Formation of Streamer Discharges from an Isolated Ionization Column at Subbreakdown Conditions

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Formation of streamer discharges from an isolated ionization column at sub-breakdown conditions

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Abstract

This paper reports a modeling study on the formation of streamer discharges from an isolated ionization column under sub-breakdown condition. Numerical simulations show that positive streamers are able to form from the tip of an ionization column in a uniform applied electric field well below the breakdown threshold field. However, even when the applied field approaches the breakdown threshold field, negative streamers fail to originate from the other tip of the ionization column after the positive streamer has propagated a certain distance. The results reported explain some puzzling observations on streamer discharges in nature such as the predominant initiation of sprites by downward propagating positive streamers and help advance the initiation theories of sprites and lightning.

INTRODUCTION

Streamer discharges are filamentary plasmas, which are driven by highly nonlinear space charge waves [1, p. 327]. In laboratory experiments, when high voltage pulses are applied to an electrode, free electrons (e.g., produced by cosmic ray radiation) near the electrode are accelerated and can acquire sufficient energy to knock out additional electrons from neutral air molecules. The number of electrons increases like avalanches, and charge separation occurs during this process, producing an increasing space charge field. When the number of free electrons in an electron avalanche reaches a critical value ($\sim 10^8$ at ground pressure), the space charge field becomes comparable to the applied field. At this point, the dynamics of the discharge is largely determined by the space charge field and the avalanche has become a streamer. To start electron avalanches, the applied field must be greater than a threshold value known as the conventional breakdown threshold field E_k, which is defined by the equality of the ionization and dissociative attachment coefficients [1, p. 135]. It is about 3.2×10^6 V/m at ground pressure in air and approximately scaled with air density. Streamer discharges are classified as positive (cathode-directed) and negative (anode-directed) streamers depending on the polarity of the charge in the streamer head (the direction of their propagation). Many studies have found that the inception condition and propagation characteristics of streamers depend on their polarity regardless of whether they develop in uniform [2, 3] or nonuniform [e.g., 3, 4] electric fields.

Streamer discharges also exist in natural plasma phenomena in the Earth's atmosphere. The most common example is lightning flashes. Although not clearly understood, streamers play important roles in initiation and propagation of lightning channels. One of the biggest mysteries in atmospheric sciences is how lightning starts in thunderclouds, where decades of electric field measurements consistently give a maximum value about an order of magnitude smaller than E_k [e.g., 5]. Another natural plasma phenomenon where streamers can be found is sprites, large electric discharges in the mesosphere and lower ionosphere, which were discovered about two decades ago [e.g., 6, 7]. They appear after intense cloud-to-ground lightning flashes that produce a strong quasi-electrostatic field for 10s-100s ms above thunderclouds. Typical sprites are initiated at $\sim$75-80 km altitudes with downward propagating positive streamers that may or may not be followed by upward propagating negative streamers [e.g., 8, 9]. It has been generally believed that the lightning electric field must reach E_k

to trigger sprites, which has been supported by large-scale modeling of sprites [e.g., 10], streamer modeling of fine resolution [e.g., 2, 11–14] and simulation of streamer formation from a descending ionization wave known as sprite halos in the lower ionosphere [15]. Those works are indeed able to explain various aspects of some sprite events. However, recent studies showed that many sprites are initiated in a lightning field about 0.2-0.6E_k [16, 17], and it is unclear how sprite streamers can be triggered in such an electric field significantly smaller than E_k. Although several previous studies found that large-scale perturbations of background neutral air density [e.g., 10, 18] or electron density [15] can facilitate sprite initiation, a powerful lightning flash capable of producing an electric field close to E_k is still required.

In this paper we report the first simulation results that unambiguously demonstrate that streamers can be initiated in an electric field close to the maximum value of observed thundercloud electric fields [5] and the field value reported in [16, 17] for sprites. The streamer initiation is enabled by introducing a small-scale isolated ionization patches (equivalent to isolated conductors or dielectrics of large dielectric constant). This work helps reveal the initiation processes of lightning and sprites, and it also explains some puzzling observations of streamer discharges in nature.

MODEL

The dynamics of a single streamer is described by drift-diffusion equations coupled with Poisson's equation in a cylindrically symmetric system [2]:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot n_e\vec{v}_e - D_e\nabla^2 n_e = (\nu_i - \nu_{a2} - \nu_{a3})n_e - \beta_{ep}n_e n_p + S_{ph}, \tag{1}$$

$$\frac{\partial n_p}{\partial t} = \nu_i n_e - \beta_{ep}n_e n_p - \beta_{np}n_n n_p + S_{ph}, \tag{2}$$

$$\frac{\partial n_n}{\partial t} = (\nu_{a2} + \nu_{a3})n_e - \beta_{np}n_n n_p, \tag{3}$$

$$\nabla^2\phi = -\frac{e}{\varepsilon_0}(n_p - n_e - n_n), \tag{4}$$

where n_e, n_p, and n_n are the electron, positive-ion, and negative-ion number densities; $\vec{v}_e$=$-\mu_e\vec{E}$ is the drift velocity of electrons, with μ_e being the absolute value of electron mobility and $\vec{E}$ electric field; ν_i is the ionization coefficient, ν_{a2}, and ν_{a3} are the two-body, and three body electron attachment coefficients, respectively; β_{ep} and β_{np} are the coefficients of

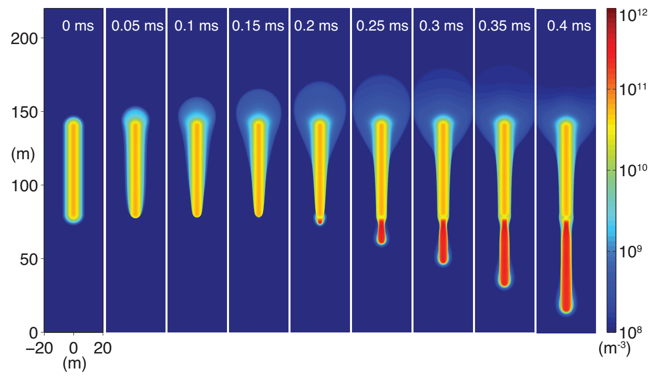

FIG. 1. Cross-sectional views of distributions of electron density during the initiation of a sprite streamer.

electron-positive ion and negative-positive ion recombination, respectively; D_e is the electron diffusion coefficient; $S_{\rm ph}$ is the rate of electron-ion pair production due to photoionization; ϕ is the electric potential, e is the absolute value of electron charge, and ε_0 is the permittivity of free space. Ions are stationary for the timescales involved in the present work because of their much larger mobilities than electrons'. The numerical methods used to solve the model equations are described in Ref. 2 except the method for calculating $S_{\rm ph}$ that is discussed in [19, 20]. A class of hypotheses for the initiation of lightning and sprites suggests that streamers are able to form around the sharp tips of conducting objects (e.g., thundercloud hydrometeors for lightning and ionospheric ionization patches for sprites) placed in an electric field much weaker than E_k. Here we model the conducting objects using an ionization column of free electrons and positive ions. The column has a hemispherical cap at each end. The plasma density in the column is uniform along the axial direction and follows the Gaussian distribution in the radial direction, while the density in the cap follows a spherical Gaussian distribution. The Gaussian distributions have the same peak density and characteristic spatial scale. The column is placed at the center of the simulation region, and the simulation is started by applying a uniform, vertical electric field $\vec{E_0}$ pointing downward modeling either the thundercloud electric field or the lightning field in the lower ionosphere.

RESULTS

Fig. 1 shows a sprite streamer successfully forms at 70 km altitude in a lightning field $E_0 \simeq 0.5E_k$. The Gaussian distributions of the initial ionization column have a characteristic spatial scale of 3 m and peak density of 6×10^{10} m^{-3}. The length of the column is 60 m. When E_0 in the lower ionosphere is suddenly established by lightning, the ionization column becomes polarized with the upward movement of free electrons. This movement exposes the positive ions in the lower end (positive end) of the column while resulting in an excess of electrons in the upper end (negative end). The continuous transport of electrons accumulates positive and negative charge (Fig. 2a) at the respective end as well as in a thin shell around the column. As the charge accumulates, it screens an increasing fraction of the external field out of the ionization column and leads to a growing enhancement of the electric field at the ends (Fig. 2b). However, asymmetric conditions are created at the two ends: a compact space charge region forms at the positive tip in contrast to a large, diffuse region at the negative tip. This is because the positive ions exposed are confined in the compact column but the excess electrons in the negative end can continuously spread outward. Consequently, the electron density distribution becomes steeper at the positive tip showing a slightly upward shifted tip and less steeper at the negative tip from 0 to 0.15 ms. During this period, the magnitude of electric field at the positive tip increases continuously but the location of the maximum field is fixed over time. At about 0.17 ms, the field reaches about $2.5E_k$ (E_k = ∼217 V/m at 70 km altitude) and the location of the maximum field starts to move forward. The electron density in the tip also quickly increases when the field becomes greater than E_k, and the high density region finally extends forward as shown by electron density distributions at 0.2 ms and later. A positive streamer is born from the positive tip of the initial ionization column. On the contrary, only a diffuse electron cloud and space charge region appear around the negative tip.

A close look at electron density, space charge density and electric field profiles along the symmetry axis of the streamer at 0.17 and 0.27 ms is shown in Fig. 3. It shows that the location of the maximum field is slightly ahead of the peak of the space charge density. At 0.17 ms, ionization of molecules in the high field region produces a visible electron density hump, which is co-located with the peak space charge density. The location of the streamer peak field starts to move at this time because a sufficient number of electrons has been produced, which, when moving into the current positive space charge region, neutralize the positive charge and leave behind a new positive space charge region. In other words, the electron density in the high field region has increased to the degree that the Maxwellian relaxation time is short enough to screen the field out of the region. The profiles at 0.27 ms

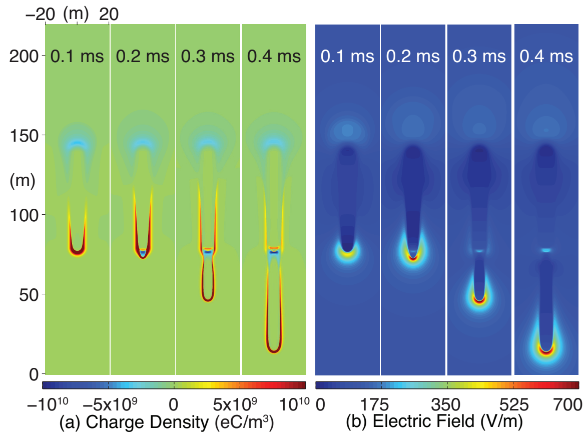

FIG. 2. Cross-sectional views of distributions of (a) charge density and (b) electric field during the initiation of the sprite streamer.

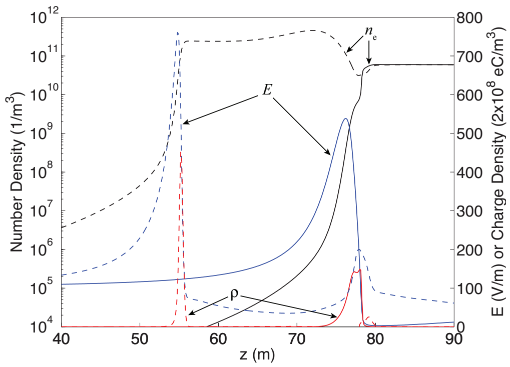

FIG. 3. Profiles of electron density, electric field and charge density along the symmetry axis at 0.17 (solid lines) and 0.27 ms (dashed lines) of the sprite streamer.

show a fully-formed streamer with a head field of $3.5E_k$ and a channel density of 2.4×10^{11} m^{-3}.

At the positive tip, the electrons ahead of the ionization region where the electric field is greater than the breakdown field E_k are produced by the photoionization process. The responsible photons are emitted from the ionization region. At the moment when the peak field starts to move, the preionization level ahead of the ionization region is still very low and the ionization time required for raising the electron density to the value capable of reducing the field is longer than Maxwellian relaxation time in the ionization region. While the location of the peak field moves forward, the magnitude of the electric field continues to increase as well as the electron density in the tip, which leads to an overshoot in the electron density profile (see the hump present on the density profile at 0.27 ms). Eventually, more and more photons are produced because the increasing field and size of the ionization region, raising the preionization level. A positive streamer with relatively constant electron density and head field begins to propagate along the direction of the electric field.

DISCUSSION

Space Charge at Ionization Column Tips. From equations (1) to (4), the following equation can be obtained for space charge density $\rho = e(n_p - n_e - n_n)$:

$$\frac{\partial \rho}{\partial t} = -\frac{e\mu_e n_e \rho}{\varepsilon_0} - \mu_e \vec{E} \cdot \nabla n_e, \tag{5}$$

where diffusion, recombination, and spatial variation of mobility are neglected as they are unimportant compared to other terms. The first term $-\frac{e\mu_e n_e \rho}{\varepsilon_0}$ on the right hand side is the dielectric relaxation term, and the second term $-\mu_e \vec{E} \cdot \nabla n_e$ describes how electron drift affects the space charge density. At the positive tip, the drift term is positive as $\vec{E}$ is opposite to the direction of electron density gradient while the same term gives an opposite sign at the negative tip. Initially, space charge density is small and the dielectric relaxation term is negligible. Due to the drift term, each polarity of charge accumulates at its end. As the space charge density increases, the dielectric relaxation term plays a more important role in the negative tip than the positive tip because the positive tip is depleted with electrons while the negative tip acquires an excess of electrons. This results in a diffuse negative tip but a compact positive tip.

Implications to Sprite Streamers. The results reported here offer an explanation to sprite streamer initiation in a lightning field about 0.2-0.6E_k as reported in [16, 17]. Ionization patches at sprite altitudes can be created by many atmospheric processes: thunderstorm and lightning related processes, meteorites, etc. In addition, it has been a puzzle why many sprites only consist of downward propagating positive streamers but no upward propagating negative streamers or if the negative streamers are present in a sprite, they often appear later and originate from existing structures at lower altitude [e.g., 8, 9]. Our aforementioned discussion indicates that asymmetric discharge conditions are created at the two tips, which make the initiation of positive streamers much easier than negative streamers. A recent, detailed experimental study showed that positive streamers can be triggered from a needle electrode at atmospheric pressure at a lower voltage than negative streamers [4]. Our results are in principle consistent with this experimental finding, although the discharge reported here originates from an isolated electrode (i.e., the column) while the electrode was connected to external circuit in [4].

As a matter of fact, with the same initial setup but an increased $E_0 \simeq 0.8E_k$, negative streamers are still absent when the positive streamer reaches the bottom boundary. However, there is a possibility that negative streamers will form eventually if the positive streamer is allowed to continue its development (see Ref. [14] for discussion on negative charging of the positive streamer trail and possible initiation of negative streamers due to this effect). It is known that a positive streamer developing from a positive electrode in such fields draws an exponentially increasing current [13]. This deposits a large amount of negative charge at the origin. It is very likely that the positive streamer developing from the ionization column also carries such a current. The current will deposit negative charge to the column faster than the dissipation from the negative tip, which may eventually lead to initiation of a negative streamer from that tip.

Streamer from Thundercloud Hydrometeors. To investigate possible streamer initiation from thundercloud hydrometeors in electric field smaller than E_k, a simulation was conducted for an applied field about $0.5E_k$ at ground pressure. The hydrometeor is modeled as an ionization column with a length of 5 mm, radius of 0.1 mm and peak density of 2×10^{20} m^{-3}. The reason why a hydrometeor can be modeled this way is their large dielectric constant ($\geq$80). The cross-sectional distributions of electron density and electric field are shown by Fig. 4. A positive streamer is successfully initiated from the positive end of the column but no negative streamer appears from the other end. The detailed dynamics of the field increase and discharges around the tips are similar to the low pressure case presented in Figs. 1-3. Our results are consistent with the experimental findings reported in [21] showing that positive streamers can form from isolated ice needles in a uniform applied field significantly below E_k. The dimension of the column is also consistent with the typical size of the ice crystals grown in [21] and falls into the upper range of the size of realistic thundercloud hydrometeors [22, p. 54 and 245]. It should be pointed out that the model hydrometeor best matches an ice needle aligned in thundercloud electric fields. Realistic hydrometeors in thunderclouds exhibit more complicated geometries, but the simple geometry chosen here allows us to find the most important factors that determine whether streamers can be

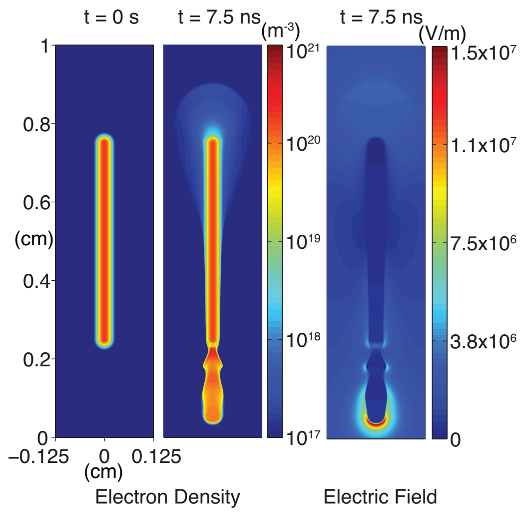

FIG. 4. Cross-sectional views of distributions of electron density and electric field of a streamer at ground pressure.

initiated in the field smaller than E_k (see the following section for estimating the required dimension of a hydrometeor to trigger streamers in an electric field below E_k). With the ionization patch, the electrode effects (such as plasma sheath, photoelectron emission and ion impact) across the discharge plasma and the ice needle are not included, but for the discharge around the positive tip the electrode effects are unimportant as electron avalanches move toward the electrode. As those effects are important for the discharge around the other tip, the discharge there can be different from what is reported here if a real ice particle is used in the simulation. Work is currently undertaken to model the discharge around a real hydrometeor and will be reported in a future paper. In addition to the application to the lightning problem, the simulation results at ground pressure are also applicable to laser-induced spark discharges in laboratory [23] and laser-induced electric discharge activities in thunderclouds [24].

Dimension and Density of Initial Ionization Column. In this section, we estimate the requirements for the dimension and density of the initial column for streamer initiation by approximating the column as a perfect conductor. Those two parameters determine the maximal field at the tip when the column is polarized. According to Ref. 25, the field at the tip of a cylindrical conductor in uniform E_0 is $E_m = [3 + 0.56(l/a)^{0.92}]E_0$, where l and a are the length and radius of the conductor, respectively. To be able to initiate a streamer, E_m should be around the streamer head field 3–5E_k if a takes a value of typical streamer radii. The value of l can then be determined as $\mathrm{l} = a[\frac{1}{0.56}(E_m/E_0 - 3)]^{1/0.92}$. The density required can be estimated by considering how much charge appears on the conductor. The linear charge density $\tau(z)$ on the conductor not too close to the ends is [26]

$$\tau(z) = \frac{4\pi\varepsilon_0 E_0 z}{\ln[4(l^2/4 - z^2)/a^2] - 2}, \tag{6}$$

where $z = 0$ is the center of the conductor. The charge density at $z = (l/2 - a)$ gives a low bound for the peak plasma density of the column:

$$n_{e0} = n_{p0} = \frac{\tau(l/2 - a)}{e\pi a^2}. \tag{7}$$

On the other hand, the requirement for the peak plasma density of the hemispherical Gaussian cap can be estimated by setting its total charge of single polarity equal to the total charge on half of an isolated conducting sphere with potential $E_0 l/2$ in a uniform field E_0:

$$n_{e0} = n_{p0} = \frac{2\varepsilon_0(3a + l)E_0}{e\pi^{0.5}a^2} \tag{8}$$

Choosing $a = 3$ m and $E_m = 3$–$5E_k$ for sprite streamers, $l = 21$–52 m and $n_{e0} = 2.1$–4.2×10^{10} m^{-3} from the above equations, which are not far from the actual values used: $l = 60$ m and $n_{e0} = 6 \times 10^{10}$ m^{-3}. In fact, simulations show that a streamer is able to form at $n_{e0} = 2\times10^{10}$ m^{-3} and $l = 60$ m but not at $n_{e0} = 1\times10^{10}$ m^{-3} and $l = 60$ m. In addition, a streamer successfully forms at $l = 24$ m and $n_{e0} = 5 \times 10^{10}$ m^{-3} but not at $l = 18$ m and $n_{e0} = 3\times10^{10}$ m^{-3}. It should be noted that the estimation from approximating the column as a cylindrical conductor may only give order of magnitude accuracy as during the polarization of the ionization column its geometry can deviate significantly from the original column. Similar calculation can be performed for streamer formation at thundercloud altitude, which can lead to the constraint for the dimension of thundercloud hydrometeors.

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