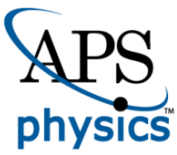

Locally Critical Resistivities from Umklapp Scattering

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Phys. Rev. Lett. **108**, 241601 — Published 13 June 2012

DOI: 10.1103/PhysRevLett.108.241601

Locally critical resistivities from umklapp scattering

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Efficient momentum relaxation through umklapp scattering, leading to a power law in temperature d.c. resistivity, requires a significant low energy spectral weight at finite momentum. One way to achieve this is via a Fermi surface structure, leading to the well-known relaxation rate $\Gamma \sim T^2$. We observe that local criticality, in which energies scale but momenta do not, provides a distinct route to efficient umklapp scattering. We show that umklapp scattering by an ionic lattice in a locally critical theory leads to $\Gamma \sim T^{2\Delta_{k_L}}$. Here $\Delta_{k_L} \geq 0$ is the dimension of the (irrelevant or marginal) charge density operator $J^t(\omega, k_L)$ in the locally critical theory, at the lattice momentum k_L. We illustrate this result with an explicit computation in locally critical theories described holographically via Einstein-Maxwell theory in Anti-de Sitter spacetime. We furthermore show that scattering by random impurities in these locally critical theories gives a universal $\Gamma \sim \left(\log \frac{1}{T}\right)^{-1}$.

CONTEXT: THE MANY FACES OF THE D.C. RESISTIVITY

The d.c. resistivity of a system with a net charge density is ultimately tied to the rate at which the charge-carrying excitations can lose their momentum. If the total momentum is conserved, the conductivity diverges at the lowest frequencies $\omega \to 0$ as

$$\sigma(\omega) = \frac{\chi^2_{\vec{J}\vec{P}}}{\chi_{\vec{P}\vec{P}}} \left(\frac{i}{\omega} + \delta(\omega) \right) . \qquad (1)$$

For instance, in a relativistic theory, Ward identities imply that the susceptibilities $\chi_{\vec{J}\vec{P}} = \rho$ and $\chi_{\vec{P}\vec{P}} = \varepsilon + P$ [1–3]. Here ρ, ϵ, P are the charge and energy densities and pressure. At low temperatures $\epsilon + P = \mu\rho$.

It is of course intuitively plausible that a net charge will accelerate indefinitely under an applied electric field. Formulae such as (1) indicate that the d.c. conductivity is a subtle observable. Despite being a low energy quantity, it is sensitive to UV data such as the charge density and translation invariance. Computations of finite d.c. conductivities at nonzero density necessarily include the effects of momentum non-conserving terms. The classic example is Fermi liquid theory in the presence of irrelevant couplings to a lattice, which leads to a universal resistivity $r \sim T^2$, as we will briefly review. Similarly, the remarkable robustness of the observed linear in (low) temperature resistivity across a range of chemically distinct unconventional materials (for an overview see [4]) may suggest that the key physics there is also universal, i.e. describable within the framework of effective field theory. In this letter we will present a new way, distinct from Fermi surface kinematics, in which the UV sensitivity of the d.c. resistivity can be subsumed into a critical effective field theory.

Any loss of momentum is a question of timescales. To invalidate the conclusion (1), and hence achieve a finite d.c. resistivity, momentum must be effectively lost on the experimental timescale. Two natural ways to achieve this are firstly if the charge carriers of interest are parametrically diluted by a bath of other degrees of freedom, and secondly if the charge carriers interact with parametrically heavier degrees of freedom. In both cases the charge carriers can dump their momentum into the other degrees of freedom and the momentum will not be returned to the charge carries within the experimental timescale. Any reliable computation of a d.c. conductivity must hinge on an approximation analogous to the two just described.

There has been some recent success realizing the former of these scenarios via the holographic correspondence. Firstly in 'probe brane' setups, where the charge carriers are parametrically diluted by critical neutral degrees of freedom [5]. An important class of these models, where the probe brane is described by the Dirac-Born-Infeld action, have been shown to have a low temperature resistivity scaling as $r \sim T^{2/z}$, in 2+1 dimensions, with z the dynamical critical exponent governing the critical neutral modes [6]. A second set of holographic models that realize similar physics are the locally critical non-Fermi liquids of [7]. Here, a parametrically small fraction of the charged degrees of freedom are fermions with non-Fermi liquid dispersion relations due to interactions with a bath of fractionalized charged degrees of freedom [8, 9]. The contribution of the non-Fermi liquid excitations to the resistivity goes like $r \sim T^{2\nu_{k_F}}$, where ν_{k_F} is related to the, UV sensitive, scaling dimension of the fermionic operator in the low energy locally critical theory of the fractionalized degrees of freedom [10].

The second scenario, involving interaction with parametrically heavy degrees of freedom, has the advantage that such degrees of freedom always exist in actual materials: as quenched random impurities and/or as a lattice of ions. It also does not depend on a large N limit in an essential way. Many of the most interesting materials appear to be very pure, and partially for that reason we will focus on scattering off an ionic lattice in this letter. A periodic lattice degrades momentum via umklapp scattering processes. In the final section we will also present a result for random impurity scattering.

MOMENTUM RELAXATION RATE DUE TO UMKLAPP SCATTERING

Consider a translationally invariant field theory at finite density such that the only conserved vector quantity is the total momentum. We recalled above that this results in an infinite d.c. conductivity. Now, perturb this theory by an irrelevant operator (so the IR is still described by the original fixed point) that breaks translational invariance:

$$H = H_0 - g\,\mathcal{O}(k_L)\,, \tag{2}$$

where k_L represents the typical lattice momentum scale.

For operators A and B, define

$$\widetilde{C}_{AB}(\omega) = \frac{T}{i\omega}\left[G^R_{AB}(\omega) - G^R_{AB}(i0)\right]\,, \tag{3}$$

where G^R is the retarded Green's function. The d.c. conductivity is given by the Kubo formula as:

$$\sigma = \lim_{\omega\to 0}\frac{\mathrm{Im}\, G^R_{\vec{J}\vec{J}}(\omega)}{\omega} = \frac{1}{T}\lim_{\omega\to 0}\widetilde{C}_{\vec{J}\vec{J}}(\omega)\,. \tag{4}$$

We would like to have a perturbative expression that captures the leading contribution coming from the leading irrelevant correction introduced in (2). The appropriate way to do this is given by the memory matrix formalism [11]. A crucial part of this formalism is the inclusion of all conserved operators that overlap with $\vec{J}$ in matrix conductivities. In our case this is only the momentum $\vec{P}$. Intuitively, the conductivity diverges because the current operator has some overlap with the momentum operator, which is conserved. Once this is considered, we can write

$$\hat{\sigma}(\omega) = \hat{\chi}\cdot\left(\hat{M}(\omega) - i\omega\hat{\chi}\right)^{-1}\cdot\hat{\chi}\,, \tag{5}$$

where hatted quantities are two dimensional matrices with indices either $\vec{J}$ or $\vec{P}$; $\hat{\chi}$ is the static susceptibility matrix and $\hat{M}$ the 'memory matrix' [11]. When $\hat{M}$ vanishes at $\omega = 0$ we obtain (1). It can be shown that to leading order in g the $M_{\vec{P}\vec{P}}$ component determines the d.c. conductivity. In this approximation

$$\begin{aligned} M_{\vec{P}\vec{P}} &= \frac{1}{T}\lim_{\omega\to 0}\widetilde{C}_{\dot{\vec{P}}\dot{\vec{P}}}(\omega) = \frac{g^2k_L^2}{T}\lim_{\omega\to 0}\widetilde{C}_{\mathcal{O}(k_L)\mathcal{O}(k_L)}\Big|_{g=0} \\ &= g^2k_L^2\lim_{\omega\to 0}\left.\frac{\mathrm{Im}\, G^R_{\mathcal{O}\mathcal{O}}(\omega,k_L)}{\omega}\right|_{g=0}\,. \end{aligned} \tag{6}$$

Here we have used that $\dot{\vec{P}} = i[H,\vec{P}] = g\,\vec{k}_L\,\mathcal{O}(k_L)$. Thus the d.c. conductivity is

$$\sigma_{\vec{J}\vec{J}} = \lim_{\omega\to 0}\frac{\chi^2_{\vec{J}\vec{P}}}{M_{\vec{P}\vec{P}}(\omega)} \equiv \frac{\chi^2_{\vec{J}\vec{P}}}{\chi_{\vec{P}\vec{P}}}\,\frac{1}{\Gamma}\,, \tag{7}$$

where $\Gamma = \lim_{\omega\to 0}\frac{M_{\vec{P}\vec{P}}(\omega)}{\chi_{\vec{P}\vec{P}}}$ is the momentum relaxation rate, as we can see from the $\vec{P}\vec{P}$ component of $\hat{\sigma}$.

For umklapp scattering by an ionic lattice, $\mathcal{O} = J^t$ and the lattice appears as a spatially dependent chemical potential. The momentum relaxation rate is then

$$\Gamma = \frac{g^2k_L^2}{\chi_{\vec{P}\vec{P}}}\lim_{\omega\to 0}\left.\frac{\mathrm{Im}\, G^R_{J^tJ^t}(\omega,k_L)}{\omega}\right|_{g=0}\,. \tag{8}$$

CRITICAL UMKLAPP WITH AND WITHOUT FERMI SURFACES

The previous section implies that the momentum relaxation rate due to perturbative umklapp scattering by an ionic lattice is given through the spectral function, $\mathrm{Im}\, G^R_{J^tJ^t}(\omega,k_L)$, with $\omega\to 0$. In order for this quantity to be captured by a critical effective field theory, with, say, a resistivity that is a power law in the temperature, it is necessary that low energy excitations exist at $k = k_L$. If no excitations are supported at the lattice momentum, for instance if $\omega\sim k^z$, then the resistivity will be due to exponentially suppressed Boltzmann states.

Systems with a Fermi surface admit critical umklapp scattering in two senses, as we now review. The first is if the umklapp momentum connects two points on the Fermi surface. Then all charge carriers involved in the umklapp scattering are critical, despite the momentum transfer. This process is mediated by the density operator at finite momentum. In 2+1 dimensions

$$J^t(\omega,k)\equiv J^t(p) = \int d^3q\,\psi^\dagger_\sigma(q)\psi_\sigma(p+q)\,. \tag{9}$$

In Fermi liquid theory this operator is relevant with dimension $\Delta = -1$. It induces an RG flow that folds the Fermi surface and gaps out the two points connected by the lattice vector. This conclusion can be averted either by tuning the gap to zero or by non-Fermi liquid physics rendering the operator (9) irrelevant. An interesting example of the first possibility is given by the 'hot spots' on a Fermi surface coupled to a critical spin density wave. Fermions at the hot spots contribute a strong power law conductivity, but can easily be short-circuited by the remainder of the 'cold' fermions [12, 13]. A renormalization group treatment [14] suggests, however, that critical umklapp scattering at the hot spots can be communicated to the rest of the Fermi surface [15]. In 1+1 dimensions, such hot spots constitute the entire Fermi 'surface' and one might expect that e.g. a half-filled Luttinger liquid could exhibit a critical resistivity in cases where the umklapp coupling is irrelevant [16]. This expectation is thwarted by additional conservation laws [17–19].

The second way in which Fermi surface kinematics enable critical umklapp scattering is through coupling the lattice to the irrelevant quartic operator

$$\begin{aligned} \mathcal{O}(p) = \int\left(\prod_{i=1}^4 d^3p_i\right)\psi^\dagger_\sigma(p_1)\psi^\dagger_{\sigma'}(p_2)\psi_\sigma(p_3)\psi_{\sigma'}(p_4) \\ \times\delta^{(3)}(p_1+p_2-p_3-p_4-p). \end{aligned} \tag{10}$$

The RG flow is towards the Fermi surface. In particular, the delta function does not scale generically [20, 21]. The entire Fermi surface will have a critical resistivity. This is a well known fact, and the corresponding momentum relaxation is easy to compute in the framework of the previous section. The operator (10) has scaling dimension $\Delta = 1$. The imaginary part of the Green's function must be odd under $\omega \to -\omega$ and therefore we can anticipate the momentum relaxation rate from (6) and (7) will have the (low) temperature dependence

$$\Gamma \sim \lim_{\omega\to 0} \frac{\mathrm{Im}\, G^R_{\mathcal{O}\mathcal{O}}(\omega, k_L)}{\omega} \sim T^2 \,. \tag{11}$$

This is the well-known Fermi liquid theory result. The above considerations can straightforwardly be generalized to cases where the excitations of the Fermi surface do not have Fermi liquid dispersion relations.

For a convex Fermi surface in 2+1 dimensions, the current $\vec{J}$ remains a conserved quantity in Fermi liquid theory, to leading order at low frequencies. Here our assumption that the current $\vec{J}$ was degraded prior to the consideration of umklapp effects does not hold. Consideration of $\vec{J}$ and $\vec{P}$ simultaneously via the entire memory matrix [22, 23] recovers a resistivity $r \sim T^2$.

Without a Fermi surface-like structure (including e.g. Fermi points), one is left with scalings towards the origin $\omega \sim k^z$. An exceptional case, however, is the limit $z \to \infty$. In this limit, time scales but space does not. In such a locally critical theory, *all* momenta become independently critical at low energies. It is immediately clear that umklapp scattering off an ionic lattice will lead to critical resistivities in such a theory. The charge density operator $J^t(\omega, k)$ will have a scaling dimension Δ_k under the critical scaling. The UV quantity k_L will then determine the IR scaling dimension Δ_{k_L} of the modes that control the loss of momentum. With this difference, that k_L appears in the operator dimension, the logic then proceeds very similarly to the Fermi liquid case. In particular, the momentum conservation delta function again does not scale, leading to $G^R_{J^t J^t}(\omega, k)$ having dimension $2\Delta_k + 1$. Therefore, the temperature dependence of the momentum relaxation rate (8) is

$$\Gamma \sim \lim_{\omega\to 0} \frac{\mathrm{Im}\, G^R_{J^t J^t}(\omega, k_L)}{\omega} \sim T^{2\Delta_{k_L}} \,. \tag{12}$$

If we require the operator $J^t(\omega, k)$ to be marginal or irrelevant in the IR theory – and if this is not the case then we have not reached the true IR and our perturbation theory is suspect – then $\Delta_{k_L} \geq 0$. As for the Fermi liquid, a marginal operator leads to a constant, T^0, momentum relaxation rate.

Locally critically theories also dovetail in an interesting way with Fermi surfaces, as one can efficiently scatter fermionic excitations with locally critical bosons. This fact is behind the non-Fermi liquid spectral functions and resistivities of [7, 9, 10]. In this work we are exploiting a different consequence of local criticality: the efficiency of umklapp scattering in such a theory, independently of the presence of Fermi surfaces.

HOLOGRAPHIC MODEL FOR LOCALLY CRITICAL UMKLAPP SCATTERING

In holography, the IR field theoretical physics is described by the far interior of the dual spacetime. In the absence of explicit charged matter in the bulk [24], it is a robust feature [25] that at zero temperature and at finite charge density, a fully regular solution to the bulk equations of motion will have an $AdS_2 \times \mathbb{R}^2$ IR geometry. It was emphasized by [7] that the isometries of this IR spacetime – time is part of the AdS_2 factor and scales while space does not – entailed an emergent local criticality. In fact, the scaling in time is part of a larger emergent $SL(2,\mathbb{R})$ symmetry of AdS_2 that strongly constrains low energy Green's functions, as we will see shortly. Here, as in the remainder, we have specialized to 2+1 dimensional field theories.

The simplest model that illustrates the physics of interest is Einstein-Maxwell theory in asymptotically Anti-de Sitter spacetime:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}\right). \tag{13}$$

We wish to compute retarded Green's functions at low temperature and frequencies $\omega, T \ll \mu$. The momentum however need not be small. It is well established that holographically the dissipative low frequency physics is captured by the near-horizon geometry, while low temperatures means that the horizon will be near-extremal (see e.g. [26]). Therefore, we can focus on the following solution to the theory, which describes a black hole in $AdS_2 \times \mathbb{R}^2$:

$$ds^2 = \frac{L^2}{6}\left(-\frac{f(r)dt^2}{r^2} + \frac{dr^2}{f(r)r^2} + dx^2 + dy^2\right). \tag{14}$$

The Maxwell potential is $A = \frac{1}{\sqrt{6}}\frac{eL}{\kappa}\left(r^{-1} - r_+^{-1}\right)dt$ and the emblackening factor $f(r) = 1 - \frac{r^2}{r_+^2}$.

To compute the retarded Green's function of J^t in this background we must perturb the time component of the Maxwell potential: δA_t. Due to the finite energy and momentum, this perturbation couples to other modes. Taking the momentum to be in the x direction, these are: $\{\delta g_{xx}, \delta g_{yy}, \delta g_{tt}, \delta g_{xt}, \delta A_t, \delta A_x\}$. All the perturbations have the form of a function of r times $e^{-i\omega t + ikx}$. A clever choice of gauge invariant variables is [27, 28]:

$$\Phi = \delta A_t' - \sqrt{\frac{3}{2}}\frac{\delta g_{tt}}{f}\,, \qquad \Psi = \delta g_{yy}\,, \tag{15}$$

and then define

$$\Phi_\pm = \Psi + \frac{r^2}{\sqrt{6}k^2}\left(1 \pm \sqrt{1+2k^2}\right)\Phi\,. \tag{16}$$

These variables are now found to satisfy

$$\Phi_\pm'' + \frac{f'}{f}\Phi_\pm' + \left(\frac{\omega^2}{f^2} - \frac{1+k^2 \pm \sqrt{1+2k^2}}{r^2 f}\right)\Phi_\pm = 0\,. \tag{17}$$

These equations can be solved in terms of hypergeometric functions. One imposes, as usual [29, 30], infalling boundary conditions at the horizon. The locally quantum critical Green's functions are obtain by expanding the solution near the boundary $r \sim 0$ of the $AdS_2 \times \mathbb{R}^2$ region

$$\Phi_\pm \propto r^{\frac{1}{2}}\left(r^{-\nu_\pm} + \mathcal{G}_\pm(\omega)\, r^{\nu_\pm}\right)\,. \tag{18}$$

Here we introduced the exponents [28]

$$\nu_\pm = \frac{1}{2}\sqrt{5 + 4k^2 \pm 4\sqrt{1+2k^2}}\,. \tag{19}$$

We have normalized the spatial coordinates x, y differently in (14) relative to [28].

The locally quantum critical Green's functions are found to be

$$\mathcal{G}_\pm(\omega) = -(\pi T)^{2\nu_\pm}\frac{\Gamma\left(1-\nu_\pm\right)\Gamma\left(\frac{1}{2} - \frac{i\omega}{2\pi T} + \nu_\pm\right)}{\Gamma\left(1+\nu_\pm\right)\Gamma\left(\frac{1}{2} - \frac{i\omega}{2\pi T} - \nu_\pm\right)}\,. \tag{20}$$

We have given this result in terms of the temperature of the black hole $T = \frac{1}{2\pi r_+}$. The expression (20) is in fact determined, up to overall normalization, by the scaling dimensions $\nu_\pm + \frac{1}{2}$ of the operators and the $SL(2,\mathbb{R})$ symmetry of the black hole in AdS_2 [26, 31, 32]. For the momentum relaxation rate due to umklapp scattering, we will be interested in the $\omega \to 0$ expansion of the imaginary part of the Green's function. This gives

$$\mathrm{Im}\,\mathcal{G}_\pm(\omega) \propto \omega\,(\pi T)^{2\nu_\pm - 1} + \cdots\,. \tag{21}$$

A well-established matching procedure, see e.g. [7, 28], shows that this is equal to the imaginary part of the full low frequency Green's function $\mathrm{Im}\,G_\pm(\omega) \propto \mathrm{Im}\,\mathcal{G}_\pm(\omega)$.

From this result we can obtain the desired density-density Green's function. The density-density Green's function is found to be a linear combination of the $G_\pm$ Green's functions [28]. The G_- Green's function is more IR singular than the G_+ Green's function, and so gives the dominant contribution. From (21) and (8) the umklapp momentum relaxation rate in this theory is

$$\Gamma \sim \lim_{\omega\to 0}\frac{\mathrm{Im}\,G^R_{J^tJ^t}(\omega, k_L)}{\omega} \sim T^{2\nu_- - 1}\,. \tag{22}$$

This result is consistent with the general expression (12) and the fact that the dimension of the frequency-space operator is $\Delta_k = \nu_- - \frac{1}{2}$.

DISCUSSION

We have found that local quantum criticality provides a new route, different to Fermi surface kinematics, to obtain critical umklapp scattering. Local criticality emerges naturally in holographic contexts, where it can be stable over a made parametrically wide intermediate energy range in a large N limit [9, 33–35].

In this work we have not touched upon the computation of optical conductivities. The optical and d.c. conductivities are deeply interconnected but behave in opposite ways. As we have seen, many low energy degrees of freedom can lead to a large resistivity, and hence small d.c. conductivity. On the other hand, the optical conductivity is essentially the spectral density for charged degrees of freedom and is therefore large when the d.c. conductivity is small. Critical optical conductivities due to umklapp scattering were recently found in an RG treatment of the quantum critical spin density wave transition in two dimensions [15].

Finally, we recall that scattering off random impurities is also naturally treated using the memory function method. A formula for the scattering rate in that case was obtained in [1, 36]. The formula is essentially just an integral of our expression (12) over momenta, which we might think of as averaging over lattice separations. For the case of charged impurities

$$\Gamma_{\rm imp} \sim \lim_{\omega\to 0}\int \frac{d^2k}{(2\pi)^2}k^2\frac{\mathrm{Im}\,G^R_{J^tJ^t}(\omega,k)}{\omega} \sim \int dk\, k^3\, T^{2\Delta_k}\,. \tag{23}$$

In the low temperature limit, this integral is dominated by the small momentum contribution. Using the concrete expression (19), the momentum is found to have a natural scale $k_\star^4 \sim \left(\log\frac{1}{T}\right)^{-1}$, which is small. A scaling argument then gives the momentum relaxation rate

$$\Gamma_{\rm imp} \sim \frac{1}{\log\frac{1}{T}}\,. \tag{24}$$

Unlike the umklapp scattering we have focussed on, this relaxation rate is completely universal in the sense that it does not depend on the UV completion of the locally critical IR theory. The power of the logarithm that appeared is sensitive to the fact that in the holographic model $\Delta_k \sim k^4$ at small k. A different power in the small momentum expansion would have led to a different power of the logarithm.

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge discussions with Chris Herzog, John McGreevy, Subir Sachdev, Eva Silverstein and especially Max Metlitski while this work was in progress. This research was supported in part by the Na-

tional Science Foundation under Grant No. NSF PHY05-51164, by DOE grant DE-FG02-91ER40654, and the Center for the Fundamental Laws of Nature at Harvard University. The work of S.A.H. is partially supported by a Sloan research fellowship.

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