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Vector boson mass generation without new fields

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Previously a model of only vector fields with a local $\mathrm{U}(1)\otimes\mathrm{SU}(2)$ symmetry was introduced for which one finds a massless U(1) photon and a massive SU(2) vector boson in the lattice regularization. Here it is shown that quantization of its classical continuum action leads to perturbative renormalization difficulties. But, non-perturbative Monte Carlo calculations favor the existence of a quantum continuum limit.

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Electromagnetic currents plus charged and neutral weak currents with the corresponding vector bosons $(\gamma, W^{\pm}, Z^0)$, are needed for a theory that accommodates simultaneously weak parity violation and electromagnetic parity conservation [1]. Explicit breaking of gauge symmetry through massive vector bosons can be avoided by the Higgs mechanism [2], which leads to desired features such as perturbative renormalizability [3].

Nevertheless, the introduction of the Higgs particle into a theory in which all matter fields are fermions with interactions mediated by vector bosons remains quite adhoc and the quadratic divergence of its self-energy causes a fine tuning problem [4]. This provides an opening for proposing new physics solutions of which supersymmetry and string theory are most popular. Though new physics should emerge on the way to the Planck scale, there appear no strong reasons to expect experimental signals for it on the LHC energy scale. Occam's razor suggests to stay with fermions and vector bosons.

Notably, the original arguments from the 1960s and early 1970s rely on perturbation theory. Our non-perturbative understanding of quantum field theory (QFT) developed later. A milestone was Wilson's [5] formulation of lattice gauge theory (LGT) in 1974 and LGT Monte Carlo (MC) calculations started in earnest during the early 1980s after pioneering papers by Creutz and others [6]. In recent years that the author suggested to address the problem of vector boson mass generation from scratch within the non-perturbative LGT framework [7]. This appears to be of theoretical interest, independently of the question whether such a scenario is eventually realized by nature or not.

Using Wilson's regularization, $V_\mu = \exp(ig_b a B_\mu)$, $B_\mu = \vec{\tau}\cdot\vec{b}_\mu/2$, where τ_i, $i=1,2,3$ are the Pauli matrices, the SU(2) lattice action reads

$$S_2 = \frac{\beta_b}{2}\sum_p \mathrm{Tr}\, V_p$$

where the sum is over all plaquettes and V_p are oriented products of SU(2) matrices around the plaquette loop. E.g., for a plaquette in the $\mu\nu$, $\mu\neq\nu$ plane V_p becomes (a lattice spacing, $x=na$ with n an integer 4-vector):

$$V_{\mu\nu}(x) \;=\; \mathrm{Tr}\,\left[V_\mu(x)\,V_\nu(x+\hat{\mu}a)\,V_\mu^\dagger(x+\hat{\nu}a)\,V_\nu^\dagger(x)\right]\,.$$

Due to non-perturbative effects there are no massless particles in the spectrum of SU(2) LGT. The self interaction of the gauge fields creates a spectrum of massive glueballs and one of them may be used to set the mass scale. Coupled fermions are confined, while the leptons are found as free particles. So, we need SU(2) LGT in a deconfined phase. This can be achieved by increasing the physical temperature [8], but in a fundamental theory we have to stay at zero temperature.

The order parameter for the deconfining phase transition is the expectation value of the Polyakov loop. Polyakov loops are traces of products of SU(2) matrices along straight lines closed by periodic boundary conditions. On a finite lattice one finds from confined to deconfined a transition between a single peak and a double peak distribution. A coupling of parallel SU(2) matrices,

$$\mathrm{Re}\,\mathrm{Tr}\,\left[V_\nu(x+\hat{\mu}a)\,V_\nu^\dagger(x)\right]\,,$$

is well suited to align Polyakov loops, but breaks SU(2) gauge invariance. To rescue local $\mathrm{U}(1)\otimes\mathrm{SU}(2)$ invariance of matter fields a concept of extended gauge invariance was introduced [7], which requires to introduce additional vector fields.

Defining the U(1) field as 2×2 matrix $U_\mu = \exp(ig_a a A_\mu)$, $A_\mu = \tau_0\, a_\mu/2$ (τ_0 unit matrix), we consider in the forthcoming the vector field lattice action

$$S = \frac{\beta_a}{2}\sum_p \mathrm{Re}\,\mathrm{Tr}\,U_p + \frac{\beta_b}{2}\sum_p \mathrm{Tr}\,V_p + \frac{\lambda}{2}\sum_{\mu\nu} S^{\mathrm{add}}_{\mu\nu}\,, \quad (1)$$

where the third sum includes identical $\mu=\nu$ indices and

$$S^{\mathrm{add}}_{\mu\nu} \;= \mathrm{Re}\,\mathrm{Tr}\,\left[U_\mu(x)\,V_\nu(x+\hat{\mu}a)\,U_\mu^\dagger(x+\hat{\nu}a)\,V_\nu^\dagger(x)\right]\,.$$

The relations $\beta_a = 1/g_a^2$ and $\beta_b = 4/g_b^2$ define β_a and β_b of (1) through the bare couplings constants and λ is a new free parameter. Properties as function of λ have been investigated by MC calculations [7]. After fixing β_a in the Coulomb phase of U(1) LGT and β_b in the scaling region of confined SU(2) LGT, one finds for small λ the same results as for $\lambda = 0$: A massless U(1) photon and confined SU(2) gauge theory with a glueball spectrum. Increasing λ, a strong first order phase transition takes place. The U(1) photon survives the transition massless, while SU(2) is then in a deconfined phase with a massive vector boson triplet. Central questions are then about 1. Perturbative Renormalizability and 2. Existence of a Quantum Continuum Limit. Both remained beyond the scope of [7]. New insights are reported here after discussing a novel way to ensure local $\mathrm{U}(1)\otimes\mathrm{SU}(2)$ invariance.

Let us consider $\mathrm{U}(1)\otimes\mathrm{SU}(2)$ field configurations $\{U'_\mu(na)\}$ for which the SU(2) part is a gauge transformation of the zero field $B_\mu(na)\equiv 0$ and $\{V'_\mu(na)\}$ for which the U(1) part is a gauge transformation of the zero field $A_\mu(na)\equiv 0$. With $G\in\mathrm{U}(1)\otimes\mathrm{SU}(2)$ each set is mapped onto itself by

$$U'_\mu(na) \;\to\; G(na)U'_\mu(na)G^{-1}(na+\hat\mu a)\,, \tag{2}$$
$$V'_\mu(na) \;\to\; G(na)V'_\mu(na)G^{-1}(na+\hat\mu a)\,, \tag{3}$$

which we call *extended gauge transformations.* Replacing U_μ and V_μ by their primed versions, the action (1) is invariant under these transformations. $S^{\rm add}_{\mu\nu}$ is the simplest example of Wilson loops that mix U'_μ and V'_μ matrices, which are now invariant operators.

The partition function of the model is

$$Z \;=\; \int \prod_n \prod_{\mu=1}^{4} dU'_\mu(na)\,dV'_\mu(na)\,e^S \tag{4}$$

where the integrations are over $\{U'_\mu(na)\}$ and $\{V'_\mu(na)\}$ defined above. They are easily implemented in a MC calculation. Let us use the notation proper parts for the U(1) factor of the U'_μ and the SU(2) factor of the V'_μ matrices and the notation gauge parts for the SU(2) factor of the U'_μ and the U(1) factor of the V'_μ matrices. The proper parts can be updated in the usual way. Specifically, a biased Metropolis-heatbath algorithm [9] was used in the simulations. Updates of the gauge parts transform all matrices emerging at a site n according to

$$U'_\mu(na)\to G_2(na)\,U'_\mu(na)\,,V'_\mu(n)\to G_1(na)\,V'_\mu(na)\,, \tag{5}$$

and all matrices on links ending at n according to

$$U_\mu(na-\hat\mu a) \;\to\; U_\mu(na-\hat\mu a)\,G_2^{-1}(na)\,,$$
$$V_\mu(na-\hat\mu a) \;\to\; V_\mu(na-\hat\mu a)\,G_1^{-1}(na)\,, \tag{6}$$

where G_2 and G_1 are, respectively, SU(2) and U(1) matrices drawn with the group measure. These updates change [10] the action (1), so that a Metropolis algorithm will have an acceptance rate in the range $(0,1]$.

Changes of the action under (5) can be undone by appropriate updates of the proper parts of the matrices. Therefore, we can calculate operators that are invariant under extended gauge transformations in any fixed gauge, i.e., omitting updates of the form (5). For MC calculations it is convenient to assign zero fields to the gauge parts, which we call *proper gauge.* The purpose of extended gauge invariance is to allow an initial Lagrangian with local $\mathrm{U}(1)\otimes\mathrm{SU}(2)$ invariance of matter fields without explicit breaking by a vector boson mass term. Including matter fields our Lagrangian in the classical continuum limit is (using Euclidean notation)

$$L = \overline{\psi}\left(i\gamma_\mu D^a_\mu - m\right)\psi + \overline{\psi}\left(i\gamma_\mu D^b_\mu - m\right)\psi\,, \tag{7}$$
$$-\frac{1}{2}\mathrm{Tr}\left(F^a_{\mu\nu}F^a_{\mu\nu}\right) - \frac{1}{2}\left(F^b_{\mu\nu}F^b_{\mu\nu}\right) - \frac{\lambda}{4}\,\mathrm{Tr}\left(F^{\rm add}_{\mu\nu}F^{\rm add}_{\mu\nu}\right)\,.$$

Here $D^a_\mu = \partial_\mu + ig_aA'_\mu$ and $D^b_\mu = \partial_\mu + ig_bB'_\mu$ are gauge covariant derivatives and the additional field tensor is

$$F^{\rm add}_{\mu\nu} \;=\; g_b\partial_\mu B'_\nu - g_a\partial_\nu A'_\mu + i\,g_ag_b\left[A'_\mu,B'_\nu\right]\,. \tag{8}$$

The fermion field ψ is assumed to be a doublet and the Lagrangian is invariant under the local $\mathrm{U}(1)\otimes\mathrm{SU}(2)$ symmetry transformations $\psi\to G\,\psi$ with the continuum limit of extended gauge transformations for the vector fields being $A'_\mu \to GA'_\mu G^{-1} + i(\partial_\mu G)G^{-1}/g_a$, $B'_\mu \to GB'_\mu G^{-1} + i(\partial_\mu G)G^{-1}/g_b$ [7]. This is the reason for the occurrence of two ψ terms in the Lagrangian.

1. In the proper gauge, $B'_\mu\to B_\mu$, $A'_\mu\to A_\mu$, one gets

$$L^{\rm add} \;=\; -\frac{\lambda\,g_a^2}{16}\left(\partial_\mu a_\nu\right)^2 \;-\frac{\lambda\,g_b^2}{16}\left(\partial_\mu b^i_\nu\right)^2\,. \tag{9}$$

These pieces are found in U(1) and SU(2) effective Lagrangians, which are usually obtained by integrating over gauge transformations with a Gaussian weighting function. With the identifications $\xi = 8/(\lambda\,g_a^2)$ and $\xi = 8/(\lambda\,g_b^2)$, respectively, Eqn. (9.56) and (16.34) of [11]. However, (16.43) comes here without the Faddeev-Popov ghost fields. Therefore [11], the tree approximation is non-unitarity because of transitions to longitudinal modes, which require massive vector bosons while there is no explicit mass term in the Lagrangian (7), while the lattice regularization is unitary to the extent that one can prove reflection positivity. On the 1-loop level the vector boson self-energy is divergent, generating an infinite mass. These properties render the model ill-defined in conventional perturbation theory.

2. One may expect that the vector boson mass am_W found in [7] is also non-perturbatively divergent. Then, the lattice regularization would not allow for a quantum continuum limit $am_W\to 0$. Instead, am_W has to stay finite $am_W\geq am_{\min}>0$ in a smoothly connected range of couplings, eventually bounded by first order phase transitions. We investigate here the line

$$\beta_a=\lambda\,,\quad \beta_b=2\lambda\,,\quad \lambda\to\infty \tag{10}$$

for which one could envision an approach to a quantum continuum limit in analogy to the behavior of asymptotically free non-Abelian gauge theories. The result is

TABLE I: Mass estimates on $14^3 N_t$ lattices and infinite volume extrapolations according to Eq. (13).

λ	$N_{\min}$	$am_W(\lambda,14)$	$am_W(\lambda,\infty)$	Q
1.1	6	0.2659 (10)	0.2658 (10)	0.13
4.0	4	0.1245 (10)	0.1180 (16)	0.31
8.0	4	0.0905 (12)	0.0876 (15)	0.44
12.0	4	0.0740 (11)	0.0719 (14)	0.31
16.0	4	0.0675 (14)	0.0653 (13)	0.36
20.0	4	0.0597 (14)	0.0552 (16)	0.42
24.0	4	0.0523 (10)	0.0500 (15)	0.11
28.0	6	0.0519 (10)	0.0487 (18)	0.26
32.0	4	0.0480 (13)	0.0448 (18)	0.96

that our fits to the scenarios $am_W \to am_{\min} > 0$ versus $am_W \to 0$ prefer the latter.

Our mass spectrum calculations were performed on lattices of size $N^3 N_t$, $N_t \gg N$. For each value of λ we have first to extrapolate the infinite volume limit $N \to \infty$ of $am_W(\lambda, N)$, denoted by $am_W(\lambda) = am_W(\lambda, \infty)$. Subsequently, we fit $am_W(\lambda)$ so that a $\lambda \to \infty$ extrapolation along the line defined by (10) can be performed.

Our masses are deduced from correlation functions $c(t)$ of suitable trial operators by performing the usual two parameter cosh fits

$$c(t) \;=\; a_1 \left[\exp(-am_W t) + \exp(-am_W(N_t - t))\right] \quad (11)$$

for a range of integers $0 \le t_1 \le t \le t_2$. Here the trial operators

$$W_{i,\mu}(x) \;=\; -i \,\mathrm{Tr}\, \left[\tau_i \, W_\mu(x)\right] \,, \qquad (12)$$

are employed, where in slight deviation from [7] a U(1) phase is included, $W_\mu(x) = U_\mu^\dagger(x)\, V_\mu(x)$ (no summation over μ). As the previously used operator, it becomes gauge invariant in combination with (static) fermion or boson fields, compare (6.20) of [12].

In addition correlations between trial operators for the U(1) photon and SU(2) glueball masses in the plaquette representations of the cubic group were calculated. Estimates of the U(1) photon mass are for all λ consistent with zero, while there are no convincing signals in the glueball channels. This is similar to the results reported in [7] for one choice of coupling constant values.

Simulations were carried out on lattices of size $N = 4, 6, 8, 10, 12, 14$ with N_t in the range [48,96] and λ values as given in table I. For each λ the 3-parameter fit

$$am_W(\lambda, N) \;=\; am_W(\lambda,\infty) + a_2(\lambda)\, \exp(-a_3(\lambda) N) \quad (13)$$

was performed to derive an infinite volume estimate $am_W(\lambda, \infty)$. These fits are shown in Fig. 1. The extrapolations are collected in table I together with the goodness Q of each fit and the estimates on our largest

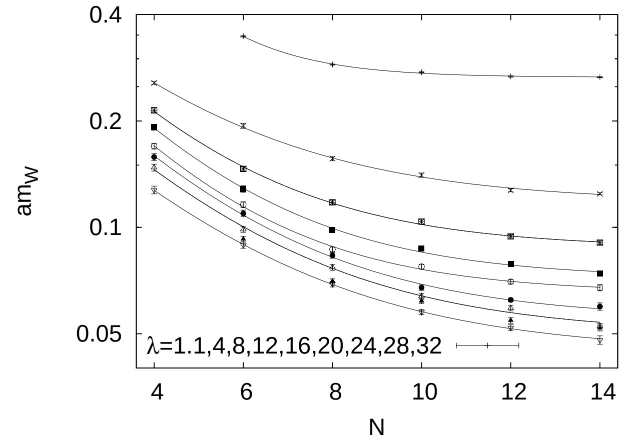

FIG. 1: Fits of $am_W(\lambda, N)$. The λ values correspond in up to down order to the curves.

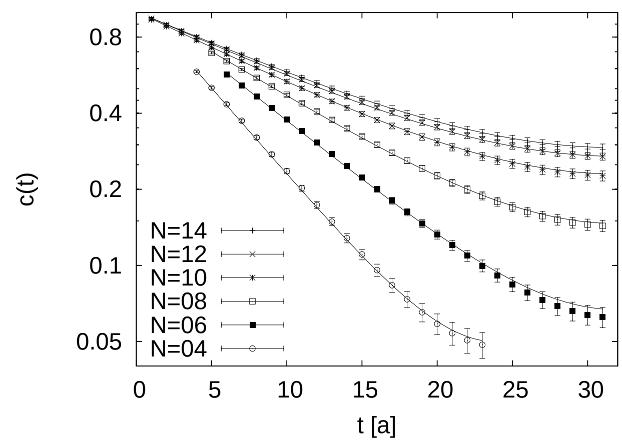

FIG. 2: Correlation functions for the $\lambda = 20$ mass estimates. The N values correspond in up to down order to the curves.

$14^3 N_t$ lattices. Error bars are given in parenthesis and refer to the last digits in the number before. In two cases data from the smallest $4^3 N_t$ lattice were omitted from the fit for consistency reasons. We indicate with $N_{\min}$ the size of the smallest lattice included in the fit.

To give an example of the numerical quality of the correlation functions (11) we depict them in Fig. 2 for our lattices at $\lambda = 20$. In stark contrast to the noise one encounters for glueball correlations in pure lattice gauge theories, these are beautiful strong correlations. One can easily follow them over more than 30 lattice spacing, though the estimates for our $14^3 N_t$ lattices are already rather time consuming. Relying on a statistics of 500,000 sweeps, they run with the present single processor code one week on an Intel i7 CPU.

We are now prepared to discuss the $\lambda \to \infty$ behavior of the $am_W(\lambda) = am_W(\lambda, \infty)$ values of table I. A num-

TABLE II: Fits of the $am_W(\lambda) = am_W(\lambda, \infty)$ values of table I. The first column gives the number of parameters, the last column the goodness of the fit.

par #	function	$am_W(\infty)$	Q
2	$f_1(\lambda) = a_1 \exp(-a_2\lambda)$	0	0
3	$f_2(\lambda) = a_0 + f_1(\lambda)$	0.05783 (66)	0
3	$f_3(\lambda) = a_1\lambda^{-a_2}\exp(-a_3\lambda)$	0	1.2×10^{-6}
4	$f_4(\lambda) = a_0 + f_3(\lambda)$	0.03 (18)	1.1×10^{-4}
4	$f_5(\lambda) = f_3(\lambda)\,(1 + a_4/\lambda)$	0	0.26
5	$f_6(\lambda) = a_0 + f_4(\lambda)$	0.02 (11)	0.20
3	$f_7(\lambda) = a_1\lambda^{-a_2}(1 + a_4/\lambda)$	0	0.036
4	$f_8(\lambda) = a_0 + f_7(\lambda)$	−0.1 (1.1)	0.065
2	$f_9(\lambda) = a_1\lambda^{-a_2}$	0	4.7×10^{-15}
10	$f_{10}(\lambda) = a_0 + f_9(\lambda)$	0.0241 (26)	2.0×10^{-4}

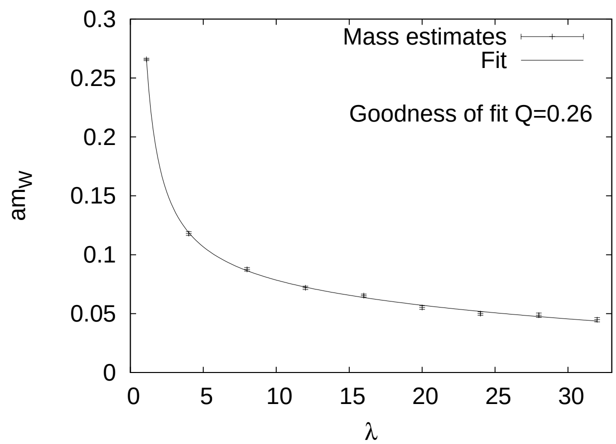

FIG. 3: Best fit of $am_W(\lambda)$.

ber of fits, either enforcing $am_W(\lambda) \to 0$ for $\lambda \to \infty$ or allowing for a free parameter $am_W(\infty)$ were tried and are compiled in table II. The fit forms $f_1(\lambda)$ to $f_4(\lambda)$ are in disagreement with the data as signaled by very small Q values in the last column. A zero means that Q is so small that the precise value cannot be tracked within our rounding errors of about 10^{-20}. Most convincing is our 4-parameter fit

$$a\,m_W(\lambda) \;=\; f_5(\lambda) \;=\; a_1\,\lambda^{a_2}\,\exp(a_3\lambda)\,(1 + a_4/\lambda) \quad (14)$$

with a well acceptable goodness of fit $Q = 0.26$. The values of its parameters are $a_1 = 0.111\,(47)$, $a_2 = -0.16\,(18)$, $a_3 = -0.0139\,(75)$, $a_4 = 1.6\begin{pmatrix}+2.1\\-0.6\end{pmatrix}$. When adding $am_W(\infty)$ as 5th parameter a_0, the function $f_6(\lambda)$ is obtained for which the goodness of fit goes slightly down, indicating that we are overfitting, and the estimated $a_0 = am_W(\infty)$ includes zero.

The functional form (14) has similarities with the asymptotic freedom behavior in LGT. As the estimated value of the factor a_3 in the exponent is small, one wonders whether a power law alone suffices to describe $m_W(\lambda)$. The functional forms $f_7(\lambda)$ to $f_{10}(\lambda)$ test this. While the last two of them are bad, this is less obvious for $f_7(\lambda)$ and $f_8(\lambda)$. Recall, under the assumption that the form of a fit is correct, Q is the probability for the discrepancy between the fit and the data.

The presented MC calculations indicate divergence of the correlation length $\xi/a \to \infty$, $\xi = m_W^{-1}$ for $\lambda \to \infty$, contradicting perturbation theory and supporting a quantum continuum limit. Of course, we cannot exclude that for larger systems and coupling constants the behavior may turn around and support $m_W(\lambda) \to m_{\min} > 0$. While this is correct, the perturbative approach is in essence vulnerable to similar criticism. Though it served us well, there is no proof that perturbation theory describes the true nature of a QFT.

Besides moving on to new horizons, we should perhaps keep an open mind for the third logical possibility that a deeper understanding of conventional QFT could unveil new models with massive vector bosons. The unexpected numerical results of this paper provide no answers, but indicate that it is worthwhile to continue this line of work.

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