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Phys. Rev. Lett. **109**, 010401 — Published 6 July 2012
DOI: 10.1103/PhysRevLett.109.010401

The Higgs mode in a two-dimensional superfluid

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We present solid evidence for the existence of a well-defined Higgs amplitude mode in two-dimensional relativistic field theories based on analytically continued results from quantum Monte Carlo simulations of the Bose-Hubbard model in the vicinity of the superfluid-Mott insulator quantum critical point, featuring emergent particle-hole symmetry and Lorentz-invariance. The Higgs boson, seen as a well-defined low-frequency resonance in the spectral density, is quickly pushed to high energies in the superfluid phase and disappears by merging with the broad secondary peak at the characteristic interaction scale. Simulations of a trapped system of ultra-cold ^{87}Rb atoms demonstrate that the low-frequency resonance is lost for typical experimental parameters, while the characteristic frequency for the onset of strong response is preserved.

PACS numbers: 05.30.Jp, 74.20.De, 74.25.nd, 75.10.-b

The emergence of low-energy excitations in systems with spontaneously broken symmetry is one of the most fascinating and fundamental subjects in physics relevant for understanding such diverse systems as solids, magnets, ultra-cold atoms, and relativistic fields. The generation of mass by the Anderson-Higgs mechanism [1, 2] is particularly important for the Standard model [3], where detection of the Higgs boson is still the missing link in revealing this mechanism, as well as for numerous superfluid/superconducting condensed-matter systems. In realistic materials the amplitude mode is often masked by other low-energy excitations. These complications are avoided by considering atomic bosonic superfluids, described by a complex order parameter field, which constitute the cleanest experimental realization.

Generic superfluids do not feature a well-defined Higgs boson (by 'well-defined' we understand a mode seen as a sharp resonance). Weakly-interacting gases do not have it because at and around the critical temperature T_c for the superfluid (SF) to normal fluid phase transition all long-wave elementary excitations are overdamped, while at $T \to 0$ the low-energy spectrum is exhausted by the Bogoliubov quasiparticle excitations where phase and density are canonical variables. Strong interactions do not necessarily change this picture. As long as the critical temperature remains large, as in ^{4}He, long-wave excitations are overdamped at $|T - T_c| \ll T_c$. At low temperature, only the Nambu-Goldstone phase modes remain at low frequencies whereas the amplitude mode is pushed to the incoherent continuum at the (large) characteristic interaction energy scale. Suppressing T_c by increasing interactions may trigger a first order transition to the solid phase and not work either. It is thus crucial to consider an experimental system with a second-order quantum critical point (QCP) where T_c for superfluidity can be tuned to near zero.

The Bose-Hubbard model

$$H = -J \sum_{<ij>} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) - \sum_i (\mu - v_i) n_i \,, \quad (1)$$

with experimentally adjustable ratios between the hopping amplitude J, on-site interaction U, chemical potential μ, and trapping potential v_i, provides an accurate description of ultra-cold bosonic atoms in optical lattices. At integer filling factor, $\nu = \langle n_i \rangle$, and zero temperature it undergoes a second-order quantum phase transition from SF to the Mott insulator (MI) phase as the interaction strength is increased [4]. The critical field theory behind this transition is Lorentz-invariant and particle-hole symmetric (while the SF-MI transition for generic values of ν belongs to the universality class of the ideal Bose gas at vanishing density and is excluded) [4, 5]. Despite the decay into two phase modes the existence of a sharp Higgs boson is guaranteed in two special limits: (i) in a three-dimensional (3D) system where the corresponding 4D quantum field theory is at the upper critical dimension with asymptotically exact mean-field behavior and vanishing decay rates (see Ref. [6] for an experimental observation in a quantum antiferromagnet); (ii) at large momentum when the relativistic time dilation effect leads to an increased quasiparticle decay time.

The most intriguing question is whether the low-frequency Higgs boson can be seen as a well-defined excitation at zero momentum at the density-driven QCP of the 2D Bose-Hubbard model and how it disappears with detuning to the SF phase. Equally important are finite temperature effects and the role of the trapping potential in experiments. A theoretical treatment of the Higgs amplitude mode is notoriously difficult and controversial. In Refs. [5, 7–9] exact scaling laws in the low-frequency limit were established, as well as arguments given that the mode is at the edge of the two-phonon continuum, rendering the mode overdamped. Huber *et al.* used a

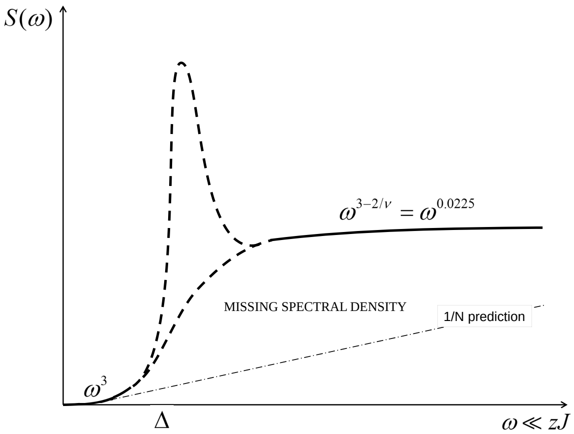

FIG. 1. Universal scaling predictions for the scalar susceptibility (solid lines). The dashed-dotted line depicts prediction of Ref. [12] and misses most of the spectral density at the relevant energy scale $\Delta \propto (1 - U/U_c)^{\nu}$. The two alternatives for connecting universal power laws are shown by dashed lines (one may also imagine multiple peaks in the crossover region).

variational Ansatz which, however, predicted a spurious first order SF-MI transition, and thus was limited to the parameter regime away from quantum criticality [10, 11]. Podolsky *et al.* generalized the field theoretical results of Ref. [7] to high frequencies and discussed in detail the response function for the order-parameter density (scalar response) within a $1/N$ and a weak coupling expansion schemes [12]. They revealed a broad peak whose maximum saturates at finite value at the QCP and concluded that close enough to the transition, it becomes impossible to identify the Higgs energy with the peak position. Their findings are in quantitative and qualitative disagreement with those reported here, as is detailed in the supplementary material [13]. The major problem with the results of Ref. [12] is the strong violation of the universal low-frequency scaling law for the scalar response function [5], $S(\omega) \propto \Delta^{3-2/\nu} F(\omega/\Delta)$, where $\Delta \propto (1 - U/U_c)^{\nu}$ is the characteristic energy scale in proximity of the quantum critical point, and $\nu = 0.6717$ the correlation length exponent. As is shown in Fig. 1, the theory misses most of the spectral density in the $\Delta < \omega < 4J$ range.

In this Letter, we employ quantum Monte Carlo simulations of the 2D model (1) in the lattice path integral representation using the worm algorithm [14–16] to study the spectral density of the kinetic energy correlation function at zero momentum, in combination with an analytic continuation method. We unambiguously demonstrate the existence of a low-energy resonance peak associated with the Higgs boson in close vicinity of the QCP by discriminating it from the second broad peak at the typical lattice-model energies. The Higgs boson energy, $\omega_{\rm H}$, obtained from the peak maximum increases with detuning nearly identically as that of the particle-hole gap Δ_{MI} in the MI phase. The spectral density associated with the Higgs boson broadens with detuning and quickly overlaps with other higher energy modes: It is no longer seen as a resonance peak for a detuning as small as 20 %, in line with the parameter regime where particle and hole masses were found to be equal on the MI side [17]. On the other hand, in close vicinity of the QCP the Higgs boson remains visible in the spectral density at temperatures as high as T_c. A peak is even seen in the normal phase; only at a temperature $T > 2T_c$ the Higgs resonance is no longer visible. However, the onset of strong response at low-frequency is barely modified. These results, further supported by simulations of realistic trapped systems, explain why the experimental protocol of extracting $\omega_{\rm H}$ from the onset of strong response [18] works even in the absence of low-frequency resonance.

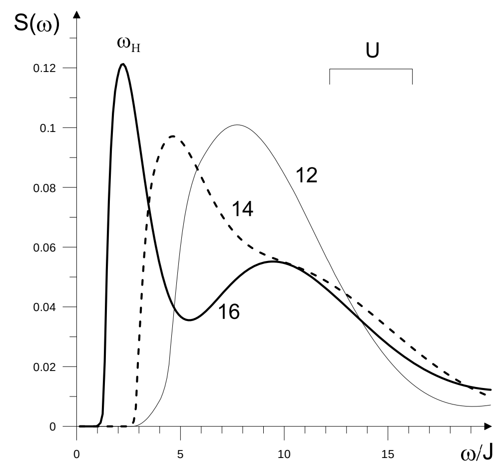

FIG. 2. Spectral density $S(\omega)$ of the kinetic energy correlation function for $U/J = 16$ (thick solid line), 14 (dashed line), and 12 (thin solid line) at low temperature $T/J = 0.1$. The Higgs amplitude mode ($\omega_{\rm H}$) emerges as a well-defined peak on approach to the quantum critical point at $U_c = 16.7424$.

A small uniform modulation of the optical lattice depth [19] leads, under mapping to the Bose-Hubbard model (1), to a perturbation proportional to the total kinetic energy of the system [20], $K = -J \sum_{\langle ij \rangle} b_i^{\dagger} b_j$,

$$V = \delta(t) K \,, \quad \delta(t) = \frac{\delta J(t)}{J} \,, \tag{2}$$

where the small $\delta J(t)$ is proportional to the lattice modulation amplitude. Within standard linear response theory one computes the corresponding correlation function $\chi(i\omega_n) = \langle K(\tau) K(0) \rangle_{i\omega_n} + \langle K \rangle$ at Matsubara frequencies, $\omega_n = 2\pi T n$, and performs an analytic continuation to obtain its spectral density $S(\omega)$. This quantity is directly proportional to the energy absorbed by the system in the experiment [18]. In the path integral representation $K(i\omega_n)$ has a straightforward Monte Carlo estimator, $\sum_k e^{i\omega_n \tau_k}$, where the sum goes over all hopping transitions in the imaginary time evolution of the system.

Thus $\chi(i\omega_n)$ is computed exactly, *i.e.,* the error bars are statistical and they can be reduced arbitrarily by increasing the simulation time. Our relative error bars for the lowest frequencies are of the order of 10^{-5}. Nevertheless, long simulations are required because finite χ is found only after the cancelation of macroscopic factors. The combination of the linear system size ($L = 20$ in practice) of our square lattice, temperature, and U/T has to be chosen such that L is always significantly larger than the correlation length by a factor of at least four in order for finite size effects to be negligible. In the supplementary material we provide details about the analytic continuation procedure which extracts the spectral density $S(\omega) = \mathrm{Im}\chi(\omega)$ from $\chi(i\omega_n)$ and show tests confirming the reliability of the results reported here [13].

Unambiguous evidence for the existence of the sharp amplitude mode in the vicinity of the quantum critical point located at $(U/J)_c = 16.7424$ [21] is provided in Fig. 2. We observe a well-defined resonance in the normalized spectral density at a scale much smaller than U, which softens on approach to the QCP in a way that is compatible with the $3D$ XY universality class, as is shown in Fig. 3. We identify the energy of the Higgs boson with the peak maximum. The width of the resonance peak narrows when $U \to U_c$, suggesting that this peak is part of the universal scaling scaling function [5] which can be rewritten as $S(\omega) \propto \omega_H^{3-2/\nu} F(\omega/\omega_H)$. Since at frequencies $\omega_H \ll \omega$ the response must be independent of ω_H we have $S(\omega \gg \omega_H) \propto \omega^{3-2/\nu} = \omega^{0.0225}$, *i.e.* it is increasing extremely slowly. The overall picture in the asymptotic limit is that of a Higgs peak superimposed on a smeared step function, see Fig. 1. When tuning away from the critical point, the Higgs mode broadens and overlaps with the second peak around the crossover scale $U/J \approx 12$. Beyond this point the Higgs boson can no longer be discerned as a separate mode, as is shown in Fig. 2. This limits the observation of the amplitude mode to the region in close vicinity of the QCP.

We also would like stress that the resonance at ω_H is seen on both sides of the QCP, *i.e.* it is also present in the MI phase close to the QCP. The similarity between the two responses for the same amount of detuning from the critical point, as is evident from Fig. 4, is expected inside the correlation volume despite obvious differences in the low-frequency part. This result provides further evidence that the analytic continuation procedure is stable.

Though our imaginary time data decaying as $\sim 1/\tau^4$ for large τ is compatible with the scaling prediction [7, 8] $S(\omega) \sim \omega^3$ for $\omega \to 0$, our errors bars at large imaginary times are too large and our system sizes too small to resolve this law unambiguously in analytic continuation.

There is substantial room for incoherent spectral weight between ω_{H} and $\omega \approx U$ which can be filled by other modes such as 'doublon' (double occupancy) excitations, pairs of phase modes with zero total momentum, as well as multi-Higgs modes. Our interpretation of the data is that higher frequency doublons ('screened' by interaction effects) overlap with lower frequency critical phase modes creating an intermediate broad peak at frequencies between U and zJ, except extremely close to QCP where the second peak saturates at about $6J$ when tuning $U \to U_c$. We associate it with pairs of phase modes with opposite momenta near the Brillouin zone

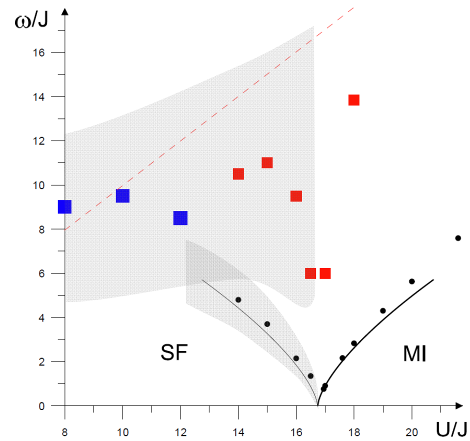

FIG. 3. (Color online). Characteristic energies in the vicinity of the quantum critical point at $(U/J)_c = 16.7424$. Black circles for $U > U_c$ and $U < U_c$ stand for particle-hole gaps in the MI phase [17] and energies of the Higgs bosons, respectively. Red squares denote the location of the broad secondary peak in $S(\omega)$ until it merges with the amplitude mode at interaction strength $U \leq 12J$ to form a single peak (blue squares). Shaded regions indicate the characteristic broadening of peaks. The thick black line is the critical law $2.25J|(U - U_c)/J|^\nu$ obtained by fitting the smallest MI gaps; its mirror reflection is shown as thin black line. The dashed red line indicates the typical interaction scale U/J.

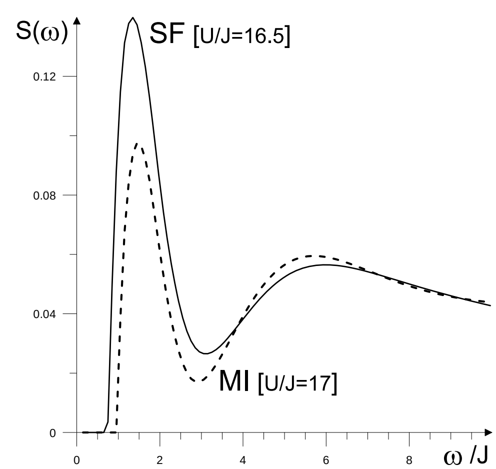

FIG. 4. Spectral densities in the SF and MI phases approximately with the same amount of detuning from quantum criticality $|U_c - U|/U \approx 0.015$.

boundary (which dominate in the integral over momentum space). This classification, however, is not rigorous in the quantum critical region [5], as is evidenced by the similarity between the SF and MI responses.

Current experiments with ultra-cold atoms are typically performed at finite temperature $T/U \geq 0.05$ (such that $T/J \sim \mathcal{O}(1)$ at $U = U_c$) and in the presence of parabolic confinement. In Fig. 5 we demonstrate that for the representative case $U/J = 16$ with $T_c/J \approx 0.45$ the Higgs mode remains clearly visible at all temperatures below the superfluid transition temperature and even slightly above it! At a temperature $T > 2T_c$ the two peaks finally merge . Nevertheless, $S(\omega)$, still levels off at the amplitude mode frequency $\omega \approx \omega_{\rm H}$ and has the same frequency for the onset of strong response, meaning that these features can be used to extract ω_H experimentally. Note that phase coherence can extend across finite systems at temperatures well above the thermodynamic T_c for the Kosterlitz-Thouless transition characterized by an exponentially divergent correlation length.

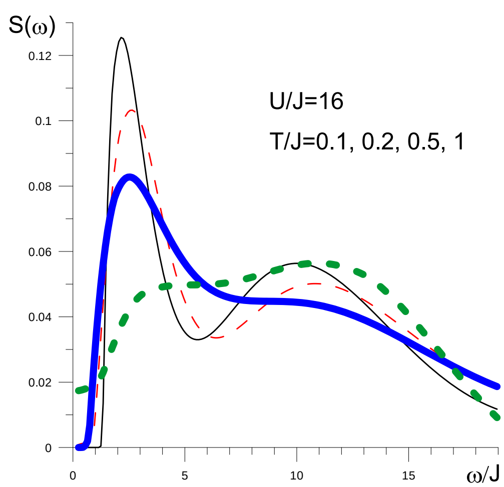

FIG. 5. (Color online). Evolution of spectral density with temperature at $U/J = 16$. As temperature is increased from $T/J = 0.1$ (thin black line), to $T/J = 0.2$ (thin dashed line), and $T/J = 0.5$ (thick blue line), the peaks get broader but remain clearly identifiable. At $T/J = 1$ (thick dashed green line) the two peaks merge.

Inhomogeneous broadening of spectral density caused by the trap has a dramatic effect on the structure of $S(\omega)$ as signals from different parts of the system are superimposed on each other. Moreover, in the presence of external potential gradients the spectral density is no longer vanishing at $\omega \to 0$ because of low-frequency sound modes (predominantly in the trap edges), in line with experimental observations [18]. Under these conditions, the Higgs mode can no longer be seen as a sharp resonance in $S(\omega)$. There is instead a broad maximum with irregular shape. Finite temperature effects further transform it into a smooth single peak. In Fig. 6 we show the comparison between the homogeneous and trapped cases. The simulation was performed for realistic experimental parameters [18] but at a variety of temperatures in order to discriminate between trap and temperature effects. Even though the resonance is lost in the total signal, the steep onset of spectral response still correlates remarkably well with the energy of the Higgs boson.

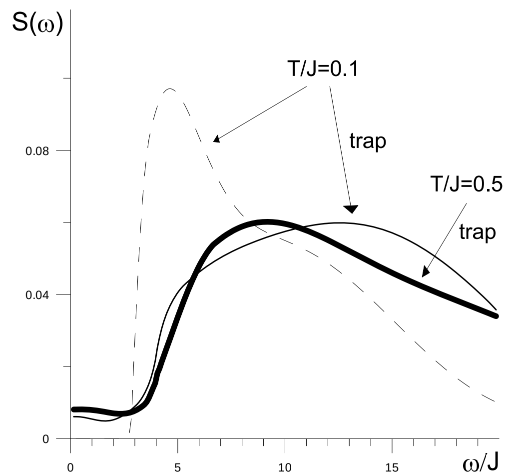

FIG. 6. Effects of trapping potential and finite temperature for $U/J = 14$ ($T_c/J \approx 1.04$). The spectral density of a homogeneous system at low temperature with Higgs resonance (dashed line) transforms into a broad (irregular) peak due to inhomogeneous broadening in a trapped system with $N = 190$ particles at unit filling factor in the middle (thin solid line). At a temperature $T/J = 0.5$ (thick solid line) we observe a smooth single peak.

In conclusion, we find that the Higgs boson is a well-defined though significantly damped excitation in close vicinity ($\sim$ 20%) of the particle-hole symmetric and Lorentz-invariant SF-MI quantum critical point in two dimensions. It is seen as a resonance in the kinetic energy correlation function which is directly probed through the modulation of the optical lattice depth in experiments with ultra-cold atoms. The energies of the amplitude mode match particle-hole gaps in the Mott insulator phase for the same amount of detuning away from quantum criticality. While temperatures at least as high as the critical temperature for superfluidity preserve the Higgs resonance, inhomogeneous broadening in small trapped systems erases resonance-type features in the spectral function. Nevertheless, it is possible to determine the energy of the amplitude mode from the onset of strong response, as is done in a recent experiment [18].

This work was supported by the National Science Foundation under grant PHY-1005543. Use was made of the maxent application [22] in the ALPS libraries [23]. We thank S. Sachdev, B. Svistunov, W. Zwerger, and the authors of Ref. [18] for insightful discussions, as well as for sharing their results with us prior to publication.

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