How Well Do We Know the Neutron Structure Function?

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How well do we know the neutron structure function?

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We present a detailed analysis of the uncertainty in the neutron F_{2n} structure function extracted from inclusive deuteron and proton deep-inelastic scattering data. The analysis includes experimental uncertainties as well as uncertainties associated with the deuteron wave function, nuclear smearing, and nucleon off-shell corrections. Correctly accounting for the Q^2 dependence of the data and calculations and restricting the nuclear corrections to microscopic models of the deuteron, we find significantly smaller uncertainty in the extracted F_{2n}/F_{2p} ratio than in previous analyses. In addition to yielding an improved extraction of the neutron structure function, this analysis also provides an important baseline that can be compared to future, model-independent extractions of neutron structure to examine nuclear medium effects in the deuteron.

Because the free neutron is an experimentally impractical scattering target, extracting its structure function, F_{2n}, requires cross section data from inclusive deep-inelastic scattering (DIS) measurements on proton and deuteron targets, together with a model describing the smearing produced by the nuclear binding in the deuteron. Previous extractions [1–3] using a variety of models of the deuteron bound state yielded a large range of F_{2n}/F_{2p} values from the same proton and deuteron data, indicating large theoretical uncertainties, particularly at high values of the Bjorken variable x — see Fig. 1. In addition to limiting our ability to extract the neutron structure function, the large spread of results, even among extractions including only traditional nuclear effects such as Fermi motion and binding, has made it difficult to identify a reliable baseline which could be used to search for more "exotic" nuclear effects such as the modification of the nucleon structure function in nuclei or non-nucleonic degrees of freedom.

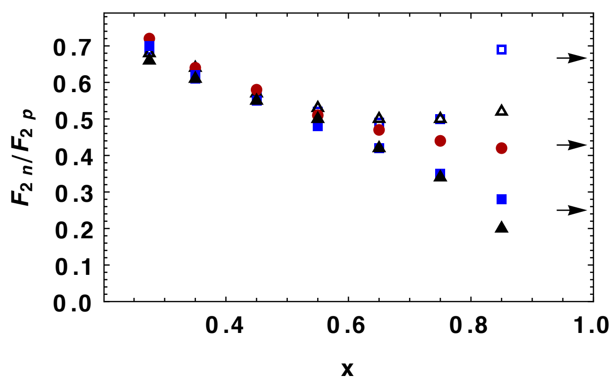

FIG. 1: (Color online) Previous extractions of F_{2n}/F_{2p}, using microscopic deuteron calculations (filled symbols) or extrapolations of nuclear effects in heavier nuclei (open symbols): (black) triangles from Ref. [1], (blue) squares from Ref. [2], (red) circles from Ref. [3]. The arrows indicate theoretical predictions for the $x \to 1$ limit (see text).

A more recent extraction of the neutron F_{2n} structure function [4] showed that some of the variation in results can be attributed to inconsistent treatment of kinematics of the data and calculations. In particular, it was vital to properly account for the Q^2 dependence of the proton and deuteron data, especially for $x \gtrsim 0.7$ [4, 5].

Accurate information on the F_{2n}/F_{2p} ratio at large x is essential for a number of reasons. These include constraining leading-twist parton distribution functions (PDFs) in the region $x \gtrsim 0.7$, which are an important input for QCD background calculations in searches of physics beyond the standard model at the Tevatron and the LHC, and in neutrino oscillation experiments. The ratio F_{2n}/F_{2p} also provides insight into the nonperturbative quark-gluon dynamics in the nucleon [3, 6]. There are several predictions for F_{2n}/F_{2p} based on symmetry arguments that determine the dominant contributions as $x \to 1$ (see Ref. [6] for details), ranging from $F_{2n}/F_{2p} = 2/3$ for exact spin-flavor SU(6) symmetry, to 3/7 assuming helicity conservation through hard gluon exchange [7], to 1/4 when SU(6) symmetry is broken through scalar diquark dominance [8, 9]. Many other models yield ratios consistent with one of the symmetry-based predictions [10, 11], while others find different $x \to 1$ limits, typically below $F_{2n}/F_{2p} = 0.4$ [6, 10, 12–14].

In this paper we extend the analysis of Ref. [4] by performing a detailed study of the model dependence of the extraction procedure, systematically assessing the uncertainties arising from the deuteron wave function at short distances, nucleon off-shell effects, and different nuclear smearing models used to compute the nuclear corrections. The goal is to determine the degree to which the F_{2n} neutron structure function can be determined at large x, using a consistent treatment of input data sets and realistic models of nuclear effects, and employing a methodology that is transparent and conceptually accessible. A similar study was recently performed which emphasized the impact of the nuclear uncertainties in the extraction of the parton distribution functions [5]. Our analysis is focused on the direct extraction of the neutron structure function, allowing us to isolate the impact of the nuclear effects by avoiding tension between the deuteron DIS data and other measurements sensitive to the d/u ratio. We also avoid uncertainties associated with decomposing the structure function into leading twist PDFs and higher-twist corrections, which leads to large uncertainties in the high-x PDFs that are not relevant when analyzing the structure function itself.

Following Ref. [4], we consider F_{2d}/F_{2p} data from SLAC, BCDMS, and NMC, as compiled in Ref. [1] for $3 < Q^2 < 230$ GeV^2, using the full Q^2 range to determine the Q^2 dependence of the ratio. The extraction of F_{2n}/F_{2p} itself is limited to data in the range $8 < Q^2 < 32$ GeV^2, and the results are interpolated to a fixed $Q^2 = Q_0^2 \equiv 16$ GeV^2. The interpolation to Q_0^2 differs from the simple average over the limited Q^2 range by at most 0.7%, with a typical correction of 0.3%. Most previous extractions of F_{2n}/F_{2p} used F_{2d}/F_{2p} from the analysis of Ref. [1], in which Q^2 varies from 4.7 to 23.6 GeV^2 over the x range of the data. However, the extractions treat the F_{2d}/F_{2p} ratios as though they were all at some average Q^2 value. Such extractions neglect the Q^2 dependence of the nuclear effects, which have been found to be significant at large x [4, 5]. By interpolating all of the source data to a common Q_0^2 and extracting F_{2n}/F_{2p} using different models evaluated at the same scale, a more systematic and meaningful assessment of the model dependence can be made.

The extraction of F_{2n}/F_{2p} proceeds by assuming that the deuteron structure function F_{2d} can be expressed as a sum of smeared proton and neutron structure functions, and an additional correction, δF_{2d}, that goes beyond the convolution approximation, $F_{2d} = \overline{F}_{2p} + \overline{F}_{2n} + \delta F_{2d}$, where $\overline{F}_{2N} = S_N\, F_{2N}$ is the smeared nucleon structure function ($N = p, n$), and S_N is the smearing ratio. The term δF_{2d} includes any corrections, such as relativistic or nucleon off-shell corrections [15, 16], that cannot be expressed as a convolution of a smearing function and the free nucleon structure function. Parameterizing this correction as a ratio to the total deuteron structure function, $\Delta = \delta F_{2d}/F_{2d}$, the F_{2n}/F_{2p} ratio can then be extracted using a modified smearing factor $\widetilde{S}_N = S_N/(1-\Delta)$,

$$\frac{F_{2n}}{F_{2p}} = \frac{1}{\widetilde{S}_n}\left(\frac{F_{2d}}{F_{2p}} - \widetilde{S}_p F_{2p}\right). \qquad (1)$$

The neutron to proton ratio can thus be extracted from F_{2d}/F_{2p} and F_{2p} data, and a model of the smearing ratios S_N and the off-shell correction Δ.

To standardize comparisons of the different calculations, all extractions use the same values for F_{2p} and F_{2n} to compute the smearing functions $\widetilde{S}_N(x)$. We use the parametrization of the world's F_{2p} data and the extracted neutron structure function from Ref. [4]. Each calculation yields a slightly different F_{2n}, but the impact of recalculating S_n using this modified F_{2n} value is small compared to the other uncertainties (F_{2n}/F_{2p} changes by less than 0.01 at $x = 0.85$), as discussed in Ref. [17].

The assessment of the model dependence of the extracted F_{2n}/F_{2p} ratio ultimately depends on the choice of nuclear models used in the analysis. The introduction of some degree of bias is therefore inevitable, although we aim to make the selection criteria as objective as possible by restricting ourselves to microscopic calculations involving high-precision NN potentials that give realistic descriptions of the deuteron bound state. Common features of the models surveyed in this analysis include the use of realistic deuteron wave functions which account for nuclear Fermi motion and binding, an exact treatment of finite-Q^2 kinematics, and allowance for possible nucleon off-shell corrections in the deuteron [3, 4, 18, 19]. We exclude models that involve extrapolations of nuclear medium modifications observed in structure functions of heavy nuclei to the deuteron [1, 2, 20, 21] which assume that $S_n = S_p$, neglect the Q^2 dependence in both the smearing functions and the initial F_{2d}/F_{2p} measurements, and also assume that the Fermi motion effect scale with density. All of these assumptions begin to fail for x above 0.5–0.6, making such approaches unreliable in the region of interest. In addition, many of these extractions invoke a very large nuclear density for the deuteron.

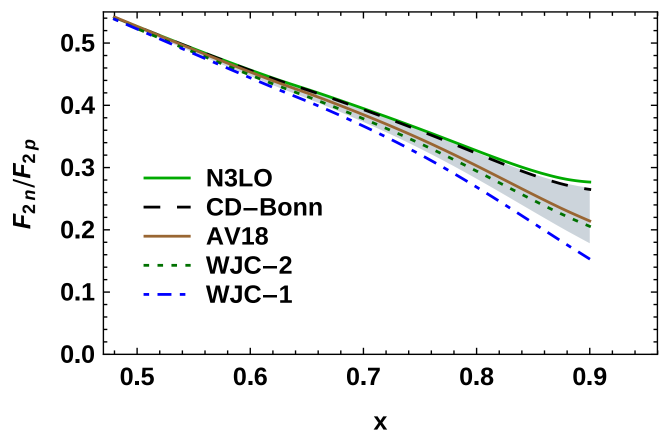

FIG. 2: (Color online) Neutron to proton structure function ratio F_{2n}/F_{2p} calculated using various deuteron wave functions (see text), within the WBA smearing model [16, 18].

The dependence on the choice of deuteron wave functions is illustrated in Fig. 2, which shows the results for F_{2n}/F_{2p} using different modern nonrelativistic (N3LO [22], CD-Bonn [23], AV18 [24]) and relativistic (WJC-1, WJC-2 [25]) deuteron wave functions. The smearing factors for each of the calculations were computed using the weak binding approximation (WBA) model [16, 18], which is derived by expanding the nucleon correlation function in the nucleus in powers of the nucleon momentum $\boldsymbol{p}$ up to order $\boldsymbol{p}^2/M^2$, where M is the nucleon mass. The gray band represents the RMS spread of the F_{2n}/F_{2p} ratios for the five wave functions considered, which is the 1σ band for the wave function uncertainty if we treat each of the NN potentials on an equal footing. The ratio F_{2n}/F_{2p} becomes increasingly sensitive to the choice of deuteron wave function at larger x values, reflecting the larger uncertainty in the NN interaction at short distances.

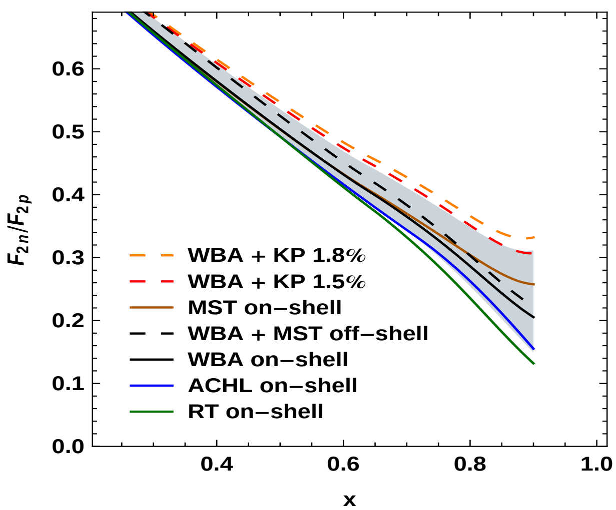

FIG. 3: (Color Online) The ratio F_{2n}/F_{2p} calculated using different smearing models, taking the average of the NN potentials from Fig. 2. The solid curves are on-shell calculations: WBA [16, 18], relativistic MST [3, 15], the light-front ACHL [4], and the RT [19] models. The dashed curves show the WBA result with various off-shell prescriptions: MST [15], and KP [16] with two different nucleon swelling parameters, 1.5% and 1.8% (see Ref. [26]). The ordering of the curves in the legend is based on the value of the extraction at $x = 0.9$.

The dependence of F_{2n}/F_{2p} on the model used for the smearing function (or smearing factor) and off-shell prescription is illustrated in Fig. 3. The curves show the results for the given calculation, averaged over the potentials shown in Fig. 2. For calculations where not all potentials were available, the WBA result was used to extrapolate to the average potential. Note that some smearing calculations, such as those used in Ref. [1], have been omitted as they represent calculations similar to those included here but with additional numerical approximations. The solid curves in Fig. 3 are the results of smearing calculations with on-shell nucleon structure functions, with their spread indicating the uncertainty associated with the smearing function. The WBA calculation (solid black curve) is a modern calculation that makes minimal approximations, and is used as the baseline for showing the results of calculations including nucleon off-shell corrections. The four WBA results (solid black curve and the three dashed curves) indicate the model dependence in the off-shell prescriptions.

To estimate the combined uncertainty, we take the RMS spread of all of the extractions of F_{2n}/F_{2p} shown in Fig. 3 at each x value, indicated by the light gray band. Similar uncertainties are obtained if the model dependence of the smearing function and off-shell contributions are extracted separately and combined in quadrature. The uncertainty associated with the smearing function is essentially negligible up to $x = 0.6$, but is comparable to the off-shell corrections for $x > 0.75$.

Figure 4 shows the combined uncertainty range (gray band) compared to the range of results shown in Fig. 1 (red hatched region). The central result is taken as the global average of the extracted F_{2n}/F_{2p} values obtained in Fig. 3. The individual uncertainties associated with the experimental systematic uncertainties (evaluated in Ref. [4]), the dependence on the deuteron wave function, and the dependence on the smearing function and off-shell effects (labeled “Model Uncertainty” in Fig. 4), are shown separately, as well as the sum of uncertainties added in quadrature.

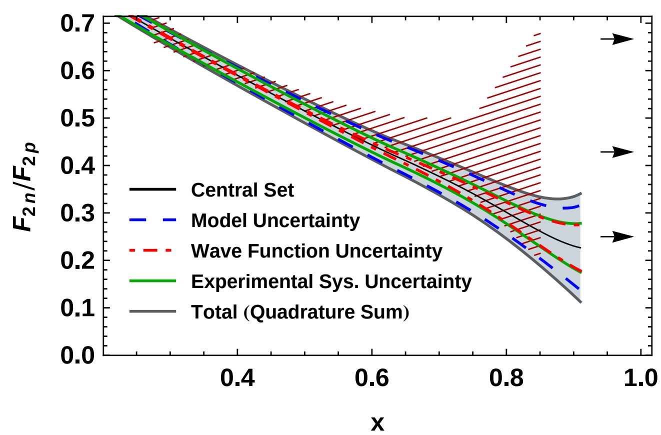

FIG. 4: F_{2n}/F_{2p} ratio together with the individual contributions to the systematic uncertainty and the quadrature sum [27]. The red hatched region corresponds to the uncertainty range in Fig. 1.

Our analysis provides a significantly narrower range of results than that evident in Fig. 1 for the full spectrum of models. At $x = 0.85$, for example, the range in Fig. 1 spans $0.2 < F_{2n}/F_{2p} < 0.7$, whereas the present analysis suggests a 1σ range of $0.18 < F_{2n}/F_{2p} < 0.32$. The tighter bounds are largely due to the exclusion of models involving extrapolation of nuclear medium effects from heavy nuclei, and would appear to exclude the SU(6) predictions of $F_{2n}/F_{2p} \to 2/3$, while favoring the lower estimates consistent with the partonic lower limit of $F_{2n}/F_{2p} \to 1/4$. If one assumes isospin symmetry and works in the parton model, the Nachtmann limit [28] implies $F_{2n}/F_{2p} > 0.25$ as a consequence of the positivity of leading order parton distribution. Since effects such as higher twist (which are implicitly included in the empirical structure functions) can modify this limit, we do not impose this constraint in the extraction of F_{2n}/F_{2p}, although our range of results is not inconsitent with this limit. If we chose to apply this condition as part of the analysis, then the region of possible results would be even smaller.

Figure 5 shows the total uncertainty bands, including experimental systematics and deuteron wave function dependence, corresponding to two different subsets of results. The solid lines show the range obtained by taking only the on-shell extractions in Fig. 3, which yield $0.16 < F_{2n}/F_{2p} < 0.28$ at $x = 0.85$. This range can be thought of as a baseline for effects beyond the on-shell convolution approximation, and comparison of these results to model-independent extractions of F_{2n}/F_{2p} can be used to isolate off-shell contributions or other more exotic nuclear effects. Again, if one imposes the Nachtmann limit and requires $F_{2n}/F_{2p} > 0.25$, then this analysis suggests a very narrow range for the on-shell extractions.

On the other hand, most modern quantitative analyses of nuclear structure functions and the nuclear EMC effect require the inclusion of some modification of the nucleon structure function in a nuclear medium [16, 29–31]. Restricting the set to only models that incorporate off-shell effects, one obtains the dashed blue band in Fig. 5. As expected, this leads to a higher range for the neutron to proton ratio, $0.25 < F_{2n}/F_{2p} < 0.36$ at $x = 0.85$.

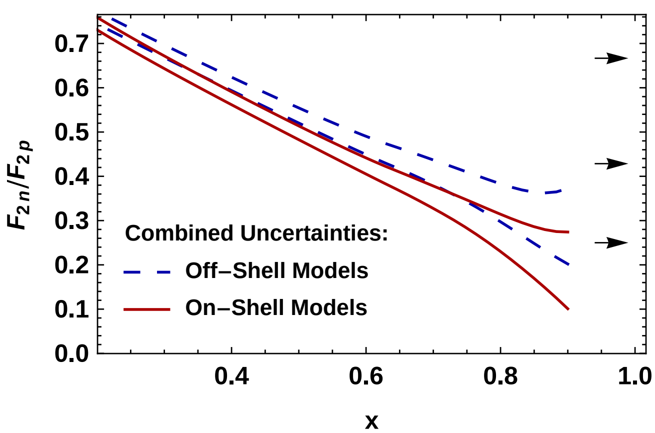

FIG. 5: F_{2n}/F_{2p} ranges for on-shell (solid) and off-shell (dashed) extractions. [27]

At leading order in the strong coupling constant, the nucleon structure functions are given by the charge-squared weighted sum of the u and d quark distributions. In this approximation the extracted value of F_{2n}/F_{2p} is directly related to the ratio of d to u quark distributions,

$$\frac{d}{u} = \frac{4F_{2n}/F_{2p} - 1}{4 - F_{2n}/F_{2p}}. \qquad (2)$$

The resulting d/u ratio is shown in Fig 6, along with the fractional uncertainty (inset), for the full range of models, as well as for the on-shell and off-shell models from Fig. 5. Higher-twist corrections do not affect this ratio, as long as they are identical for the proton and neutron (see, however, Ref. [32]), and target mass corrections also cancel to a large extent, although some residual prescription dependence survives as $x \to 1$ [33]. One can see from the inset that the *fractional* uncertainty in the d quark distribution becomes very large for $x \gtrsim 0.7$, yielding large uncertainties on quantities sensitive to $d(x)$, such as the PDF inputs for d quark dominated cross section calculations in high-energy collisions. However, the *absolute* uncertainty on the d quark PDF is small compared to the overall size of the u ($< 10\%$ for all x values shown), giving a fairly precise measure of the relative contributions from u and d quarks and for observables which are not especially sensitive to the d quark contributions.

While the extracted d/u ratio in Fig. 6 is illustrative of the reduced uncertainty from the restricted range of nuclear corrections considered here, the primary goal of the present work is an extraction of the total F_{2n} rather than a separation of the PDFs from target mass and higher twist contributions. A complementary study which quantified the effects of nuclear corrections on PDFs within a global QCD analysis was recently performed by the CJ Collaboration [26]. The largest impact of the nuclear effects, which were computed using the WBA smearing function with nucleon off-shell corrections from the modified KP model (see Fig. 3), was found for the d quark PDF at large x. The uncertainties in the resulting d/u ratio were similar to those in Fig. 6, although with a somewhat larger spread. In particular, the range in Ref. [26] for the neutron to proton ratio at $x = 0.85$ was found to be $0.32 < F_{2n}/F_{2p} < 0.50$ at $Q^2 = 16$ GeV^2, where the upper and lower limits were obtained using models with the minimum and maximum nuclear corrections, respectively. This is a larger range than found here, although it represents the full range of results, rather than an estimated 1σ error band. Taking a linear sum of all uncertainties from Fig. 4 yields the range $0.04 < F_{2n}/F_{2p} < 0.42$. The upper limit is in reasonable agreement with the CJ global fit, while the lower limit is significantly smaller. This is because PDFs in global QCD analyses are constrained to be positive *a priori*, forcing the overall d/u bands (and consequently F_{2n}/F_{2p}) to lie higher than those in Fig. 6.

In summary, we have extracted the F_{2n}/F_{2p} structure function ratio from a range of microscopic models of the deuteron structure, and estimated the uncertainties associated with the choice of nuclear smearing model, deuteron wave function, nucleon off-shell corrections, and experimental uncertainties. After correctly accounting for the Q^2 dependence of the convolutionm, neglected in several previous extractions, the various microscopic models evaluated here provide a narrower range of results, providing reliable baseline for the neutron structure assuming only traditional nuclear effects. With this, future model-independent extractions of the neutron structure function [34–36] can both improve our knowledge of the neutron structure and have the potential to provide signatures for more exotic nuclear effects.

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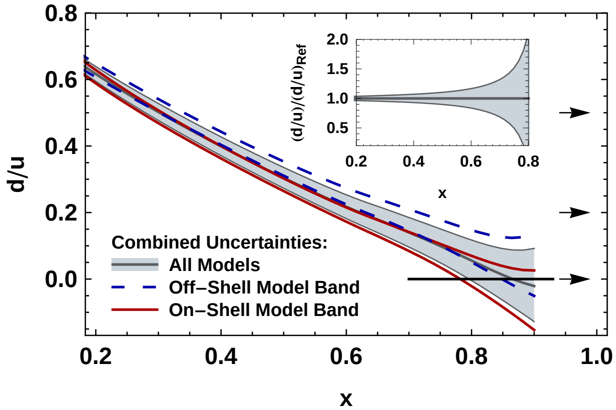

FIG. 6: Ratio d/u extracted from F_{2n}/F_{2p} at $Q^2 = 16$ GeV^2 using Eq. (2), with the gray band indicating the total uncertainties as shown in Fig. 4, with the dashed (solid) curves representing the on-shell (off-shell) nuclear model results. The inset shows the growing fractional uncertainty on d/u as the ratio becomes smaller for $x \to 1$.

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[1] L. W. Whitlow, E. M. Riordan, S. Dasu, S. Rock, and A. Bodek, Phys. Lett. B **282**, 475 (1992).
[2] A. Bodek, S. Dasu, and S. E. Rock, Lepton/Photon Symp. **v.1**, 160 (1991).
[3] W. Melnitchouk and A. W. Thomas, Phys. Lett. B **377**, 11 (1996).
[4] J. Arrington, F. Coester, R. J. Holt, and T.-S. H. Lee, J. Phys. G **36**, 025005 (2009).
[5] A. Accardi et al., Phys. Rev. D **81**, 034016 (2010).
[6] R. J. Holt and C. D. Roberts, Rev. Mod. Phys. **82**, 2991 (2010).
[7] G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. **35**, 1416 (1975).
[8] R. P. Feynman, *Photon Hadron Interactions* (Benjamin, Reading, Massachusetts, 1972).
[9] F. E. Close, Phys. Lett. B **43**, 422 (1973).
[10] F. Close and W. Melnitchouk, Phys. Rev. C **68**, 035210 (2003).
[11] A. W. Schreiber, A. Signal, and A. W. Thomas, Phys. Rev. D **44**, 2653 (1991).
[12] L. Chang, I. Cloet, C. Roberts, and H. Roberts, AIP Conf.Proc. **1354**, 110 (2011), arXiv:1101.3787.
[13] J. Soffer, AIP Conf. Proc. **1369**, 165 (2011).
[14] H. Mineo, W. Bentz, N. Ishii, and K. Yazaki, Nucl.Phys. **A703**, 785 (2002).
[15] W. Melnitchouk, A. W. Schreiber, and A. W. Thomas, Phys. Lett. B **335**, 11 (1994).
[16] S. Kulagin and R. Petti, Nucl. Phys. **A 765**, 126 (2006).
[17] I. R. Afnan et al., Phys. Rev. C **68**, 035201 (2003).
[18] Y. Kahn, W. Melnitchouk, and S. A. Kulagin, Phys. Rev. C **79**, 035205 (2009).
[19] A. Rinat and M. Taragin, Phys. Lett. B **551**, 284 (2003).
[20] L. L. Frankfurt and M. I. Strikman, Phys. Rept. **160**, 235 (1988).
[21] L. Weinstein et al., Phys. Rev. Lett. **106**, 052301 (2011).
[22] D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003).
[23] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
[24] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[25] F. Gross and A. Stadler, Phys. Lett. B **657**, 176 (2007).
[26] A. Accardi et al., Phys. Rev. D **84**, 014008 (2011).
[27] See EPAPS Document No. ??? for the extracted F_{2n}/F_{2p} ratios and uncertainties shown in Figs. 4 and 5.
[28] O. Nachtmann, Nucl.Phys. **B63**, 237 (1973).
[29] J. R. Smith and G. A. Miller, Phys. Rev. Lett. **91**, 212301 (2003).
[30] J. R. Smith and G. A. Miller, Phys. Rev. C **65**, 055206 (2002).
[31] I. Cloet, W. Bentz, and A. W. Thomas, Phys. Lett. B **642**, 210 (2006).
[32] S. I. Alekhin, S. A. Kulagin, and S. Liuti, Phys. Rev. D **69**, 114009 (2004).
[33] L. Brady, A. Accardi, T. Hobbs, and W. Melnitchouk, Phys. Rev. D **84**, 074008 (2011).
[34] S. Bültmann et al., Jefferson Lab expt. E12-10-102.
[35] J. Gomez et al., Jefferson Lab expt. E12-10-103.
[36] N. Baillie et al. (2011), arXiv:1110.2770.