, we could not uncover this dependence. Using this slope, we find that B_K is increased by 1.5% when changing L_P from its value on ensemble C3 to its physical value. Since this is a small effect, and since we only have results for the slope on the coarse ensembles, we do not adjust the central value of B_K, but instead quote the 1.5% as a systematic error.

We also need to correct for the mismatch between am_s and am_s^{phys}. Here we follow SW-1, and assume the SU(3) ChPT form of the m_s dependence. Then we find that, on C3, the correction from using a sea-quark mass which is 40% too large is 1.3%. This approach is more conservative than that used in SW-1. Again, we do not adjust the central value, but quote the shift as a systematic error.

We collect all our errors in Table II. The statistical error has been reduced by a factor of 3 compared to SW-1, and now is smaller than many other sources of error. The dominant error comes from our use of one-loop matching. We estimate this as $\delta B_K/B_K = \alpha_s^2$, with α_s evaluated at the scale of our finest lattice. It is thus reduced from the 5.5% estimate of SW-1 by the addition of the U1 ensemble. The discretization error is nearly unchanged from SW-1, despite the addition of the ultrafine lattices, because of the increased uncertainty in the continuum extrapolation. The only errors not discussed above are the "r_1" and "f_π" errors, which are estimated essentially as discussed in SW-1.

Adding systematic errors in quadrature, we find

$$\hat{B}_K = 0.727 \pm 0.004(\mathrm{stat}) \pm 0.038(\mathrm{sys})\,.$$

The central value is almost unchanged from SW-1, but the significant improvements we have made have both re-

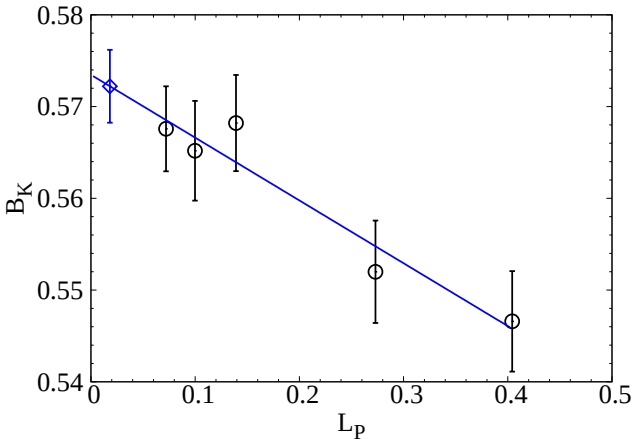

FIG. 3. $B_K(\mathrm{NDR}, 2\ \mathrm{GeV})$ vs. L_P on the coarse ensembles. The solid line shows a linear fit, with the extrapolation to physical pion mass given by the diamond.

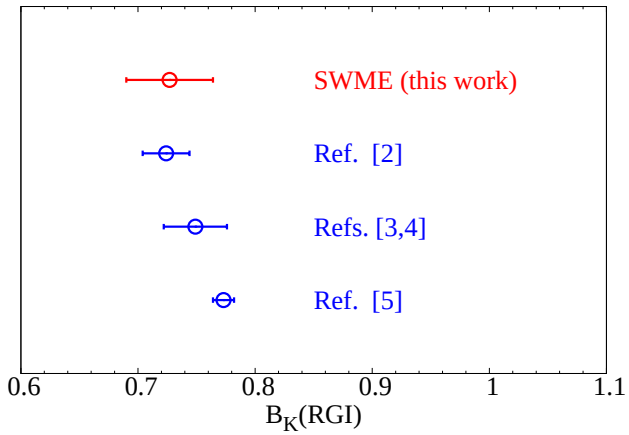

FIG. 4. Comparison of our result for $\widehat{B}_K = B_K(\mathrm{RGI})$ with other results obtained with $N_f = 2 + 1$ flavors.

TABLE II. Error budget for B_K using SU(2) SChPT fitting.

cause	error (%)	memo
statistics	0.6	see text
matching factor	4.4	$\Delta B_K^{(2)}$ (U1)
discretization	1.9	diff. of constant and constrained fits
X-fits	0.33	varying Bayesian priors (S1)
Y-fits	0.07	diff. of linear and quadratic (C3)
am_l extrap	1.5	diff. of (C3) and linear extrap
am_s extrap	1.3	diff. of (C3) and linear extrap
finite volume	0.5	diff. of $V = \infty$ fit and FV fit
r_1	0.14	r_1 error propagation (C3)
f_π	0.4	132 MeV vs. 124.4 MeV

duced the error and solidified our error estimates. Our result is consistent with other $N_f = 2+1$ results [2, 4, 5], as shown in Fig. 4. The largest difference is that our result lies 1.4σ below that of Ref. [5] (the most accurate result). This consistency with results obtained using different fermion discretizations is our most significant conclusion. It is important, however, to further reduce errors in all lattice calculations to check that this consistency holds up. Work is in progress to reduce our dominant systematic using two-loop matching and non-perturbative renormalization.

Lattice results for B_K now allow the venerable experimental result for ϵ_K to be used to constrain the parameters of the SM. Indeed, we have now reached the situation that the accuracy of B_K calculations is such that errors from other sources dominate—in particular those from V_{cb} and the Wilson coefficient η_{cc} (see, e.g., Ref. [15]).

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