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Domain-wall dynamics in translationally non-invariant nanowires: theory and applications

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We generalize domain-wall dynamics to the case of translationally non-invariant ferromagnetic nanowires. The obtained equations of motion make the description of the domain-wall propagation more realistic by accounting for the variations along the wire, such as disorder or change in the wire shape. We show that the effective equations of motion are very general and do not depend on the model details. As an example of their use, we consider an hourglass-shaped nanostrip in detail. A transverse domain wall is trapped in the middle and has two stable magnetization directions. We study the switching between the two directions by short current pulses. We obtain the exact time dependence of the current pulses required to switch the magnetization with the minimal Ohmic losses per switching. Furthermore, we find how the Ohmic losses per switching depend on the switching time for the optimal current pulse. As a result, we show that as a magnetic memory this nanodevice may be 10^5 times more energy efficient than the best modern devices.

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During last two decades there has been a significant progress in describing magnetization and, in particular, domain-wall (DW) dynamics in magnetic nanostructures [1–35]. The interest to these studies has been inspired not only by the fundamental physics questions but also by the potential applications for the spintronic memory and logic nanodevices [36–38]. However, recently this progress has been staggered due to the inability to make perfect translationally invariant nanowires from one side and the difficulty to apply the theories made for translationally invariant systems to successfully describe some of the phenomena in the experimental systems from the other side. There have been attempts to consider analytically and numerically [27–31] the systems with rough surfaces or other disorder but the general theory for translationally non-invariant magnetic systems is still lacking.

In this Letter we generalize current and field induced DW dynamics to the case of translationally non-invariant ferromagnetic nanowires. This generalization makes the description of the DW motion more realistic by accounting for variations along the wire, such as disorder or change in the wire shape. We show that the effective equations of DW motion are very general and do not depend on the details of the model Hamiltonian. These equations are the main theoretical result of this Letter.

As an example of the application of this theory, we study current-induced magnetization switching in a thin hourglass-shaped nanostrip. We show that a transverse DW trapped in the middle of a curved inward nanostrip can serve as a magnetic memory device, see Fig. 1. At zero current, the magnetization in the transverse DW can have either of the two equilibrium directions in the plane of the nanostrip. We study the switching between these two magnetization directions mediated by short current pulses with the requirement that the DW returns to the initial position after switching. The main energy loss during the switching is due to Ohmic heating of the wire. The minimum energy required per switch depends on the switching time. We obtain the exact time dependence of the current pulses required to switch the magnetization in the

most efficient way (with the minimal Ohmic losses per switching for a given switching time). Furthermore, we find how the Ohmic losses depend on the switching time for this optimal current pulse.

The two equilibrium magnetization directions can serve as "zero" and "one" of a memory bit. We show that based on this prototype, it is possible to design a nonvolatile memory device with an extremely short writing time, which is only limited by the spin-wave frequency. The energy required per switch for such a nanodevice is much lower than that for the state-of-the-art memory devices.

Equations of motion. We study the DW dynamics by employing the Landau-Lifshitz-Gilbert (LLG) equation with current terms [19, 28]. In general, a nontrivial solution of the static LLG equation, $\mathbf{S}_0 \times \delta\mathcal{H}_0/\delta\mathbf{S}_0 = 0$, has a continuous symmetry. It means that this solution can be parametrized by an even-dimensional vector $\boldsymbol{\xi}$ such that $\mathbf{S}_0(z, \boldsymbol{\xi})$ is the solution for any $\boldsymbol{\xi}$ in a continuous interval. Then the DW dynamics due to a correction to Hamiltonian \mathcal{H}_0 or an electric current can be described by the time-dependent parameter $\boldsymbol{\xi}(t)$. The equations for $\boldsymbol{\xi}(t)$ are called the effective equations of motion. Below we sketch the general derivation of such effective equations.

To consider the dynamics we assume that there is a perturbation \mathbf{h} , such that the full LLG equation takes the form

$$\dot{\mathbf{S}} = \mathbf{S} \times \frac{\delta\mathcal{H}_0}{\delta\mathbf{S}} + \mathbf{h}, \quad (1)$$

where the time is measured in units of the gyromagnetic ratio $\gamma_0 = g|e|/(2mc)$ and $\mathbf{S} = \mathbf{M}/M$ with M being the saturation magnetization. We note that \mathbf{h} is not only a correction to Hamiltonian and may also contain other contributions such as dissipation, adiabatic and nonadiabatic current terms. We look for a solution of this equation in the form $\mathbf{S}(z, t) = \mathbf{S}_0(z, \boldsymbol{\xi}(t)) + \mathbf{s}$, where the dependence $\boldsymbol{\xi}(t)$ is weak and \mathbf{s} is small and orthogonal to \mathbf{S}_0 at each point.

For current-driven magnetization dynamics the current is assumed to be uniform in the nanowire. The correction \mathbf{h} then

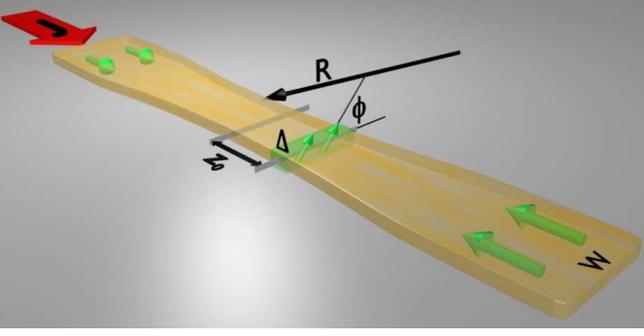


FIG. 1. (color online) Hourglass-shaped nanostrip as a prototype of a magnetic memory nanodevice.

can be written as a sum of two terms:

$$\mathbf{h} = \mathbf{S}_0 \times \frac{\delta \mathcal{H}_\delta}{\delta \mathbf{S}_0} + \mathbf{h}_\alpha, \quad \mathbf{h}_\alpha = \alpha \mathbf{S}_0 \times \dot{\mathbf{S}}_0 - j \partial \mathbf{S}_0 + \beta j \mathbf{S}_0 \times \partial \mathbf{S}_0. \quad (2)$$

The first term is the correction to the Hamiltonian, \mathcal{H}_δ , which turns the zero modes into soft modes as well as couples the magnetization to an external magnetic field. The second term, \mathbf{h}_α , is due to dissipation and current terms. Here α is the Gilbert damping constant, j is an electric current in the units of velocity, and β is the non-adiabatic spin torque constant.

Taking the scalar product of Eq. (1) with $\mathbf{S}_0(z) \times \partial_{\xi_j} \mathbf{S}_0(z)$ and integrating over the space, one can find the following equation for the collective coordinates ξ_i :

$$2\dot{\xi}_i = -\epsilon^{ij} \partial_{\xi_j} E - \epsilon^{ij} \int dz \mathbf{S}_0(z) \times \partial_{\xi_j} \mathbf{S}_0(z) \cdot \mathbf{h}_\alpha(z), \quad (3)$$

where $E(\xi) = \mathcal{H}_\delta[\mathbf{S}_0(z, \xi)]$ is the energy of the domain wall [39] as a function of the soft modes ξ .

For thin nanowires, a DW is a rigid spin texture. Its slow dynamics can be described in terms of only two collective coordinates corresponding to zero modes of motion. These zero modes are the DW position z_0 and its conjugate variable – the tilt angle ϕ for the transverse DW. For the vortex DW, ϕ can serve as the magnetization angle defining the transverse position of the vortex in the wire [40]. Using the definition of \mathbf{h}_α , Eq. (2), and the fact that $\dot{\mathbf{S}}_0 = -\dot{z}_0 \partial_z \mathbf{S}_0 + \dot{\phi} \partial_\phi \mathbf{S}_0$ we obtain

$$\mathbf{h}_\alpha = \alpha \dot{\phi} \mathbf{S}_0 \times \partial_\phi \mathbf{S}_0 - j \partial_z \mathbf{S}_0 + (\beta j - \alpha \dot{z}_0) \mathbf{S}_0 \times \partial_z \mathbf{S}_0 - \mathbf{S}_0 \times \mathbf{H} \quad (4)$$

Here we included uniform magnetic field \mathbf{H} in \mathbf{h}_α by adding the term $-\mathbf{S}_0 \times \mathbf{H}$ in (4).

Up to the leading order in small dissipation (α and β) the equations of motion become

$$\dot{z}_0 = -\frac{1}{2} \frac{\partial E}{\partial \phi} + j, \quad (5)$$

$$\dot{\phi} = \frac{1}{2} \frac{\partial E}{\partial z_0} - \frac{\alpha a_{zz}}{2} \frac{\partial E}{\partial \phi} + H + (\alpha - \beta) a_{zz} j. \quad (6)$$

Here for simplicity we have taken \mathbf{H} to be along the wire (in the z direction) and $a_{zz} = \frac{1}{2} \int dz (\partial_z \mathbf{S}_0)^2$. These equations

are rather universal and can also be applied to describe the dynamics of vortex domain walls [41] in terms of two collective coordinates associated with the DW degrees of freedom. The $1/2$ in the first term on the right-hand side of Eqs. (5) and (6) is a consequence of the Poisson bracket of the conjugated variables z_0 and ϕ . The most general derivation of these equations is based on Poisson brackets and energy dissipation and will be presented elsewhere [42].

Equations (5) and (6) do not depend on details of the microscopic model. The only required input is the energy of a *static* DW as a function of two parameters z_0 and ϕ . This function can be either calculated by means of an analytical approximate model, micromagnetic simulations, or can be measured experimentally for a given wire by a method analogous to the one described in Ref. 39.

The energy of a static DW, $E(z_0, \phi) = \mathcal{H}[\mathbf{S}_0(z; z_0, \phi)]$, where \mathbf{S}_0 is a solution of a static LLG, in general depends on both z_0 and ϕ . The main dependence of E on the angle ϕ comes from the anisotropy in the transverse plane, $E(\phi) = -\kappa \cos(2\phi)$ [43]. The dependence of E on the DW position z_0 may come from different sources such as, z -dependence of the wire shape, nonuniform concentration of impurities, wire surface roughness, nanofabricated notches, etc. Equations (5) and (6) can be used to study DW propagation in disordered wires as well as DW depinning dynamics [31, 33] under the action of time-dependent magnetic fields and currents.

Taking $H = \partial_{z_0} E = 0$ in Eq. (6) one recovers the DW dynamics equations for a translationally invariant nanowire with no magnetic field applied [25]. In this case the DW moves with a constant velocity $\beta j / \alpha$ for small currents j , whereas above the critical current $j_c = \kappa \alpha / |\alpha - \beta|$ in addition to moving along the wire its angle ϕ rotates around the wire axis.

Memory device. As an example of the use of equations (5) and (6) we consider a magnetic memory device based on a flat hourglass-shaped nanostrip (shown in Fig. 1) with the constant thickness h . We propose a nonvolatile device, which employs the magnetization direction within the DW as the information storage [44]. Without current, the transverse DW stays at the place where the nanostrip's cross-section is the narrowest. When a particular current pulse is applied, the DW magnetization angle ϕ flips from 0 to π . At the intermediate step of this flipping process, the DW also deviates from the narrowest position of the nanostrip and at the end it comes back but with the opposite magnetization direction (for computational details see Fig. 3 below). The same current pulse at a later time can move it back to the original configuration [45].

The time it takes to switch the magnetization depends on the current pulse shape. During this process the main energy loss in a realistic wire is the Ohmic loss. How much energy is needed for a single switching also depends on the parameters of the current pulse.

We, thus, aim to solve the following problems: 1) What is the optimal (requiring the least amount of energy) current pulse shape for a given switching time? 2) How the minimal required energy per flip depends on the switching time? The

answers to these questions are given in Figs. 2 and 3. Here we sketch the calculation.

For a smooth nanostrip, we can approximate its width by a parabolic shape $w(z) = w_0 + z^2/R$ and its thickness $h \ll w_0$, see Fig. 1. Then the correction to the DW energy due to the shift from the center of the strip becomes $E_0(z_0) = \frac{\gamma_0 J}{M w_0 \Delta R} z_0^2$, where J is the exchange constant, Δ is the DW width, and R is the curvature radius of the nanostrip.

Taking $H = 0$ (no magnetic field applied) and $a_{zz} = 1/\Delta$ [46], and rescaling variables as $t \rightarrow \kappa t/\Delta$, $z_0 \rightarrow z_0/\Delta$, and $j \rightarrow j/\kappa$ to make all of them dimensionless, Eqs. (5) and (6) become

$$j = \dot{z}_0 + \sin 2\phi, \quad (7)$$

$$\Omega(t) \equiv \dot{\phi} - (\alpha - \beta)\dot{z}_0 + \beta \sin 2\phi - \sigma z_0 = 0. \quad (8)$$

Here we have used the fact that in dimensionless variables the z_0 -dependence of the DW energy is $E_0(z_0) = \sigma z_0^2$ with

$$\sigma = \frac{\gamma_0 J \Delta}{M w_0 \kappa R}. \quad (9)$$

We can estimate σ for materials such as Permalloy using the gyromagnetic ratio $\gamma_0 = 1.76 \times 10^{11} \text{ s}^{-1} \text{ T}^{-1}$, the exchange constant $J = 1.3 \times 10^{11} \text{ J/m}$, the saturation magnetization $M = 8 \times 10^5 \text{ A/m}$, and $\kappa = j_c(\alpha - \beta)/\alpha$ where $(\alpha - \beta)/\alpha \approx 1$ and the critical current measured in units of velocity $j_c \approx 100 \text{ m/s}$. It gives $\sigma \approx \Delta/(w_0 R) \times 10 \text{ nm}$. Further taking for realistic nanowires $R = 100 \text{ nm}$, $w_0 = 100 \text{ nm}$, and $\Delta \approx 10 \text{ nm}$, we find that $\sigma \approx 0.01$ which makes it of the order of α and β . It also justifies neglecting the terms proportional to $\alpha\sigma$ and $\beta\sigma$ in Eqs. (7) and (8).

Our goal is to minimize the Ohmic losses [26, 47] per one switch of the memory bit. It corresponds to the flipping of the DW angle ϕ between two stable values 0 to π , which are defined by the minima of transverse anisotropy. To achieve this, one has to minimize j^2 during the switching time T while keeping the constraint (8).

The fact that the DW is at rest at the angle $\phi = 0$ before the current pulse and at rest again at the angle $\phi = \pi$ immediately after the current pulse is taken into account by the boundary conditions $\phi(0) = 0$, $z_0(0) = 0$, $\dot{z}_0(+0) = 0$, and $\phi(T) = \pi$, $z_0(T) = 0$, $\dot{z}_0(T-0) = 0$.

In order to find minimum of the power $\int_0^T j^2 dt$ with constraint (8), which must hold at all times, we use the Lagrange multiplier $\dot{J}(t)$ and minimize the functional

$$\int_0^T [j^2 - \dot{J}(t)\Omega(t)] dt = \int_0^T [(\dot{z}_0 + \sin 2\phi)^2 - \dot{J}(t)\Omega(t)] dt$$

with respect to three functions $z_0(t)$, $\phi(t)$, and $\dot{J}(t)$ with $J(0) = 0$. Then, in addition to Eq. (8), we obtain two equations

$$\ddot{J} - 2\beta\dot{J} \cos 2\phi + 4(\dot{z}_0 + \sin 2\phi) \cos 2\phi = 0, \quad (10)$$

$$j = j_0 + \frac{\sigma}{2} J - \frac{\alpha - \beta}{2} \dot{J}, \quad (11)$$

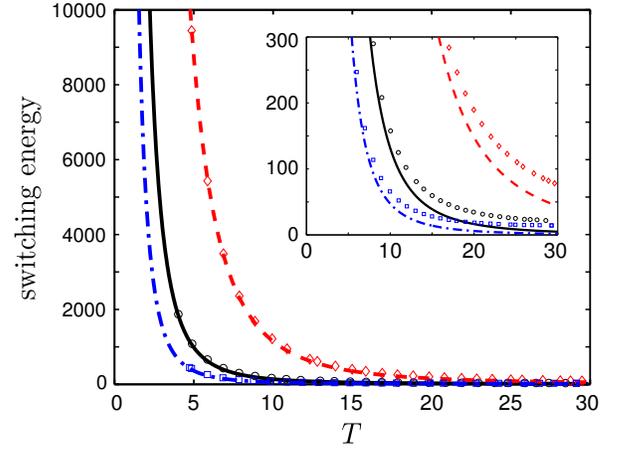


FIG. 2. (color online) Dependence of the switching energy in units of $\int_0^T j^2(t) dt$ on the switching time T for $\sigma = 0.01$ (red dashed line), $\sigma = 0.03$ (black solid line) and $\sigma = 0.05$ (blue dot-dashed line). The inset shows the same dependence for smaller range of the switching energies. One can clearly see the deviations of the simulated data from the analytical solution for the small T limit.

where to write the last equation we used Eq. (7), and j_0 is an integration constant. It can be seen that $j_0 = j(0)$ if $\dot{J}(0) = 0$.

First, we consider the case when the current is absent, $j = \dot{z}_0 + \sin \theta = 0$. Then $J = 0$ and equation (8) gives

$$\ddot{\phi} + 2\alpha\dot{\phi} \cos 2\phi + \sigma \sin 2\phi = 0. \quad (12)$$

For small angles ϕ , this equation describes a damped harmonic oscillator. For $\alpha \ll \sqrt{\sigma}$ the motion is underdamped and the DW performs oscillations with the frequency $\omega_0 = \sqrt{2\sigma}$. For $\alpha \gg \sqrt{\sigma}$ the motion is overdamped.

If one kicks the DW with a very narrow current pulse $j(t) = A\delta(t)$, integrating $\dot{z}_0 + \sin 2\phi = A\delta(t)$ from $-\epsilon$ to ϵ with the initial condition $z_0(-\epsilon) = 0$ and taking the limit $\epsilon \rightarrow 0$, we find $z_+ = \lim_{\epsilon \rightarrow 0} z_0(\epsilon) = A$. The same procedure for equation (8) gives $\phi_+ = (\alpha - \beta)z_+ = (\alpha - \beta)A$. After this pulse, the motion will be described by equation (12). Depending on the characteristic time scales in this equation, $1/\sqrt{\sigma}$ or $1/\alpha$, the motion will be underdamped or overdamped.

Let us consider the limiting case $\alpha = \beta = 0$ of the system without dissipation. Using Eqs. (7), (8), and (11) we find

$$z_0 = \frac{\dot{\phi}}{\sigma}, \quad (13)$$

$$J = \frac{2}{\sigma} \left(\frac{\ddot{\phi}}{\sigma} + \sin 2\phi - j_0 \right). \quad (14)$$

Substituting these equations into Eq. (10) we also obtain

$$\left(2\sigma \cos 2\phi + \frac{\partial^2}{\partial t^2} \right) \left(\sigma \sin 2\phi + \frac{\partial^2}{\partial t^2} \phi \right) = 0. \quad (15)$$

There is a trivial solution of this equation, $\phi(t) = 0$, but it does not satisfy the boundary conditions: the angle does not flip in this case. However, in the limit when the switching process is

fast, or alternatively σ is small, one can find an approximate solution. If the switching time T satisfies $T\sqrt{\sigma} \ll 1$, all the terms except for the fourth derivative can be neglected and we obtain $\partial^4 \phi / \partial t^4 = 0$. Using the boundary conditions for both ϕ and z_0 one then finds

$$\phi(t) = 3\pi(t/T)^2 - 2\pi(t/T)^3 \quad (16)$$

which gives

$$j(t) = \frac{1}{\sigma} \ddot{\phi} + \sin 2\phi \approx \frac{1}{\sigma} \ddot{\phi} = \frac{6\pi}{\sigma} \left(1 - \frac{2t}{T}\right). \quad (17)$$

The switching energy measured in dimensionless units $\int_0^T j^2(t) dt$ is shown in Fig. 2. For small T , according to Eq. (17), it is given by $12\pi^2/(\sigma^2 T^3)$. The actual units of the switching energy can be estimated as $\rho J_c^2 \Delta / (j_c h)$, where $\rho \approx 1.4 \times 10^{-7} \Omega\text{m}$ is Permalloy resistivity, the critical current density $J_c \approx 10^{12} \text{ A/m}^2$, $\Delta \approx h \approx 10 \text{ nm}$; and the units of time are $\Delta/j_c \approx 10^{-10} \text{ s}$. This gives the units of the switching energy to be $\approx 10^{-16} \text{ J}$. For comparison, best MRAM devices typically consume 10^{-10} J per switching a bit [48] in 5 ns. This shows that our proposed memory device is about 10^5 times more energy efficient for the same switching times. Figure 2 also gives the estimate of the best switching time achievable for any particular energy supplied per switch.

The full set of equations with realistic α and β parameters can be solved numerically. The comparison of the analytical solution (17) for small T with the numerics for a range of T is shown in Fig. 3. We find that the linear current coincides with the simulation result for the short switching times, i.e. $T \lesssim 1/\sqrt{\sigma}$. For long switching times, the optimal current $j(t)$ flips the DW and then lets it relax with some oscillations to $z_0 = 0$ and $\phi = \pi$ within the required time T .

Summary. We have generalized the DW dynamics to the case of translationally non-invariant ferromagnetic nanowires. The obtained equations of motion make the description of the DW propagation closer to experiment by accounting for smooth surface roughness and other disorder effects. We have also considered an hourglass-shaped nanostrip with a transverse DW trapped in the middle as a prototype of a magnetic memory device. The exact time-dependence of the current pulses required to switch the magnetization with the minimal Ohmic losses per switching has been obtained. Furthermore, we find how the switching time depends on the Ohmic losses per switching for the optimal current pulse. Our estimates show that this hourglass-shaped nanodevice may be 10^5 times more energy efficient for the same switching times as used in the best modern memory devices.

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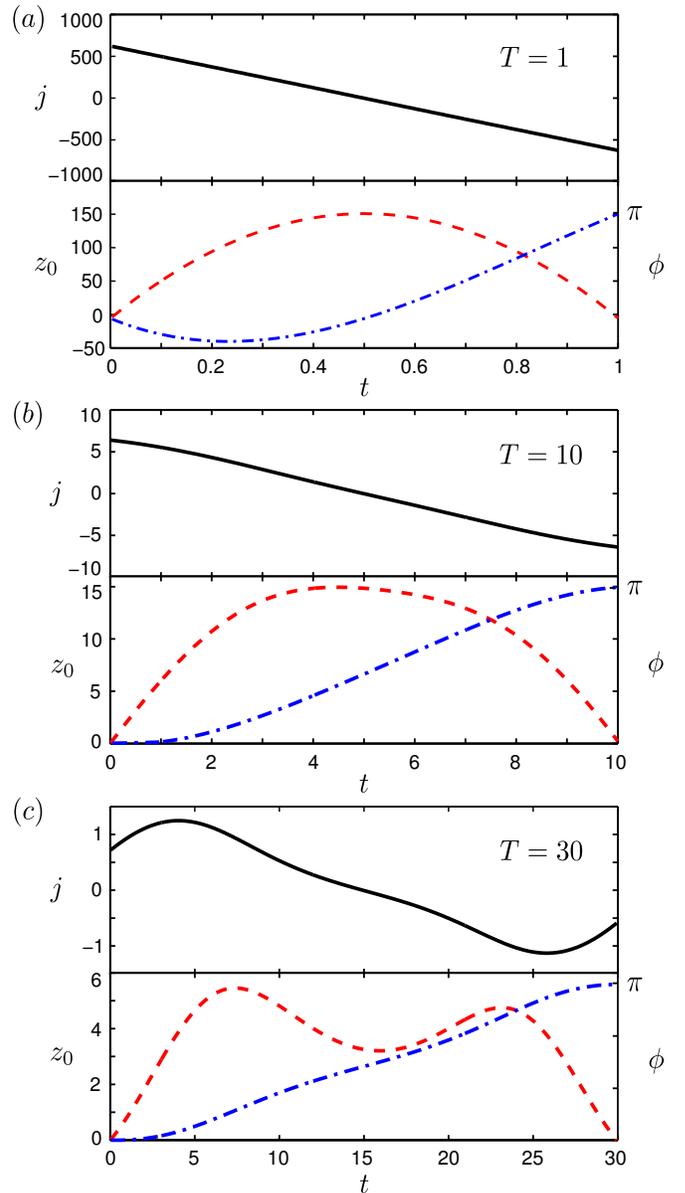


FIG. 3. (color online) The time dependence of the DW position and angle for the optimal current pulse with the parameters $\alpha = 0.01$, $\beta = 0.02$, and $\sigma = 0.03$. Optimal current $j(t)$ (black solid line), $z_0(t)$ (red dashed line) and angle $\phi(t)$ (blue dash-dotted line) as functions of time t for (a) $T = 1$, (b) $T = 10$, (c) $T = 30$.

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- [45] For a movie demonstrating the mechanism of this memory element see the supplementary material.
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