Quantum Electron Star

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A quantum electron star

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We construct and probe a holographic description of state of matter which results from coupling a Fermi liquid to a relativistic conformal field theory (CFT). The bulk solution is described by a quantum gas of fermions supported from collapse into the gravitational well of AdS by their own electrostatic repulsion. A physical interpretation of our result is that, in the probe limit studied here, the Landau quasiparticles survive this coupling to a CFT.

INTRODUCTION

Holographic duality can be used to study metallic states of matter which are not described by Landau's Fermi liquid theory [1–14] (for a summary, see *e.g.* [5, 15, 16]). In particular, the work of [2, 4] constructs controlled non-Fermi liquid fixed points. In their setup, however, the fermions are only a parametrically small fraction ($\sim 1/N^2$) of all degrees of freedom. The large bath of additional degrees of freedom is responsible for the destruction of the Landau quasiparticle, and for the short transport lifetime and linear-T resistivity obtained in the special case of the marginal Fermi liquid, as described in [5, 6].

This large bath is locally critical, *i.e.* has dynamical critical exponent $z = \infty$, and has non-zero entropy at zero temperature, which indicates fine tuning and an instability towards a lower energy state. Various instabilities have been suggested; an intrinsic one, arising from the density of fermions itself, was pointed out in [8]: the fermions screen the gauge flux which supports the AdS_2 throat, leading to a Lifshitz geometry with large but finite dynamical exponent $z \sim N^2$, which is a better approximation to the ground state. Such 'electron star' states, comprising a gravitating gas of charged fermions in the bulk, have been further studied in detail [9–14]. These states represent an improvement over the work of [2, 4] in that the fermions contribute at leading order in N^2 to the construction of the geometry.

However, in these states, the single-fermion response exhibits *many* Fermi surfaces [12, 14], and the conformal dimension of the boundary fermionic operator is much larger than one. Such unphysical features are not intrinsic to holographic states supported by fermions, but rather they are a consequence of the approximations used in constructing the states, in particular the Thomas-Fermi approximation. We would like to construct a holographic non-Fermi liquid with more realistic features, and this requires going beyond this approximation.

A point of departure is provided by [17][1], where the Thomas Fermi approximation is replaced by the Hartree-Fock approximation. The state constructed there can be understood as a Fermi liquid coupled to a rough representation of confining gauge theory. In the bulk, the confinement gap is introduced as an artificial 'hard wall' termination of the geometry at a fixed value of the RG coordinate, and appears essential for the construction of the fermionic ground state.

This paper makes a further step towards the above goal of a realistic holographic non-Fermi liquid by constructing a bulk Fermi liquid state without artificial cutoffs in the geometry. More precisely, we show that, with due improvements of the definition and construction of the fermionic ground state, the bulk state conceived in [17] survives the limit where the hard-wall cutoff is removed. The problem we solve can also be viewed as the fermion analog of the probe holographic superconductor calculation in [24]. We name the resulting state a *quantum electron star*, following [12], in the sense that we are describing a degenerate quantum gas of charged fermions hovering above the Poincaré horizon.

Technically, we had to introduce some improvements over previous methods. In particular, in order to properly define the fermionic ground state, we had to introduce a short-distance (UV) completion of the bulk system. We accomplished this by putting the bulk system on a lattice.

Next, we describe the problem, and outline our method of solution. Results and discussion follow.

SETUP OF THE PROBLEM

We consider a system defined by the action

$$S = \int \mathrm{d}^{d+1}x\sqrt{-g}\left[\frac{R-2\Lambda}{16\pi G_N} - \frac{1}{4q^2}F^2\right] + W[A]\,, \quad (1)$$

where

$$e^{iW[A]} = \int \mathcal{D}\psi\; e^{iS_f[\psi,A]}\,, \quad (2)$$

$$S_f[\psi, A] = \int \mathrm{d}^{d+1}x\sqrt{-g}\;\left[-i\bar{\psi}\left(\Gamma^M\mathcal{D}_M + m\right)\psi\right]\,, \quad (3)$$

$\bar{\psi} \equiv \psi^\dagger\Gamma^{\underline{t}}$ and $\mathcal{D}_M \equiv \partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} + iA_M$, with ω_{abM} the spin connection.

[1] Earlier work which studies quantum spinor fields in a holographic context includes [18, 19]. Recent provocative work towards this goal includes [20–23].

We study this system in a probe limit, $G_N \to 0$ at fixed Λ, where the geometry is not dynamical. In the dual language, we are studying a CFT where the fraction of degrees of freedom that carry charge is small.

In particular, we specialize to the AdS$_4$ metric

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(-\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right), \tag{4}$$

and we consider a gauge field of the form $A = \Phi(z)\mathrm{d}t$. The equation of motion for the potential Φ is

$$-\Phi''(z) = q^2 z^{-3}\left\langle \hat{\psi}^\dagger \hat{\psi}\right\rangle . \tag{5}$$

To compute the expectation value on the right hand side, we expand the spinor field operator $\hat{\psi}$ in eigenfunctions of the Dirac Hamiltonian. The Dirac equation is $(\not{D} + m)\psi = 0$, and we make the following ansatz for the eigenfunction ψ:

$$\psi = z^{3/2} e^{-i\omega t + i\vec{k}\cdot\vec{x}}\Psi(z) . \tag{6}$$

For any fixed $\vec{k}$, the Dirac equation can be block-diagonalized [4]. Without loss of generality we set $\vec{k} = k\hat{x}$, and we let $\Psi = (\Psi_+, \Psi_-)^t$. Using the following basis for the Clifford algebra:

$$\Gamma^{\underline{z}} = \sigma^3 \otimes \mathbb{1},\ \Gamma^{\underline{t}} = i\sigma^1 \otimes \mathbb{1},\ \Gamma^{\underline{x}} = \sigma^2 \otimes \sigma^3,\ \Gamma^{\underline{y}} = \sigma^2 \otimes \sigma^1, \tag{7}$$

we have

$$\left[i\sigma^2\partial_z - \sigma^1\frac{m}{z} \pm k\sigma^3 + \Phi(z)\right]\Psi_\pm = \omega\Psi_\pm . \tag{8}$$

Eq. (8) has two linearly independent solutions, whose asymptotic behavior near the AdS boundary is

$$\Psi_\pm \overset{z\to 0}{\sim} a z^{-mL}\begin{pmatrix}0\\1\end{pmatrix} + b z^{mL}\begin{pmatrix}1\\0\end{pmatrix} \tag{9}$$

plus terms subleading in z. We demand that the non-normalizable solution be zero[2]: $a = 0$. Moreover, as in [17], we impose hard-wall boundary conditions at an IR cutoff $z = z_m$: the upper component of both $\Psi_\pm(z_m)$ must vanish.

With these boundary conditions, the Dirac Hamiltonian is a self-adjoint differential operator with spectrum $\omega_{n,\vec{k},s}$, labelled by a discrete index n, by the momentum $\vec{k}$ and by the sign $s = \pm$ that distinguishes the upper components from the lower components. We denote the eigenfunctions with $\Psi_{n,\vec{k},s}$.

In order to give definite meaning to the expectation value in (5), for a given profile of Φ, we fill all the states with $\omega_{n,k,s} < 0$, and we subtract the same expectation value for $\Phi = 0$. That is, we solve

$$-\Phi''(z) = q^2\left[n(z)|_\Phi - n(z)|_{\Phi=0}\right] \equiv q^2 \Delta n(z) , \tag{10}$$

with

$$n(z) = \sum_{\vec{k},n,s} \theta(-\omega_{\vec{k},n,s})\Psi^\dagger_{n,\vec{k},s}(z)\Psi_{n,\vec{k},s}(z) . \tag{11}$$

Clearly (11) involves two sums that need to be regulated. We regulate the sum over n by discretizing the z coordinate, with lattice spacing Δz, and we impose a hard cutoff $\vec{k}^2 < \Lambda_k^2$ on the sum over momenta. After the subtraction in (10), and appropriate renormalization of the charge q, the problem has a well-defined limit as $\Delta z \to 0$, $\Lambda_k \to \infty$. Additional information on the subtraction and renormalization can be found in the supplementary material.

Eq (10) also needs to be complemented with appropriate boundary conditions on Φ. We want a finite chemical potential in the boundary theory, so we set $\Phi(0) = -\mu$, and we also impose $\Phi'(z_m) = 0$. We expect that the boundary condition at z_m becomes unimportant as $z_m \to \infty$, and we verified this by exploring mixed boundary conditions as well, without finding a significant influence in the interior, for large enough z_m.

We solve the integro-differential system formed by (8) and (10) by an iterative method, whereby the number density computed with a given a profile of Φ is used to update Φ through (10) and the new profile is used to update the number density, until convergence.

THE RESULTING GROUNDSTATE

Fig. 1 and 2 display typical profiles for $\Phi(z)$ and $n(z)$. They also show that the problem possesses a well-defined limit as $z_m \to \infty$, $\Lambda_k \to \infty$. We also verified that the profiles are insensitive to the discretization of the z coordinate, for small enough lattice spacing. Once the cutoffs are removed, the only remaining scale in the problem is the chemical potential μ, and, without loss of generality, we can set $\mu = 1$.

The profile of the potential approaches a constant as $z \to \infty$[3], and we would like to argue that this constant is zero.

The asymptotic behavior of the wavefunction for large z depends on the sign of

$$p^2 \equiv (\omega - \Phi(\infty))^2 - k^2 . \tag{12}$$

[2] Allowing the mass to range over $(-\frac{1}{2}, \infty)$, this includes the alternative quantization.

[3] If the electric field $\propto \Phi'$ does not vanish at the Poincaré horizon, an argument similar to §7.4 of [8] indicates that there will be a value of z beyond which backreaction cannot be ignored, no matter how small the Newton's constant.

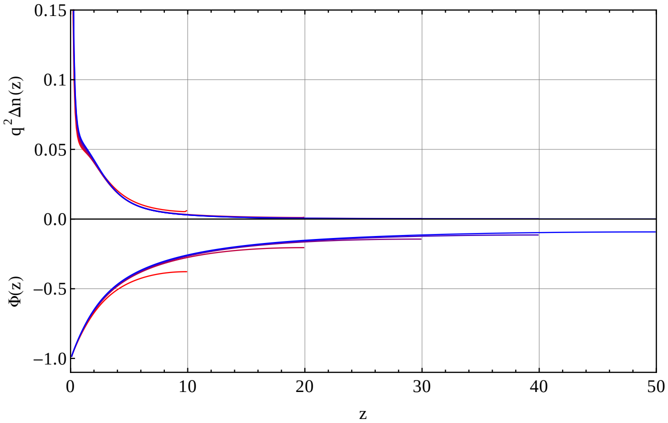

FIG. 1. This plot demonstrates the existence of the limit $z_m \to \infty$ with, from red (light) to blue (dark), $z_m =$ 10, 20, 30, 40, 50. The associated values of the boundary charge density are $\rho = 0.1710$, 0.1669, 0.1638, 0.1627, 0.1622. $m = 0.3$; $q = 2.0$; $dz = 0.1$; $\Lambda_k = 20$.

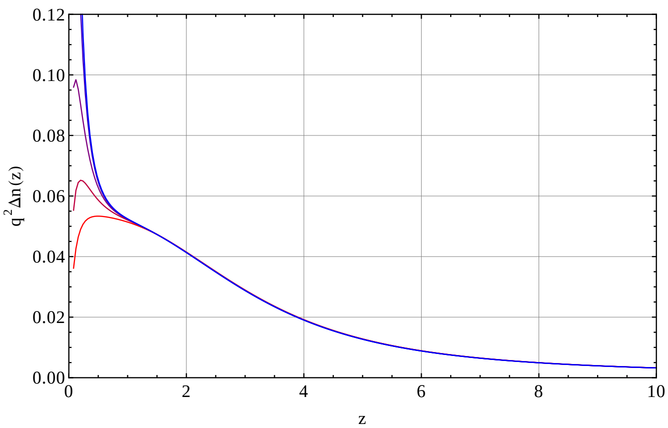

FIG. 2. This plot displays how the limit $\Lambda_k \to \infty$ is approached, with, from red (light) to blue (dark), $\Lambda_k =$ 2, 3, 5, 10, 20. See the appendix for further discussion of the surface charge density. Corresponding values of the boundary charge density are $\rho = 0.1721$, 0.1717, 0.1714, 0.1710, 0.1708 (note that here $z_m = 20$, so these numbers should be compared with the first value in fig. 1). Positive k represent the spectrum of the $m = 0.3$; $q = 2.0$; $z_m = 20$; $dz = 0.04$.

For $p^2 < 0$ the wavefunction is exponentially decaying, whereas for $p^2 > 0$, $\Psi(z) \sim e^{ipz}$. Let us call the region $p^2 > 0$ the infrared (IR) lightcone. As we increase z_m, the gap between the bands inside IR light cone decreases like $1/z_m$. Hence, for $z_m \to \infty$, a continuum develops inside the IR light cone (fig. 3). The contribution to the number density coming from each state within the IR light cone also decreases like $1/z_m$, and is constant in z, for large z. Therefore, any finite portion of the lightcone that lies below $\omega = 0$ gives a finite, z-independent contribution to the number density.

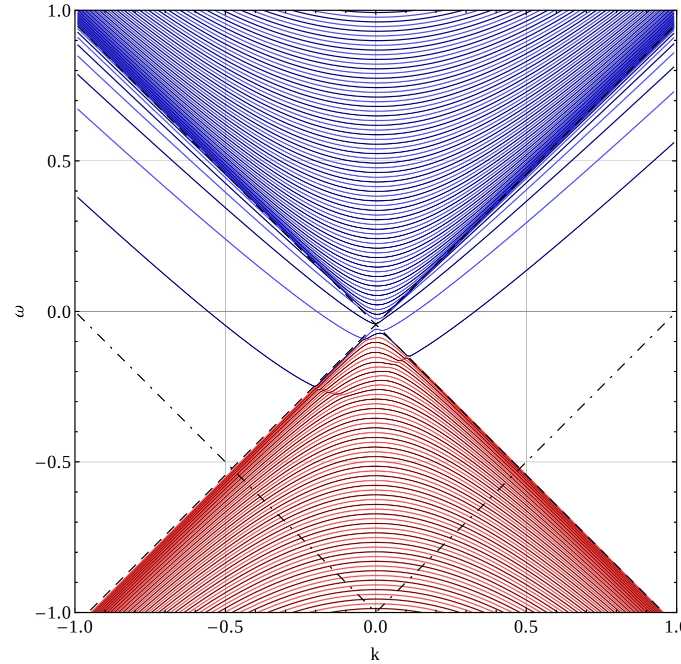

FIG. 3. The portion of the spectrum of the Dirac operator nearest to the chemical potential. For $k > 0$ we display $\omega_{n,k,+}$, for $k < 0$ we display $\omega_{n,k,-}$. Also shown are the IR lighcone (dashed lines) and the UV lightcone (dash-dotted lines). It is clear from the figure that a continuum is developing inside the IR lightcone. $m = 0.3$; $q = 2.0$; $z_m = 200$; $dz = 0.2$; $\Lambda_k = 20$.

If $\Phi(\infty) < 0$, after the subtraction of (10), the lower half of the light cone does not contribute any net number density, but the portion of the upper half of the light cone that lies below $\omega = 0$ gives, at large z, a finite, z-independent contribution to the number density. This is incompatible with the potential going to a constant as $z \to \infty$, due to Gauss law. A similar argument obtains for the case $\Phi(\infty) > 0$. The only possibility is for the potential to go to zero, so that the states within the IR light cone do not contribute at all to the number density. Then, the only contribution comes from the few bands that lie outside the IR light cone. The corresponding wavefunctions are exponentially decaying, they don't contribute to the number density at large z, and hence they are compatible with the potential approaching a constant (zero) as $z \to \infty$.

Determining numerically the exact nature of the falloff of the number density at large z is very difficult, but it is quite manifestly subexponential. A possible explanation for this is that the bands that lie outside of the IR light cone, for a certain range of k, skirt the edge of the cone (follow the dashed line), as can be seen in fig. 3. When the state is close to the edge of the cone, the rate of decay is very weak. Since the distance from the edge is a decreasing function of z_m, it is conceivable to obtain a subexponential decay of the number density as $z_m \to \infty$.

From the holographic point of view, the IR region of the geometry is dual to a relativistic CFT, which only has spectral weight inside the lightcone $|\omega| < ck$. One interesting quantity that can be extracted from our computation is the boundary charge density ρ. This is the response to the chemical potential, and it is given by

$$\rho = \Phi'(0) = q^2 \int \mathrm{d}z \; \Delta n(z) \,, \qquad (13)$$

where the second equality is a consequence of Gauss law. The charge density in the boundary is equal to the total charge in the bulk. Since μ is the only scale in the problem, its dependence on μ is determined by dimensional analysis to be $\rho = A\mu^2$, for some constant $A(q, m)$.

Let us also point out that (13) guarantees Luttinger's theorem in the boundary [17]. Luttinger's theorem states that, for interacting fermions, the area of the Fermi surface is proportional to the number density, with the same factor as for free fermions. As will be discussed in more detail in the next section, there is a Fermi surface wherever one of the bands in fig. 3 crosses $\omega = 0$. According to our construction of $\Delta n(z)$, each k mode within the Fermi surface contributes 1 to integral on the RHS of (13), thereby ensuring Luttinger's theorem.

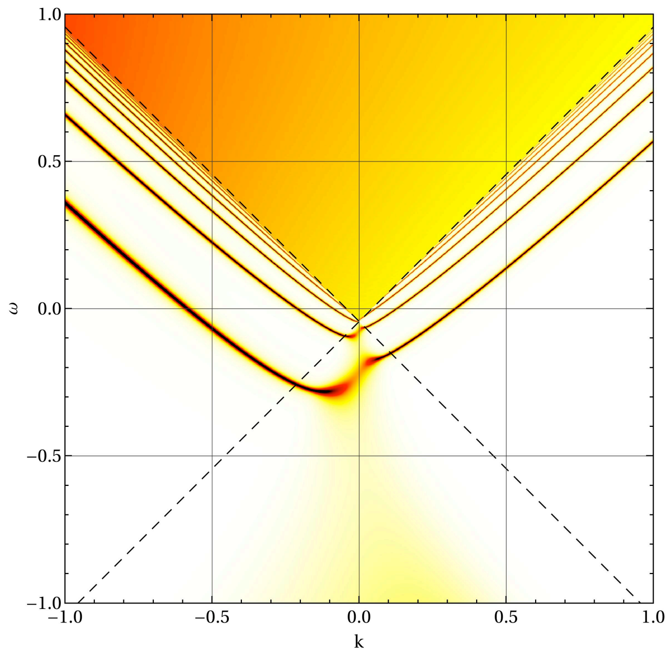

FIG. 4. Density plot of the spectral density $\mathrm{Im}\, G_R(\omega, k)$, displaying the spectra of the stable quasiparticles, which coincide with the bands in fig. 3, and the continuum inside the IR light cone. Notice the increased width of the quasiparticle peaks as they enter the light cone. $m = 0.3$; $q = 2.0$; $z_m = 200$; $dz = 0.2$; $\Lambda_k = 20$.

GREEN'S FUNCTIONS

We compute the photoemission response of our state, proportional to the single-fermion spectral density $\mathrm{Im}\, G_R(\omega, k)$, where G_R is the retarded single-fermion Green's function. To compute the retarded function, we impose in-falling boundary condition for the Dirac field at the Poincaré horizon, and include a source at the UV boundary in (9), $a \neq 0$. Then $G_R(\omega, k) = b/a$. See fig. 4.

The lifetimes of quasiparticles in holographic Fermi surfaces can be understood in terms of interactions with other gapless degrees of freedom [4, 6, 7, 25]. The degrees of freedom into which a Fermi surface quasiparticle might decay are those inside the IR lightcone described above.

Outside of the IR light cone, for $\omega = \omega_{n,k}$, there exists a solution to the Dirac equation that is real, decays exponentially at large z and is normalizable in the UV. This means that the infalling boundary condition is trivially satisfied, because the wavefunction is zero at the horizon, and we have a finite response $b \neq 0$ for zero a. As a consequence, $G_R(\omega, k)$ has a pole at $\omega = \omega_{n,k}$, and hence there is a delta function singularity in $\mathrm{Im}\, G_R$. This delta function implies the existence of an exactly stable quasiparticle in the boundary theory. In particular, the points where $\omega_{n,k} = 0$ are Fermi surfaces with stable excitations. To detect such infinitely-narrow resonances in the numerics, we add a small imaginary part to the frequency, so as to move the pole away from the $\mathrm{Re}(\omega)$ axis, and convert the delta function to a narrow Lorentzian.

On the other hand, inside the IR lightcone, the asymptotic behavior of the wavefunction is $\Psi \sim e^{\pm ipz}$, and hence, in general, a finite and complex $G_R(\omega, k)$ is needed to satisfy infalling boundary conditions. Consequently, quasiparticle excitations have finite width, because they can decay into the gapless CFT excitations. This phenomenon is visible in fig. 4 where the bound state bands enter the IR lightcone.

DISCUSSION

The state we have constructed can be described semi-holographically [4, 7] as arising from a Fermi liquid coupled to a relativistic CFT. The study of Fermi surfaces coupled to critical systems has a long history, *e.g.* [26–33]. The conclusion of our holographic calculation is that the coupling between these sectors described here is an irrelevant deformation of the Landau theory. In fact, according to our discussion about the stability of the quasiparticles, and the location of the IR light cone, the only possible singularities at $\omega = 0$, $k \neq 0$ are delta function peaks, which indicate exactly stable quasiparticles. This fact is very likely a consequence of the probe limit

$G_N \to 0$.

The nature of the coupling between the FS and the CFT that one infers for the semi-holographic picture is a hybridization between a fermionic operator of the CFT and the electron operator, as in [4, 7]. Possibly-relevant couplings between the fermion density and relevant bosonic operators of the CFT of the kind considered in [34] are suppressed in our large-N limit.

It will be very interesting to study the effect of the screening by the fermions on the boundary gauge theory dynamics. On general grounds we expect that, even beginning with a confining solution at $\mu = 0$, when the fermion density becomes large enough, the gauge theory will deconfine. Holographically, this requires taking into account the gravitational backreaction of the bulk fermions. Progress in this direction will be reported elsewhere. Resolving the problem confronted in this paper – the question of the state of the bulk fermions in the presence of a horizon – was an essential prior step.

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