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The Muon Anomaly and Dark Parity Violation

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The muon anomalous magnetic moment exhibits a 3.6σ discrepancy between experiment and theory. One explanation requires the existence of a light vector boson, Z_d (the dark Z), with mass $10 - 500$ MeV that couples weakly to the electromagnetic current through kinetic mixing. Support for such a solution also comes from astrophysics conjectures regarding the utility of a $U(1)_d$ gauge symmetry in the dark matter sector. In that scenario, we show that mass mixing between the Z_d and ordinary Z boson introduces a new source of "dark" parity violation which is potentially observable in atomic and polarized electron scattering experiments. Restrictive bounds on the mixing $(m_{Z_d}/m_Z)\delta$ are found from existing atomic parity violation results, $\delta^2 < 2 \times 10^{-5}$. Combined with future planned and proposed polarized electron scattering experiments, a sensitivity of $\delta^2 \sim 10^{-6}$ is expected to be reached, thereby complementing direct searches for the Z_d boson.

For a number of years, there has been a persistent disagreement between the experimental value of the muon anomalous magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$

$$a_\mu^{\rm exp} = 116\ 592\ 089(63) \times 10^{-11} \quad (1)$$

and the theoretical $SU(3)_C \times SU(2)_L \times U(1)_Y$ Standard Model (SM) prediction

$$a_\mu^{\rm SM} = 116\ 591\ 802(49) \times 10^{-11}. \quad (2)$$

The above 3.6σ discrepancy [1]

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 287(80) \times 10^{-11} \quad (3)$$

could be indicative of problems with the theoretical calculations and/or experimental measurements. Alternatively, it could be a harbinger of "new physics" effects beyond SM expectations [2]. One possibility, receiving support from dark matter conjectures [3, 4], envisions the existence of a relatively light $U(1)_d$ gauge boson, Z_d, coming from the "dark" sector that indirectly couples to our world via $U(1)_Y \times U(1)_d$ kinetic mixing [5], parameterized by ε such that [6]

$$\mathcal{L}_{\rm int} = -e\varepsilon Z_d^\mu J_\mu^{em}, \quad J_\mu^{em} = \sum_f Q_f \bar{f}\gamma_\mu f, \quad (4)$$

where Q_f is the electric charge of fermion f. The coupling of Z_d to the weak neutral current, from kinetic mixing, is highly suppressed at low energies because of a cancellation between the ε dependent field redefinition and Z-Z_d mass matrix diagonalization effects induced by ε [6]. (We do not consider here the possibility that some or all of our fermions may have explicit $U(1)_d$ charges.)

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The $Z_d\mu\bar{\mu}$ vector current coupling in Eq. (4) gives rise to an additional one loop contribution [7, 8] to a_μ

$$\Delta a_\mu^{Z_d}(\text{vector}) = \frac{\alpha}{2\pi}\varepsilon^2 F_V\left(m_{Z_d}/m_\mu\right) \quad (5)$$

$$F_V(x) \equiv \int_0^1 dz \frac{2z(1-z)^2}{(1-z)^2 + x^2 z}, \quad F_V(0) = 1. \quad (6)$$

The effect in Eq. (5) has the right algebraic sign, such that for 10 MeV $\lesssim m_{Z_d} \lesssim$ 500 MeV and ε^2 roughly in the range $10^{-6} - 10^{-4}$, the discrepancy Δa_μ in Eq. (3) can be eliminated. We plot [9] in Fig. 1 the band in (m_{Z_d}, ε^2) space that reduces the discrepancy to within 90% CL, *i.e.*

$$\Delta a_\mu^{Z_d} = 287 \pm 131 \times 10^{-11}. \quad (7)$$

There, we also give a (roughly) 90% CL bound from the electron anomalous magnetic moment [10, 11] constraint $|a_e^{Z_d}| < 10^{-11}$ using m_e in place of m_μ in Eq. (5) as well as a 3σ $a_\mu^{Z_d}$ bound. Constraints from other direct experimental searches for Z_d are also given [12, 13]. However, those bounds are somewhat model dependent since they assume the Z_d decays primarily into e^+e^- or $\mu^+\mu^-$ pairs. They will be diluted if, for example, Z_d decays primarily into light "dark particles" that escape the detector as $Z_d \to$ missing energy [6].

Recently [6], we generalized the $U(1)_d$ kinetic mixing scenario to include possible Z-Z_d mass mixing by introducing the 2×2 mass matrix

$$M_0^2 = \begin{pmatrix} 1 & -\varepsilon_Z \\ -\varepsilon_Z & m_{Z_d}^2/m_Z^2 \end{pmatrix} m_Z^2 \quad (8)$$

where m_{Z_d} and m_Z (with $m_{Z_d}^2 \ll m_Z^2$) represent the "dark" Z and SM Z masses (before diagonalization). The off-diagonal mixing is parametrized by

$$\varepsilon_Z = \frac{m_{Z_d}}{m_Z}\delta, \quad 0 \le |\delta| < 1 \quad (9)$$

where the m_{Z_d}/m_Z factor allows a smooth $m_{Z_d} \to 0$ limit for nonconserved current amplitudes and δ is expected

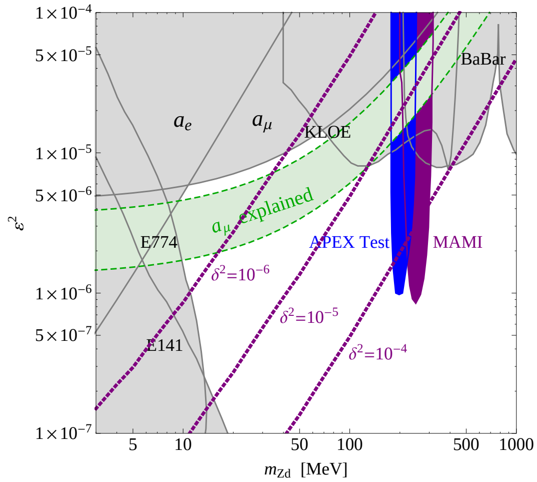

FIG. 1: Dark Z boson exclusion regions (partly adapted from Ref. [9]) in the (m_{Z_d}, ε^2) plane along with the band that explains the Δa_μ discrepancy (90% CL) and exclusion regions from atomic parity violation (above the lines) for Z-Z_d mixing δ values.

to be a small quantity that depends on the Higgs scalar sector of the theory [6]. Z-Z_d mixing induced by ε_Z leads to an additional coupling of Z_d to fermions via the weak neutral current

$$\begin{aligned} \mathcal{L}_{\rm int} &= -\frac{g}{2\cos\theta_W}\varepsilon_Z Z_d^\mu J_\mu^{NC} \\ J_\mu^{NC} &= \sum_f (T_{3f} - 2Q_f \sin^2\theta_W)\bar{f}\gamma_\mu f - T_{3f}\bar{f}\gamma_\mu\gamma_5 f \end{aligned} \tag{10}$$

with $T_{3f} = \pm 1/2$ and $\sin^2\theta_W \simeq 0.23$ the SM weak mixing angle. Because of its axial-vector coupling, this new interaction violates parity and current conservation. As a result, it can lead to potentially observable effects in atomic parity violation (APV) and polarized electron scattering experiments, as well as rare flavor changing K and B decays or Higgs decays ($H \to ZZ_d$) to longitudinally polarized Z_d bosons (phase space permitting). We pointed out in Ref. [6] that the nonobservation of such effects already leads to bounds $|\delta| \lesssim 10^{-2} - 10^{-3}$ depending on m_{Z_d} and in some cases ε. Here, we further explore such constraints, but focus on that part of parameter space 10 MeV $\lesssim m_{Z_d} \lesssim$ 500 MeV and $|\varepsilon| \approx 10^{-3} - 10^{-2}$ favored by a Z_d explanation of the Δa_μ discrepancy in Eq. (3). Also, to keep our analysis independent of the Z_d decay properties, we concentrate on low energy parity violation, *i.e.* atomic and polarized electron scattering experiments. A variety of direct searches for Z_d have been discussed in the literature [6, 9, 12, 13].

We begin by considering changes to $\Delta a_\mu^{Z_d}$ due to $\delta \neq 0$. The additional $Z_d\mu\bar{\mu}$ vector coupling in Eq. (10) modifies the contribution in Eq. (6) via the replacement

$$\varepsilon^2 \to \left(\varepsilon + \varepsilon_Z \frac{1 - 4\sin^2\theta_W}{4\sin\theta_W\cos\theta_W}\right)^2 \simeq (\varepsilon + 0.02\varepsilon_Z)^2 \tag{11}$$

where $\sin^2\theta_W \simeq 0.24$ appropriate for low $Q^2 \simeq m_\mu^2$ scales [14] has been employed. For the Δa_μ favored range of m_{Z_d} and ε^2 in Fig. 1, the shift in Eq. (11) is small ($\lesssim 2\%$) for all δ and can be ignored.

The axial-vector part of the $Z_d\mu\bar{\mu}$ coupling in Eq. (10) gives rise to a negative $\Delta a_\mu^{Z_d}$ contribution [8]

$$\begin{aligned} \Delta a_\mu^{Z_d}(\text{axial}) &= -\frac{G_F m_\mu^2}{8\sqrt{2}\pi^2}\delta^2 F_A\left(m_{Z_d}/m_\mu\right) \\ &\simeq -117\times 10^{-11}\delta^2 F_A\left(m_{Z_d}/m_\mu\right) \end{aligned} \tag{12}$$

$$F_A(x) \equiv \int_0^1 dz \frac{2(1-z)^3 + x^2 z(1-z)(z+3)}{(1-z)^2 + x^2 z}, \tag{13}$$

where $G_F \simeq 1.166\times 10^{-5}$ GeV^{-2}, $F_A(0) = 1$, and $F_A(\infty) = 5/3$. For $\delta^2 \lesssim 0.1$ (a mild requirement [6]), that contribution is also negligible throughout the Δa_μ favored region in Fig. 1. So, we conclude that the effect of Z-Z_d mass mixing plays little direct role in any discussion of the Δa_μ discrepancy and its interpretation as due to ε^2.

Next, we examine constraints on the m_{Z_d}, ε, δ parameter space coming from low energy parity violating experiments and their implications for a Z_d interpretation of the Δa_μ discrepancy.

It is well known that the classic Cesium atomic parity violation experiment [15] provides a stringent constraint on heavy Z' bosons [16] that violate parity, often implying $m_{Z'} \gtrsim \mathcal{O}(1\text{ TeV})$. However, its application to relatively light gauge bosons such as Z_d has been less explored. Such a connection was first made by Bouchiat and Fayet [17] for a light "U-boson" with very general parity violating couplings to fermions. They found strong constraints and argued against axial-vector couplings. We recently [6] revisited the application of low energy parity violation experimental constraints within the general Z-Z_d mass mixing formalism of Eq. (8). We updated the Cesium constraint to include more recent atomic theory [18], expanded the analysis to polarized electron scattering [19] and applied our study specifically to the "dark" Z boson. Here, we focus on the connection of that analysis with the Δa_μ discrepancy and its interpretation via 10 MeV $\lesssim m_{Z_d} \lesssim$ 500 MeV with $\varepsilon^2 \sim 10^{-6} - 10^{-4}$.

The additional parity violation from Eq. (10) manifests itself as replacements in low energy SM parity violating weak neutral current amplitudes [6]

$$G_F \to \rho_d G_F, \quad \sin^2\theta_W \to \kappa_d \sin^2\theta_W, \tag{14}$$

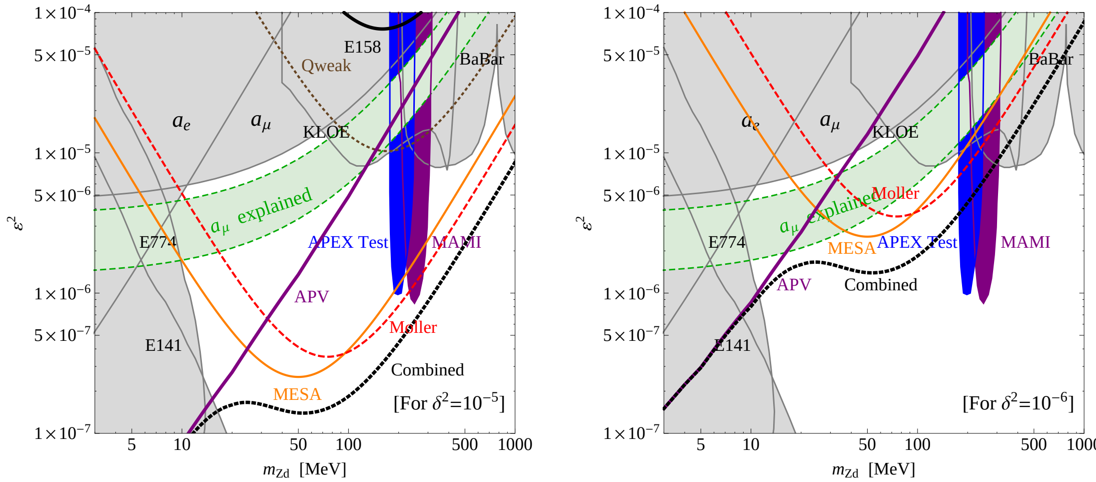

FIG. 2: Dark Z boson exclusion regions from various parity violating experiments (existing and proposed) and their combined sensitivity for $\delta^2 = 10^{-5}$ (Left) and 10^{-6} (Right) at 90% CL.

where for (momentum transfer) $Q^2 = -q^2$

$$\rho_d = 1 + \delta^2 f\left(Q^2/m_{Z_d}^2\right), \tag{15}$$

$$\kappa_d = 1 - \varepsilon\delta \frac{m_Z}{m_{Z_d}} \frac{\cos\theta_W}{\sin\theta_W} f\left(Q^2/m_{Z_d}^2\right) \tag{16}$$

giving rise to

$$\Delta \sin^2\theta_W \simeq -0.42\varepsilon\delta \frac{m_Z}{m_{Z_d}} f\left(Q^2/m_{Z_d}^2\right). \tag{17}$$

As pointed out in Ref. [17], for parity violation in heavy atoms, such as Cesium, there is a correction factor $f = K(\mathrm{Cs})$ relevant for very small m_{Z_d}. For example, $K(\mathrm{Cs}) \simeq 0.5$ at $m_{Z_d} \simeq 2.4$ MeV, which sets the typical momentum transfer $\langle Q\rangle$ in this case, whereas $K(\mathrm{Cs}) \simeq 0.74,\ 0.98$ at $m_{Z_d} \simeq 10,\ 100$ MeV. In the case of polarized electron scattering asymmetries, the Z_d propagator effect gives

$$f\left(Q^2/m_{Z_d}^2\right) = \frac{1}{1 + Q^2/m_{Z_d}^2} \tag{18}$$

with $\langle Q\rangle$ ranging from $50 - 170$ MeV for the experiments we consider.

Currently, the SM prediction for the weak nuclear charge $Q_W(Z,N) \simeq -N + Z(1 - 4\sin^2\theta_W)$ in the case of ${}^{133}_{55}\mathrm{Cs}$ (including electroweak radiative corrections) [20]

$$Q_W^{\mathrm{SM}}({}^{133}_{55}\mathrm{Cs}) = -73.16(5) \tag{19}$$

is in excellent agreement with experiment (including the most up-to-date atomic theory) [15, 18]

$$Q_W^{\mathrm{exp}}({}^{133}_{55}\mathrm{Cs}) = -73.16(35). \tag{20}$$

The 90% CL bound on the difference

$$|\Delta Q_W(\mathrm{Cs})| = |Q_W^{\mathrm{exp}}({}^{133}_{55}\mathrm{Cs}) - Q_W^{\mathrm{SM}}({}^{133}_{55}\mathrm{Cs})| < 0.6 \tag{21}$$

can be compared with the potential Z_d contribution [6]

$$\Delta Q_W({}^{133}_{55}\mathrm{Cs}) = \left(-73.16\delta^2 + 220\varepsilon\delta\frac{m_Z}{m_{Z_d}}\sin\theta_W\cos\theta_W\right)K(\mathrm{Cs}). \tag{22}$$

In principle, there could be a cancellation between the two terms in Eq. (22) for $\varepsilon(m_Z/m_{Z_d}) \sim 0.8\delta$. However, for the Δa_μ preferred band in Fig. 1, $|\varepsilon(m_Z/m_{Z_d})| \gtrsim 2$; the second term in Eq. (22) always dominates. In fact, a conservative self-consistent assessment of the bound (at 90% CL) from Eqs. (21) and (22) yields

$$|\delta^2 - 2\delta| < 0.008 \quad \rightarrow \quad \delta^2 < 2\times 10^{-5} \tag{23}$$

for the entire Δa_μ motivated band in Fig. 1. That means the first term in Eq. (22) can be neglected and the $Q_W({}^{133}_{55}\mathrm{Cs})$ bound becomes for arbitrary ε^2 and m_{Z_d} essentially a bound

$$\varepsilon^2 < \frac{4\times 10^{-5}}{\delta^2 K^2}\left(\frac{m_{Z_d}}{m_Z}\right)^2 \tag{24}$$

on the allowed $\sin^2\theta_W$ shift. The atomic parity violation bound on ε^2 is illustrated in Fig. 1 for various values of δ^2. Note that for $\delta^2 \gtrsim 2\times 10^{-5}$, the entire Δa_μ discrepancy motivated band is already ruled out. Alternatively, if a light Z_d is responsible for the Δa_μ discrepancy, the Z-Z_d mixing $|\varepsilon_Z| = |(m_{Z_d}/m_Z)\delta|$ must be very tiny ($\delta^2 < 2\times 10^{-5}$). Of course, the Δa_μ discrepancy may have nothing to do with Z_d. In that case, larger δ^2 values can be accommodated by going to smaller ε^2 or larger m_{Z_d} values, although other constraints [6] then come into play.

Atomic parity violation already provides a powerful constraint on δ^2 over an interesting m_{Z_d} range. Future experiments employing ratios of isotopes could in principle eliminate the atomic theory uncertainty and further

Experiment	$\langle Q\rangle$	$\sin^2\theta_W(m_Z)$	Bound on dark Z (90% CL)
Cesium APV	2.4 MeV	0.2313(16)	$\varepsilon^2 < \frac{39\times10^{-6}}{\delta^2}\left(\frac{m_{Z_d}}{m_Z}\right)^2\frac{1}{K(m_{Z_d})^2}$
E158 (SLAC)	160 MeV	0.2329(13)	$\varepsilon^2 < \frac{62\times10^{-6}}{\delta^2}\left(\frac{(160\text{ MeV})^2+m_{Z_d}^2}{m_Z\, m_{Z_d}}\right)^2$
Qweak (JLAB)	170 MeV	±0.0007	$\varepsilon^2 < \frac{7.4\times10^{-6}}{\delta^2}\left(\frac{(170\text{ MeV})^2+m_{Z_d}^2}{m_Z\, m_{Z_d}}\right)^2$
Moller (JLAB)	75 MeV	±0.00029	$\varepsilon^2 < \frac{1.3\times10^{-6}}{\delta^2}\left(\frac{(75\text{ MeV})^2+m_{Z_d}^2}{m_Z\, m_{Z_d}}\right)^2$
MESA (Mainz)	50 MeV	±0.00037	$\varepsilon^2 < \frac{2.1\times10^{-6}}{\delta^2}\left(\frac{(50\text{ MeV})^2+m_{Z_d}^2}{m_Z\, m_{Z_d}}\right)^2$
Combined		±0.00021	$\varepsilon^2_{\text{comb}} < \frac{1}{\sum_i(1/\varepsilon_i^2)}$

TABLE I: Existing and possible future constraints on dark Z from various parity violating experiments

probe Z_d mass and mixing as well as other "new physics" scenarios [21].

Another type of low energy parity violating experiment involves polarized electron scattering on electrons, protons or other targets. They measure the parity violating asymmetry [19] $A_{LR} \equiv \sigma_L - \sigma_R/\sigma_L + \sigma_R$ due to γ-Z interference at low Q^2. In some cases, such as ee and ep, those experiments are particularly sensitive to $\sin^2\theta_W$ at low Q^2 where the effective $\sin^2\theta_W$ is expected [14] to be about 0.24, thereby leading to very small asymmetries (proportional to $1-4\sin^2\theta_W$). Already, experiment E158 at SLAC has measured [22] (evolving to $Q^2 = m_Z^2$)

$$\sin^2\theta_W(m_Z)_{\overline{\text{MS}}} = 0.2329(13) \quad \text{(E158 at SLAC)} \qquad (25)$$

which is to be compared with the Z pole average [1]

$$\sin^2\theta_W(m_Z)_{\overline{\text{MS}}} = 0.23125(16). \qquad (26)$$

The relatively good agreement between Eqs. (25) and (26) already constrains many types of "new physics" at a sensitivity similar to APV. In the case of Z_d at low masses, Cesium APV has the advantage of a low [17] $\langle Q\rangle \simeq 2.4$ MeV while for E158, $\langle Q\rangle^{\text{E158}} \simeq 160$ MeV such that Z_d propagator effects suppress the sensitivity by $m_{Z_d}^2/(Q^2+m_{Z_d}^2)$ at the amplitude level.

A comparison of E158 constraints using (see Eq. (17))

$$\varepsilon^2 < \frac{6\times10^{-5}}{\delta^2}\left(\frac{0.026\text{ GeV}^2+m_{Z_d}^2}{m_Z m_{Z_d}}\right)^2 \qquad (27)$$

with APV is illustrated in Fig. 2. The one-sided 90% CL coefficient in that bound has been increased due to the $\sim 1\sigma$ difference between Eqs. (25) and (26). For a given δ^2, the bounds at large m_{Z_d} are similar, but APV is superior for $m_{Z_d} \lesssim 160$ MeV.

An ongoing polarized ep experiment [9, 23], Qweak at JLAB aims to measure $\sin^2\theta_W$ to ±0.0007 at $\langle Q\rangle \simeq 170$ MeV. That represents an improvement by about a factor of 2 over E158, but the similar $\langle Q\rangle$ means that it also lacks low m_{Z_d} sensitivity. In the longer term, a new polarized ee (Moller) [24] experiment at JLAB would measure $\sin^2\theta_W$ to ±0.00029 at $\langle Q\rangle \simeq 75$ MeV and a very low energy polarized ep experiment at a new proposed MESA facility [25] in Mainz, Germany would measure $\sin^2\theta_W$ to ±0.00037 for $\langle Q\rangle$ perhaps as low as 50 MeV. The sensitivities of these (proposed) experiments are also illustrated in Fig. 2, using the constraints in Table I derived from Eq. (17).

In Fig. 2, we give a combined sensitivity bound for $\delta^2 = 10^{-5}$ and $\delta^2 = 10^{-6}$ from all existing and proposed low energy parity violating experiments. That plot illustrates the complementarity of atomic and polarized electron scattering experiments. In addition to providing overlapping probes of new physics, collectively they span a large range of (m_{Z_d}, ε^2) space and probe down to δ^2 of $\mathcal{O}(10^{-6})$. Of course, it is possible that a light Z_d exists that is consistent with the Δa_μ discrepancy and will be discovered. For example, if $m_{Z_d} \simeq 75$ MeV, $|\varepsilon| \simeq 3\times10^{-3}$ and $|\delta| \simeq 2\times10^{-3}$, the proposed Moller and MESA experiments should find shifts $|\Delta\sin^2\theta_W| \simeq 0.0015$ and 0.0021 respectively, corresponding to about 5σ discovery sensitivities.

In conclusion, we have found that existing atomic parity violating results already require $\delta^2 \lesssim 2\times10^{-5}$ for the entire range of (m_{Z_d}, ε^2), *i.e.* 10 MeV $\lesssim m_{Z_d} \lesssim$ 500 MeV, $\varepsilon^2 \simeq 10^{-6} - 10^{-4}$, favored by the Z_d interpretation of the Δa_μ discrepancy. That requirement calls into question the Z_d interpretation of the Δa_μ unless Z-Z_d mixing is naturally small, for example, if the mass m_{Z_d} is primarily generated by an $SU(2)_L\times U(1)_Y$ Higgs singlet [6]. Future polarized electron scattering experiments will provide additional Z_d sensitivity, particularly for $m_{Z_d} \gtrsim 75$ MeV (where 5σ effects are possible) and will nicely complement atomic parity violation experiments as well as direct Z_d searches.

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